Horizontal mergers, Quality choice, and Welfare

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Abstract

We analyze horizontal mergers of firms producing vertically differentiated goods when the average costs of quality are convex and either fixed or variable in production. We show that in a duopoly: (i) the merged firm produces only one quality if production costs are fixed; and (ii) the merged firm produces the two qualities if production costs are variable. We demonstrate that the social planner chooses the same variety of qualities as the merged firm. In the case of variable production costs, both the social planner and the merged firm choose the same qualities. We find that social welfare is reduced in both cases. We show that this welfare reduction is higher when the production costs are fixed and lower when these costs are variables when compared to horizontal mergers where only prices adjustments are considered and not qualities'. We give implications for efficiencies defence.

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I. Introduction

A merger is an operation by which independent firms join to become one entity. There are several types of mergers: the mergers of firms of the same industry producing identical or similar product lines called horizontal mergers; the mergers of a firm producing an intermediate goods and a firm producing a final good which uses the intermediate goods as input called vertical mergers; and finally the mergers of firms producing both different and independent goods called conglomerate mergers.

The effects of horizontal mergers have been studied for homogeneous goods (Farrell et Shapiro, 1990) and horizontally differentiated goods (Deneckere et Davidson, 1985). Our study relates to horizontal mergers of firms producing quality differentiated goods, i.e. all consumