

A Tax Reform Analysis of the Laffer Argument

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Abstract

This paper shows that tax reform techniques are well-suited to an examination of the Laffer argument, i.e., the possibility that an increase in a tax rate may reduce tax revenues (and vice versa). Our methodology allows us to examine the Laffer argument directly, without deriving the Laffer curve, which in turn allows us to conduct the analysis in a very general setting. Despite the high level of generality, we are able to reach some clear conclusions that provide formal support for the established intuitions that the Laffer effect requires: (i) a ‘high’ labour-income tax rate, and (ii) a ‘large’ labour supply response to wage changes. The notions of ‘high’ and ‘large’ are made precise in our framework. The analysis also provides indirect support for the intuition that it is never optimal for a government to operate on the downward-sloping segment of the Laffer curve. Finally, we show that our methods provide a theoretical framework for an empirical investigation.

Keywords: Laffer argument, tax reform.

JEL classifications: H2, H6.

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1 Introduction

The possibility that an increase in a tax rate may actually decrease total tax receipts (and vice versa) has featured prominently in many tax policy debates ever since Arthur Laffer famously drew his curve on a napkin in a Washington restaurant in the mid 1970s.¹ Indeed, recent data indicating expanding tax revenues in the US despite the Bush administration's (controversial) 2003 tax cuts has breathed new life into the debate.² The Laffer argument is well known and quite simple: a tax rate of zero will (obviously) yield zero tax receipts, while a tax rate of 100 percent will also yield zero tax receipts as the private sector will not generate a tax base for no return. Thus an inverted U-shaped curve is obtained in tax rate–tax revenue space, with tax revenues first increasing in the tax rate, reaching a peak, and then decreasing to reach zero at a tax rate of 100 percent. The increasing portion of the curve is known as the ‘normal’ segment, while the decreasing portion is known as the ‘prohibitive’ segment. Tax policy debates revolve around whether the economy is currently thought to be situated on the normal or prohibitive segment.

Despite the fact that the Laffer argument has received considerable attention from politicians, journalists, and analysts in the public domain, it has been subjected to relatively little formal analysis. An early study by Fullerton [1982] estimates the Laffer curve for the US economy. Hansson and Stuart [2003] provide relatively recent estimates of the peaks of Laffer curves for a sample of OECD countries.³ These and other empirical studies tend to be based on a simple analytical framework derived from some back-of-an-envelope (it might be more apt to say back-of-a-napkin!) calculations that establish a relationship between tax revenues, the labour-income tax rate, and the labour supply

¹So the story goes, see Fullerton [1982].

²See, for example, the editorial in the *Wall Street Journal*, 19 June 2005. For an opposing view, see Krugman, *New York Times*, 11 July 2005.

³Some other recent studies have, in effect, focused on the Laffer argument indirectly by estimating ‘taxable income elasticities’, i.e., how reported levels of taxable income respond to tax rate changes. These estimates not only capture, for example, the labour supply response, but also capture possible increased tax evasion and avoidance in response to tax rate hikes. See Gruber and Saez [2002], Carroll and Hrungr [2005], and Kopczuk [2005]. The estimated taxable income elasticity can then be used to examine the Laffer argument.

elasticity. Put simply, tax revenues can be written as:

$$T = \phi w L(\varpi) \tag{1.1}$$

where T is tax revenues, ϕ is the labour-income tax rate, w is the producer price of labour, and L is labour supply as a function of the consumer wage $\varpi = (1 - \phi)w$. Differentiating (1.1) with respect to ϕ :

$$\frac{\partial T}{\partial \phi} = w [L(\varpi) - \phi w L'(\varpi)] \tag{1.2}$$

The Laffer effect occurs when $\partial T / \partial \phi < 0$, which from (1.2) is when $\phi w L'(\varpi) > L(\varpi)$. Dividing both sides of this inequality by $\phi L(\varpi)$ yields:

$$w \frac{L'(\varpi)}{L(\varpi)} > \frac{1}{\phi} \tag{1.3}$$

Multiplying both sides of (1.3) by $(1 - \phi)$ and using the fact that $\varpi = (1 - \phi)w$ we obtain:

$$\varpi \frac{L'(\varpi)}{L(\varpi)} > \frac{1 - \phi}{\phi} \tag{1.4}$$

where the left-hand side of (1.4) is the labour supply elasticity. Based on something like the simple analysis above, the canonical thinking is that the Laffer effect requires a high labour-income tax rate and/or a high labour supply elasticity, relative to observed labour-income tax rates and generally accepted estimates of the labour supply elasticity. For example, if the labour-income tax rate is 30 percent, based on (1.4) the Laffer effect would require a labour supply elasticity of at least $2\frac{1}{3}$. Empirical estimates of labour supply elasticities are typically no greater than 0.5.⁴

In order to estimate empirical Laffer curves, some restrictive assumptions regarding preferences, technologies, and the nature of the economy must, of course, be made. In a theoretical study, Malcomson [1986] uses a simple general equilibrium model with three goods (one consumption good, one public good, and labour) and identical consumers.

⁴See Hansson and Stuart [2003].

Assuming well-behaved functional forms (but not necessarily the specific forms used in empirical studies), Malcomson [1986] shows that the Laffer curve may not be continuous and may not reach an interior maximum. The curve can be increasing in the labour-income tax rate until reaching a discontinuity at 100 percent, where tax receipts fall to zero. In this case, there is no prohibitive segment of the Laffer curve and there is no Laffer effect, except when the tax rate is increased to 100 percent. In a follow-up paper, Gahvari [1989] shows that the discontinuity identified by Malcomson [1986] disappears if government expenditures take the form of cash transfers to consumers, rather than being used to provide a public good.

Guesnerie and Jerison [1991] generalise the theoretical analysis further still by using a low-dimensional version (one consumption good, one public good, and labour) of the classic Diamond and Mirrlees [1971] general equilibrium tax model. Unlike Malcomson [1986] and Gahvari [1989], the model has heterogeneous consumers, although the public good is assumed to be separable from the consumption good and labour in each consumer's utility function. Guesnerie and Jerison [1991] show that the Laffer curve may have multiple local maxima, and in some cases it may never slope downwards.⁵ They also note that it is not clear if their results can be extended to a model with many commodities, which gives some significance to their use of a simplified version of the Diamond-Mirrlees model.

In this paper, we show that an unrestricted version of the Diamond-Mirrlees model can be used by undertaking a tax reform style analysis of the Laffer argument. Tax reform analysis takes the existing tax system and its (possible) imperfections as its starting point, and examines the conditions under which there exist *small* changes in taxes that are equilibrium preserving and Pareto improving.⁶ We take a similar approach to the examination of the Laffer argument. Starting in an arbitrary tax equilibrium, we characterise the conditions under which a small increase in the labour-income tax

⁵Guesnerie and Jerison [1991] also address the normative question of how relevant the Laffer argument is for social choice amongst tax equilibria.

⁶Tax reform therefore differs from optimal tax analysis, which pays no attention to the existing tax system and implicitly assumes that the government is free to implement large changes in taxes to obtain an optimum. The tax reform approach was pioneered by Guesnerie [1977], and developed further by Diewert [1978] and Weymark [1979].

rate necessarily results in lower tax revenues.⁷ Thus, we can directly examine the Laffer argument without deriving the Laffer curve, which requires consideration of large changes in taxes. In our analysis, if the conditions for the Laffer effect to occur are satisfied, then roughly speaking it could be said that the economy is situated on the prohibitive segment of the Laffer curve.

Despite the model's high level of generality, we are able to obtain some clear conclusions, in particular:

- If the status quo tax system is Pareto efficient, an increase in the labour-income tax rate cannot result in the Laffer effect;
- The labour-income tax rate must be 'high' in an economy that is subject to the Laffer effect;
- Labour supply must be 'very sensitive' to changes in wages in an economy that is subject to the Laffer effect.

These conclusions are consistent with back-of-the-envelope calculations and current thinking about the Laffer argument; thus we establish that such thinking passes a rigorous general equilibrium test. Moreover, we are able to:

- Give precise meaning to the notions that the labour-income tax rate must be 'high' and that labour supply must be 'very sensitive' to changes in wages;
- Characterise precisely what an economy must 'look like' if the Laffer effect is to occur, where the characterisation exercise provides a theoretical framework for an empirical investigation into the possibility of the Laffer effect in real economies.

A recent literature, e.g., Agell and Persson [2001] and Novales and Ruiz [2002], has used endogenous growth models to examine if the Laffer effect is more likely to occur in the longer run. Our aims are in the same spirit, in the sense that we are interested in how the Laffer argument is affected by changes in models and modelling techniques.

⁷In the concluding section we discuss some problems that arise if an attempt is made to characterise when a decrease in the labour-income tax rate increases tax revenues.

Our analysis is also related to a recent literature that uses tax reform techniques to revisit fundamental questions in the theory of taxation. For example, Fleurbaey [2006] uses tax reform techniques to re-examine the desirability of consumption versus income taxation. Tax reform techniques have also been used by Murty and Russell [2005] to analyse externalities, and by Krause [2007] to analyse the incidence of capital taxation.

The remainder of the paper is organised as follows. Section 2 describes the model, while Section 3 discusses the tax reform methodology we employ and presents the results. Section 4 presents a numerical example of an economy that is subject to the Laffer effect, which illustrates how one would apply our methodology to test the Laffer argument in real economies. Section 5 contains some concluding comments, including a discussion of some weaknesses with our approach. Proofs and many of the mathematical details are relegated to an appendix.

2 The Model

The economy has k consumers, indexed by $i = 1, \dots, k$. Consumer i chooses his (net of endowment) consumption vector $x_i \in \mathbb{R}^n$, and his labour supply $l_i \in [0, 1]$,⁸ to solve the following programme:

$$V_i(q, \omega, g) = \max_{x_i, l_i} \{U_i(x_i, l_i, g) \mid qx_i \leq \omega l_i\} \quad (2.1)$$

where $V_i(\cdot)$ is the indirect utility function, $U_i(\cdot)$ is the direct utility function with $\nabla_{x_i} U_i(\cdot) \gg 0^{(n)}$, $\nabla_{l_i} U_i(\cdot) < 0$, and $\nabla_g U_i(\cdot) > 0$ where g is a public good provided by the government.⁹ The consumer price vector corresponding to the commodities is $q = p + t$, where p is the producer price vector corresponding to the commodities and t is a vector of specific commodity taxes. The consumer wage is $\omega = w - \tau$, where w is the producer price of labour and τ is the specific tax on labour income. The consumers have no profit income, as we make the common assumption that the government taxes

⁸We assume that each consumer is endowed with one unit of time. Time not used to supply labour is consumed as leisure.

⁹Vector notation: $z \geq \bar{z} \iff z_j \geq \bar{z}_j \forall j$, $z > \bar{z} \iff z_j \geq \bar{z}_j \forall j \wedge z \neq \bar{z}$, $z \gg \bar{z} \iff z_j > \bar{z}_j \forall j$.

away all pure profit.¹⁰ Under standard assumptions regarding preferences (namely, local non-satiation and strict convexity), the solution to (2.1) yields each consumer's (net) commodity demand and labour supply functions:

$$x_i(q, \omega, g) \quad \text{and} \quad l_i(q, \omega, g) \quad (2.2)$$

The production of the n private commodities is undertaken by a single, aggregate, profit-maximising firm, as there are no aggregation problems on the supply side in the absence of production externalities (as is assumed). Accordingly, there is no loss in generality by assuming a single firm. The firm's closed and strictly convex technology set is denoted by $Y \subset \mathbb{R}^{n+1}$. The firm's profit maximisation problem can be stated as:

$$\pi(p, w) = \max_{x, l} \{px - wl \mid \langle x, l \rangle \in Y\} \quad (2.3)$$

The firm's profit function is $\pi(\cdot)$. Application of Hotelling's Theorem to the profit function yields the firm's output-supply and input-demand functions:

$$\nabla_p \pi(\cdot) = x(p, w) \quad \text{and} \quad \nabla_w \pi(\cdot) = -l(p, w) \quad (2.4)$$

where x is the (net) supply vector of private commodities, and l is the firm's demand for labour.

The government uses commodities and labour to produce the public good according to the following technology:

$$g \leq f(x_g, l_g) \quad (2.5)$$

where $f(\cdot)$ is strictly concave and increasing in all its arguments, and x_g and l_g denote the employment of commodities and labour to produce the public good.

¹⁰Guesnerie and Jerison [1991] also make this assumption. Alternatively, one could assume that the production side of the economy is characterized by constant returns to scale, which implies zero profits in equilibrium.

Equilibrium is obtained if and only if:

$$\sum x_i(q, \omega, g) + x_g - x(p, w) \leq 0^{(n)} \quad (2.6)$$

$$l(p, w) + l_g - \sum l_i(q, \omega, g) \leq 0 \quad (2.7)$$

$$g - f(x_g, l_g) \leq 0 \quad (2.8)$$

The simplicity of the model, as summarised in equations (2.6) – (2.8), is reflective of its generality. In particular, we place no restrictions on the number of commodities or the forms of the demand and supply functions (although we will assume that they are differentiable). Equations (2.6) and (2.7) are market clearing conditions for the n private commodities and labour. Equation (2.8) requires that the provision of the public good be technically feasible. It is shown in the Appendix that if all the equations in (2.6) and (2.7) are satisfied as equalities, the government’s budget is exactly balanced. But if some of these equations are satisfied as inequalities, the government’s budget will be in surplus. An equilibrium is said to be *tight* when (2.6) – (2.8) are satisfied as equalities, and *non-tight* when some of these equations are satisfied as inequalities.

3 The Laffer Argument

Consider an arbitrarily given tight equilibrium of our economy,¹¹ where the corresponding tax system may or may not be optimal in any sense of the word. We are interested in whether a small (modelled as differential) increase in the labour-income tax rate *necessarily* moves the economy to a neighbouring equilibrium which has a lower level of tax revenues (holding all other taxes constant). To this end, we define a *policy reform* as a vector $dP := \langle dp, dt, dw, d\tau, dg, dx_g, dl_g \rangle$, where the government has direct control over the taxes t and τ , as well as over the level (g) and method of production (x_g and l_g) of the public good. Changes in these instruments may induce changes in producer

¹¹We always assume that the status quo equilibrium is tight so that the system (2.6) – (2.8) can be differentiated.

prices, p and w , according to the equilibrium conditions.¹² Specifically, a policy reform is equilibrium preserving if and only if:

$$-\nabla Z dP \geq 0^{(n+2)} \quad (3.1)$$

where ∇Z is the $(n + 2) \times (3n + 4)$ Jacobian matrix (with respect to dP) associated with equations (2.6) – (2.8) and is defined explicitly as:

$$\nabla Z := \begin{bmatrix} \sum \nabla_q x_i - \nabla_p x & \sum \nabla_q x_i & \sum \nabla_\omega x_i - \nabla_w x & -\sum \nabla_\omega x_i & \sum \nabla_g x_i & \mathbf{I}^{(n \times n)} & 0^{(n)} \\ \nabla_p l - \sum \nabla_q l_i & -\sum \nabla_q l_i & \nabla_w l - \sum \nabla_\omega l_i & \sum \nabla_\omega l_i & -\sum \nabla_g l_i & 0^{(n)} & 1 \\ 0^{(n)} & 0^{(n)} & 0 & 0 & 1 & -\nabla_{x_g} f & -\nabla_{l_g} f \end{bmatrix}$$

where all derivatives are evaluated in the status quo equilibrium.

The set of policy reforms that include no changes in the commodity taxes are those that satisfy:

$$\begin{bmatrix} 0^{(n \times n)} & \mathbf{I}^{(n \times n)} & 0^{(n)} & 0^{(n)} & 0^{(n)} & 0^{(n \times n)} & 0^{(n)} \end{bmatrix} dP = 0^{(n)} \quad (3.2)$$

The average (and marginal) labour-income tax rate is τ/w . Thus, the set of policy reforms that include an increase in the labour-income tax rate are those that satisfy:

$$\begin{bmatrix} 0^{(n)} & 0^{(n)} & -1 & 1 & 0 & 0^{(n)} & 0 \end{bmatrix} dP > 0 \quad (3.3)$$

At this point it is important to mention that, following convention, we have specified taxes in the model in terms of specific taxes rather than tax rates. This is purely a standard modelling convenience.¹³ We also hold the specific commodity taxes, rather

¹²More formally, dP gives the government $3n + 4$ instruments, but it must satisfy the $n + 2$ equilibrium equations in (2.6) – (2.8). This suggests by the Implicit Function Theorem $2n + 2$ degrees of freedom in picking tight equilibria in the neighbourhood of the status quo equilibrium. There are only $2n$ degrees of freedom, however, as two degrees of freedom are lost due to the consumers' demand and supply functions being homogenous of degree zero in consumer prices, and the firm's supply and demand functions being homogenous of degree zero in producer prices. But intuitively there are more than $2n$ degrees of freedom, because we also allow the economy to move from a tight equilibrium to a non-tight equilibrium. See chapter 2 in Guesnerie [1995] for a detailed discussion of these issues.

¹³For a recent exposition and discussion of the analytical equivalence between specific taxes and tax

than commodity tax rates, constant for simplicity. We consider an increase in the labour-income tax rate, rather than in the specific tax on labour, only because discussions of labour taxation vis-à-vis the Laffer argument are almost always in terms of the labour-income tax rate. None of our results would change if we worked entirely in terms of specific taxes or tax rates, although some of the equations in Lemma 1 (below) would change slightly.

The set of policy reforms that do not include a decrease in the level of the public good are those that satisfy:

$$\begin{bmatrix} 0^{(n)} & 0^{(n)} & 0 & 0 & 1 & 0^{(n)} & 0 \end{bmatrix} dP \geq 0 \quad (3.4)$$

Suppose there does not exist a policy reform that satisfies (3.1) – (3.4). Then all policy reforms that satisfy (3.1) – (3.3) must violate (3.4). That is, all policy reforms that are equilibrium preserving, that involve no changes in the commodity taxes, and that involve an increase in the labour-income tax rate, must also involve a lower level of the public good. Given that the government is free to change the combination of inputs x_g and l_g in order to minimise the cost of producing the public good, the lower level of the public good must necessarily be the result of lower tax revenues accruing to the government. Put another way, if there does not exist a policy reform that satisfies (3.1) – (3.4), then all equilibria in the neighbourhood of the status quo equilibrium that are attainable by an increase in the labour-income tax rate necessarily involve a lower level of tax revenues than does the status quo equilibrium. Therefore, an economy in which there does not exist a policy reform that satisfies (3.1) – (3.4) can be interpreted as an economy in which an equilibrium-preserving increase in the labour-income tax rate necessarily results in lower tax revenues, i.e., the Laffer effect. Such an economy could be interpreted as being situated on the prohibitive segment of the Laffer curve, although, strictly speaking, this is not correct since we have not derived a Laffer curve.

By Motzkin’s Theorem,¹⁴ there does not exist a policy reform dP that satisfies (3.1)

rates, see Blackorby and Murty [2007].

¹⁴See the Appendix for a statement of this theorem.

– (3.4) if and only if there exist real numbers $\mu \geq 0^{(n+2)}$, $\theta \in \mathbb{R}^{(n)}$, $\alpha > 0$, and $\delta \geq 0$ such that:

$$\begin{aligned} & \theta \begin{bmatrix} 0^{(n \times n)} & \mathbf{I}^{(n \times n)} & 0^{(n)} & 0^{(n)} & 0^{(n)} & 0^{(n \times n)} & 0^{(n)} \end{bmatrix} + \\ & \alpha \begin{bmatrix} 0^{(n)} & 0^{(n)} & -1 & 1 & 0 & 0^{(n)} & 0 \end{bmatrix} + \delta \begin{bmatrix} 0^{(n)} & 0^{(n)} & 0 & 0 & 1 & 0^{(n)} & 0 \end{bmatrix} = \mu \nabla Z \end{aligned} \quad (3.5)$$

The system of equations in (3.5) characterises what the economy must ‘look like’ for it to be subject to the Laffer effect. Expanding (3.5) yields the following lemma:

Lemma 1 *Consider any tight equilibrium of our economy. An increase in the labour-income tax rate necessarily results in lower tax revenues if and only if there exist real numbers $\mu \geq 0^{(n+2)}$, $\theta \in \mathbb{R}^{(n)}$, $\alpha > 0$, and $\delta \geq 0$ such that:*

$$0^{(n)} = \langle \mu_1, \dots, \mu_n \rangle \left[\sum \nabla_q x_i - \nabla_p x \right] + \mu_{n+1} \left[\nabla_p l - \sum \nabla_q l_i \right] \quad (3.6)$$

$$\theta = \langle \mu_1, \dots, \mu_n \rangle \sum \nabla_q x_i - \mu_{n+1} \sum \nabla_q l_i \quad (3.7)$$

$$-\alpha = \langle \mu_1, \dots, \mu_n \rangle \left[\sum \nabla_\omega x_i - \nabla_w x \right] + \mu_{n+1} \left[\nabla_w l - \sum \nabla_\omega l_i \right] \quad (3.8)$$

$$\alpha = - \langle \mu_1, \dots, \mu_n \rangle \sum \nabla_\omega x_i + \mu_{n+1} \sum \nabla_\omega l_i \quad (3.9)$$

$$\delta = \langle \mu_1, \dots, \mu_n \rangle \sum \nabla_g x_i - \mu_{n+1} \sum \nabla_g l_i + \mu_{n+2} \quad (3.10)$$

$$0^{(n)} = \langle \mu_1, \dots, \mu_n \rangle - \mu_{n+2} \nabla_{x_g} f \quad (3.11)$$

$$0 = \mu_{n+1} - \mu_{n+2} \nabla_{l_g} f \quad (3.12)$$

where all derivatives are evaluated in the status quo equilibrium.

It is worth mentioning that Lemma 1 provides a theoretical foundation for an empirical investigation into the possibility of the Laffer effect in real economies. Indeed, it is often argued, e.g., by Fleurbaey [2006], that criteria such as that in Lemma 1 are of more value to policy-makers than theoretical descriptions of optimal tax systems. The information requirements of (3.6) – (3.9) are estimates of aggregate demand and supply price derivatives (or elasticities) for the commodities and labour. In principle, these can be estimated using market data and econometric techniques. Equation (3.10) requires estimates of how commodity demand and labour supply vary with the level of the public

good. A standard separability assumption on preferences, however, could be made to ensure that $\nabla_g x_i(\cdot) = 0^{(n)}$ and $\nabla_g l_i(\cdot) = 0$, which would remove the need for such estimates (although at the cost of some loss in generality).¹⁵ Equations (3.11) and (3.12) require estimates of the marginal productivities of commodities and labour in producing the public good, which in principle can also be estimated. Once such data are obtained, the task then would be to check (say with a computer) whether the Lemma 1 conditions can be satisfied. A simple numerical example of an economy that satisfies (3.6) – (3.12) is provided in the next section.

From Lemma 1 we obtain the following results (all proofs are in the Appendix):

Theorem 1 *Consider a tight equilibrium of our economy in which the tax system is Pareto efficient. In such an economy, an increase in the labour-income tax rate cannot result in the Laffer effect.*

Theorem 1 provides formal support—albeit indirectly—for the intuition that it is not optimal for a government to operate on the prohibitive segment of the Laffer curve.¹⁶ An optimal tax system (which we take to satisfy the minimum condition of Pareto optimality) is necessarily characterised by the marginal benefit of a change in each tax being equated to its marginal cost. For example, a tax decrease which reduces the consumer price of some good will boost consumption of that good and welfare. This is the benefit. The cost is that the increase in demand must be met by an increase in supply by transferring resources from other sectors of the economy. Theorem 1 implies that the tax system of an economy subject to the Laffer effect cannot be characterised by the condition that marginal benefit equals marginal cost for each tax.¹⁷ In particular, the labour-income tax rate is ‘too high’ in the following sense:

Theorem 2 *In an economy subject to the Laffer effect, the labour-income tax rate is at the highest level consistent with the status quo levels of the commodity taxes and public*

¹⁵Recall that Guesnerie and Jerison [1991] maintain such a separability assumption throughout their paper.

¹⁶Although in models of tax competition this intuition may not hold. See Hindriks [2001] and Keen and Kotsogiannis [2003].

¹⁷Of course, if the status quo tax system were optimal, there would be no reason for the government to increase the labour-income tax rate. The question of the Laffer effect is, however, a positive question, rather than a normative question.

good, and satisfaction of (2.6) – (2.8) such that the equilibrium is tight.

In other words, the economy looks *as if* the government has chosen the highest possible labour-income tax rate, subject to the equilibrium constraints, $t = \hat{t}$, and $g = \hat{g}$, where \hat{t} and \hat{g} denote the status quo levels of the commodity taxes and public good. Theorem 2 tells us that out of the entire set of tight equilibria with commodity taxes equal to \hat{t} and the public good equal to \hat{g} , the equilibrium with the highest labour-income tax rate is the one in which the Laffer effect would occur. Such an equilibrium cannot be Pareto efficient, as any other equilibrium within this set of equilibria would be Pareto superior.

Theorem 3 *In an economy subject to the Laffer effect, the labour supply response to a change in the consumer wage is large in the sense that:*

$$\sum \nabla_{\omega} l_i > \frac{\nabla_{x_g} f}{\nabla_{l_g} f} \sum \nabla_{\omega} x_i \quad \iff \quad \nabla_{l_g} f \sum \nabla_{\omega} l_i > \nabla_{x_g} f \sum \nabla_{\omega} x_i$$

where all derivatives are evaluated in the status quo equilibrium.

The ratio $\nabla_{x_g} f(\cdot)/\nabla_{l_g} f(\cdot)$ is a vector of technical rates of substitution of each commodity for labour in production of the public good. Theorem 3 states that the Laffer effect requires that the labour supply response to an increase in the consumer wage, valued at the marginal product of labour in public good production, must be greater than the commodity demand response to an increase in the consumer wage, valued at the marginal products of commodities in public good production. The marginal productivities in public good production are used to put the labour supply response and commodity demand response in comparable units.

Theorem 3 provides formal support for the popular thinking that labour supply must be ‘very sensitive’ to changes in wages in order for Laffer-like phenomena to occur. To interpret Theorem 3, consider first the effect of an increase in the labour-income tax rate on commodity demand. Suppose the aggregate value of commodity demand is increasing in the consumer wage, i.e., $\nabla_{x_g} f(\cdot) \sum \nabla_{\omega} x_i(\cdot) > 0$.¹⁸ An increase in the labour-income tax rate (which reduces the consumer wage) causes a decrease in the

¹⁸This condition can be assumed to hold in almost all economies, unless there are ‘too many’ inferior goods.

demand for commodities. All else equal, this results in excess supply of the commodities, with a corresponding shift in the government's budget from balance to surplus.¹⁹ Now consider labour supply. Standard results in consumer theory ensure that the substitution effect of an increase in the labour-income tax rate is negative, and if leisure is a normal good the income effect is positive. If the substitution effect dominates, an increase in the labour-income tax rate reduces labour supply and, all else equal, creates excess demand in the labour market, with a corresponding shift in the government's budget from balance to deficit. Therefore, put simply, the Laffer effect requires that an increase in the labour-income tax rate reduce labour supply by more than it reduces commodity demand, so that the net effect is for the government's budget to move from balance to deficit. With public good production held constant, the move from balance to deficit is explained by lower tax revenues, i.e., the Laffer effect. The government must, however, reduce public good production to correspond to the lower level of tax revenues, in order to restore budget balance.

4 A Numerical Example

In order to illustrate the empirical applicability of Lemma 1, we present a simple example of an economy in which an increase in the labour-income tax rate would result in the Laffer effect. The economy has two commodities ($n = 2$) and k consumers.²⁰ The key features of the economy are summarised in the Jacobian matrix:

$$\nabla Z = \begin{bmatrix} -2 & 0 & -1 & 0 & 2 & -1 & 0 & 1 & 0 & 0 \\ 0 & -2 & 0 & -1 & 2 & -1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & -4 & 3 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & -2 \end{bmatrix}$$

¹⁹Recall that in a tight equilibrium the government's budget is balanced, and in a non-tight equilibrium the government's budget is in surplus.

²⁰It does not matter how many consumers (and for that matter producers) there are, because the information requirements of Lemma 1 are in terms of *aggregate* demand and supply derivatives.

By Lemma 1, there does not exist a policy reform dP such that:

$$-\nabla Z dP \geq 0^{(4)}$$

$$\left[0^{(2 \times 2)} \quad \mathbf{I}^{(2 \times 2)} \quad 0^{(2)} \quad 0^{(2)} \quad 0^{(2)} \quad 0^{(2 \times 2)} \quad 0^{(2)} \right] dP = 0^{(2)}$$

$$\left[0^{(2)} \quad 0^{(2)} \quad -1 \quad 1 \quad 0 \quad 0^{(2)} \quad 0 \right] dP > 0$$

$$\left[0^{(2)} \quad 0^{(2)} \quad 0 \quad 0 \quad 1 \quad 0^{(2)} \quad 0 \right] dP \geq 0$$

if and only if there exist real numbers $\mu \geq 0^{(4)}$, $\theta \in \mathbb{R}^{(2)}$, $\alpha > 0$, and $\delta \geq 0$ such that:

$$\begin{aligned} & \theta \left[0^{(2 \times 2)} \quad \mathbf{I}^{(2 \times 2)} \quad 0^{(2)} \quad 0^{(2)} \quad 0^{(2)} \quad 0^{(2 \times 2)} \quad 0^{(2)} \right] + \\ & \alpha \left[0^{(2)} \quad 0^{(2)} \quad -1 \quad 1 \quad 0 \quad 0^{(2)} \quad 0 \right] + \delta \left[0^{(2)} \quad 0^{(2)} \quad 0 \quad 0 \quad 1 \quad 0^{(2)} \quad 0 \right] = \mu \nabla Z \end{aligned}$$

The reader can check that $\mu = \langle \frac{1}{4}, \frac{1}{4}, \frac{1}{2}, \frac{1}{4} \rangle$, $\theta = \langle -\frac{1}{4}, -\frac{1}{4} \rangle$, $\alpha = 1$, and $\delta = \frac{1}{4}$ satisfy the required conditions.

It should be noted that there is nothing especially unusual about the economy described in ∇Z . For example, the aggregate demand and supply derivatives satisfy the appropriate signs. For simplicity we have assumed that demand for each commodity is dependent upon only its own price. (This would be true, for example, if the consumers' preferences were Cobb-Douglas.) We have also implicitly assumed that commodities and labour are separable from the public good in the consumers' utility functions. This ensures that $\nabla_g x_i(\cdot) = 0^{(2)}$ and that $\nabla_g l_i(\cdot) = 0$ for all i . The only unusual feature of the economy is that $\sum \nabla_\omega l_i(\cdot) > 0$ is relatively large, as reflected in the large (absolute) values of -4 and 3 in columns five and six of ∇Z . As popular thinking and Theorem 3 suggest, a necessary condition for the Laffer effect is that labour supply be very sensitive to wage changes. We also know that the labour-income tax rate is 'too high' relative to its Pareto-efficient level (Theorem 2), but this is not observable in the data required by Lemma 1.

5 Concluding Comments

In this paper we have characterised the conditions under which an increase in the labour-income tax rate necessarily results in lower tax revenues, but we have not attempted to characterise when a *decrease* in the labour-income tax rate *increases* tax revenues. At first thought one might expect that the analysis could simply be reversed, in that it is characterised when a decrease in the labour-income tax rate requires an increase in the public good. However, this cannot be interpreted as the generation of extra tax revenues. An increase in the public good could be obtained with the same, or even lower, level of tax revenues if the status quo method of producing the public good were inefficient. For this reason, our analysis considers only when an increase in the labour-income tax rate can necessarily result in lower tax revenues.

Nevertheless, we believe our model and methodology provide important insights into the Laffer argument. At its core, the Laffer argument asks the following question: Given the current state of the economy and its tax system, will a small increase (decrease) in a tax rate yield lower (higher) tax revenues? The very nature of the tax reform approach makes it well-suited to answer this question. Moreover, we can examine the Laffer argument in a very general setting, which ensures that our results are valid for all well-behaved classes of preferences and technologies. This also makes our characterisation result (Lemma 1) directly applicable to empirical testing. Such characterisations are often argued to be of more use to governments than, say, optimal tax recommendations, as political constraints typically make it impossible for the government to implement the large tax changes required to reach an optimum. We have also obtained a number of specific results that provide formal support for the intuition that the Laffer effect requires an economy with a ‘high’ labour-income tax rate and a labour-supply response that is ‘very sensitive’ to wage changes. Finally, although we have not derived a Laffer curve, our analysis supports the intuition that it is not optimal for a government to operate on the prohibitive segment of the Laffer curve.

6 Appendix

The Government's Budget

The government's budget surplus BS can be written as:

$$BS = \sum (q - p)x_i + \sum (w - \omega)l_i + \pi(p, w) - px_g - wl_g \quad (\text{A.1})$$

where the first term represents receipts from commodity taxation, the second term is receipts from taxing labour, the third term is from the total taxation of pure profits, and the last two terms represent the government's expenditures on commodities and labour to produce the public good. Rewriting the profits term in (A.1) yields:

$$BS = \sum (q - p)x_i + \sum (w - \omega)l_i + px - wl - px_g - wl_g \quad (\text{A.2})$$

Under the assumption of local non-satiation, the consumers will satisfy their budget constraints with equality, which implies that $qx_i = \omega l_i$ for all i . Thus (A.2) reduces to:

$$BS = -p \sum x_i + w \sum l_i + px - wl - px_g - wl_g \quad (\text{A.3})$$

Rearranging (A.3) yields:

$$BS = p(x - x_g - \sum x_i) + w(\sum l_i - l_g - l) \quad (\text{A.4})$$

Market clearing requires that the terms in parentheses in (A.4) be non-negative. Thus, the government's budget is exactly balanced in a tight equilibrium, and the government's budget is in surplus in a non-tight equilibrium.

Motzkin's Theorem of the Alternative

Let A , C , and D be $n_1 \times m$, $n_2 \times m$, and $n_3 \times m$ matrices, respectively, where A is non-vacuous (not all zeros). Then *either*

$$Az \gg 0^{(n_1)} \quad Cz \geq 0^{(n_2)} \quad Dz = 0^{(n_3)}$$

has a solution $z \in \mathbb{R}^m$, or

$$y_1 A + y_2 C + y_3 D = 0^{(m)}$$

has a solution $y_1 > 0^{(n_1)}$, $y_2 \geq 0^{(n_2)}$, and y_3 sign unrestricted, *but never both*. A proof of Motzkin's Theorem can be found in Mangasarian [1969].

Proof of Theorem 1

Step 1: We first derive the equations that implicitly characterise a Pareto-efficient tax system. A policy reform increases the welfare of consumer i if and only if $dV_i(\cdot) = \nabla V_i dP > 0$, where ∇V_i is the gradient of consumer i 's indirect utility function with respect to dP , i.e., $\nabla V_i := \langle \nabla_q V_i, \nabla_q V_i, \nabla_\omega V_i, -\nabla_\omega V_i, \nabla_g V_i, 0^{(n)}, 0 \rangle$. Starting in an initial tight equilibrium, if there does not exist a policy reform that is equilibrium preserving and Pareto improving, then the status quo equilibrium is Pareto efficient. Let ∇V be the $k \times (3n + 4)$ matrix formed by the vectors ∇V_i . By Motzkin's Theorem, if there does not exist a policy reform dP such that $-\nabla Z dP \geq 0^{(n+2)}$ and $\nabla V dP \gg 0^{(k)}$, then there exist two vectors of real numbers $\lambda > 0^{(k)}$ and $\bar{\mu} \geq 0^{(n+2)}$ such that:

$$\lambda \nabla V = \bar{\mu} \nabla Z \tag{A.5}$$

The system of equations (A.5) characterises the set of Pareto-efficient equilibria. Expanding (A.5) yields:

$$\sum \lambda_i \nabla_q V_i = \langle \bar{\mu}_1, \dots, \bar{\mu}_n \rangle \left[\sum \nabla_q x_i - \nabla_p x \right] + \bar{\mu}_{n+1} \left[\nabla_p l - \sum \nabla_q l_i \right] \tag{A.6}$$

$$\sum \lambda_i \nabla_q V_i = \langle \bar{\mu}_1, \dots, \bar{\mu}_n \rangle \sum \nabla_q x_i - \bar{\mu}_{n+1} \sum \nabla_q l_i \tag{A.7}$$

$$\sum \lambda_i \nabla_\omega V_i = \langle \bar{\mu}_1, \dots, \bar{\mu}_n \rangle \left[\sum \nabla_\omega x_i - \nabla_w x \right] + \bar{\mu}_{n+1} \left[\nabla_w l - \sum \nabla_\omega l_i \right] \tag{A.8}$$

$$-\sum \lambda_i \nabla_\omega V_i = -\langle \bar{\mu}_1, \dots, \bar{\mu}_n \rangle \sum \nabla_\omega x_i + \bar{\mu}_{n+1} \sum \nabla_\omega l_i \tag{A.9}$$

$$\sum \lambda_i \nabla_g V_i = \langle \bar{\mu}_1, \dots, \bar{\mu}_n \rangle \sum \nabla_g x_i - \bar{\mu}_{n+1} \sum \nabla_g l_i + \bar{\mu}_{n+2} \tag{A.10}$$

$$0^{(n)} = \langle \bar{\mu}_1, \dots, \bar{\mu}_n \rangle - \bar{\mu}_{n+2} \nabla_{x_g} f \tag{A.11}$$

$$0 = \bar{\mu}_{n+1} - \bar{\mu}_{n+2} \nabla_{l_g} f \quad (\text{A.12})$$

where all derivatives are evaluated in the status quo equilibrium. Equations (A.6) – (A.12) can be interpreted as the first-order conditions for a Pareto optimum. Each number λ_i can be interpreted as the welfare weight of consumer i , and the vector $\langle \bar{\mu}_1, \dots, \bar{\mu}_{n+2} \rangle$ can be interpreted as the multipliers attached to the $n + 2$ equilibrium constraints in (2.6) – (2.8). That is, if the government were to choose its policy instruments to maximise a Bergson-Samuelson social welfare function $W(V_1, V_2, \dots, V_k)$ subject to (2.6) – (2.8), then $\lambda_i = \partial W(\cdot) / \partial V_i$ and $\langle \bar{\mu}_1, \dots, \bar{\mu}_{n+2} \rangle$ is the vector of multipliers attached to the $n + 2$ constraints in (2.6) – (2.8).

Step 2: We show that there does not exist an equilibrium of our economy in which both (3.6) – (3.12) and (A.6) – (A.12) can be simultaneously satisfied. Suppose $\mu_{n+2} = 0$. Then from (3.11) and (3.12) we have $\langle \mu_1, \dots, \mu_n \rangle = 0^{(n)}$ and $\mu_{n+1} = 0$. Then from (3.9) we have $\alpha = 0$ which is a contradiction. Hence $\mu_{n+2} > 0$. From (3.11) and (3.12) we have:

$$\nabla_{x_g} f = \frac{\langle \mu_1, \dots, \mu_n \rangle}{\mu_{n+2}} \quad \text{and} \quad \nabla_{l_g} f = \frac{\mu_{n+1}}{\mu_{n+2}} \quad (\text{A.13})$$

Suppose $\bar{\mu}_{n+2} = 0$. Then from (A.11) and (A.12) we have $\langle \bar{\mu}_1, \dots, \bar{\mu}_n \rangle = 0^{(n)}$ and $\bar{\mu}_{n+1} = 0$. Then from (A.10) we have $\sum \lambda_i \nabla_g V_i = 0$ which is a contradiction. Hence $\bar{\mu}_{n+2} > 0$. From (A.11) and (A.12) we have:

$$\nabla_{x_g} f = \frac{\langle \bar{\mu}_1, \dots, \bar{\mu}_n \rangle}{\bar{\mu}_{n+2}} \quad \text{and} \quad \nabla_{l_g} f = \frac{\bar{\mu}_{n+1}}{\bar{\mu}_{n+2}} \quad (\text{A.14})$$

Let $\beta := \mu_{n+2} / \bar{\mu}_{n+2}$. From (A.13) and (A.14) we have $\langle \mu_1, \dots, \mu_n \rangle = \beta \langle \bar{\mu}_1, \dots, \bar{\mu}_n \rangle$ and $\mu_{n+1} = \beta \bar{\mu}_{n+1}$. Equation (3.9) can now be written as:

$$\frac{\alpha}{\beta} = - \langle \bar{\mu}_1, \dots, \bar{\mu}_n \rangle \sum \nabla_{\omega} x_i + \bar{\mu}_{n+1} \sum \nabla_{\omega} l_i \quad (\text{A.15})$$

The right-hand sides of (A.15) and (A.9) are identical, but the left-hand side of (A.15) is positive while the left-hand side of (A.9) is non-positive, yielding a contradiction. ■

Proof of Theorem 2

Consider the following hypothetical maximisation problem. Choose p, t, w, τ, g, x_g , and

l_g to maximise $\alpha(\tau - w)$ subject to (i) $t = \hat{t}$, (ii) $g = \hat{g}$, and (iii) the equilibrium conditions (2.6) – (2.8), where $\alpha > 0$ and \hat{t} and \hat{g} denote the status quo levels of the commodity taxes and public good. Let $\theta = \langle \theta_1, \dots, \theta_n \rangle$ denote the multipliers on constraint (i), let δ denote the multiplier on constraint (ii), and let $\mu = \langle \mu_1, \dots, \mu_{n+2} \rangle$ denote the multipliers on constraint (iii). The Lagrangian can be written as:

$$L(\cdot) = \alpha(\tau - w) + \theta(t - \hat{t}) + \delta(g - \hat{g}) - \langle \mu_1, \dots, \mu_n \rangle \left[\sum x_i(q, \omega, g) + x_g - x(p, w) \right] \\ - \mu_{n+1} \left[l(p, w) + l_g - \sum l_i(q, \omega, g) \right] - \mu_{n+2} [g - f(x_g, l_g)]$$

The first-order conditions on p , t , w , τ , g , x_g , and l_g are identical to equations (3.6) – (3.12), respectively. Therefore, an economy that is subject to the Laffer effect is an economy that ‘looks like’ the government has attempted to maximise the labour-income tax rate, subject to the equilibrium constraints, $t = \hat{t}$, and $g = \hat{g}$. ■

Proof of Theorem 3

From equation (3.9) we obtain:

$$-\langle \mu_1, \dots, \mu_n \rangle \sum \nabla_{\omega} x_i + \mu_{n+1} \sum \nabla_{\omega} l_i > 0 \quad (\text{A.16})$$

In proving Theorem 1 we showed that $\mu_{n+2} > 0$. Thus:

$$\frac{-\langle \mu_1, \dots, \mu_n \rangle \sum \nabla_{\omega} x_i + \mu_{n+1} \sum \nabla_{\omega} l_i}{\mu_{n+2}} > 0 \quad (\text{A.17})$$

Using equations (3.11) and (3.12) we obtain:

$$-\nabla_{x_g} f \sum \nabla_{\omega} x_i + \nabla_{l_g} f \sum \nabla_{\omega} l_i > 0 \quad (\text{A.18})$$

Rearranging (A.18) completes the proof. ■

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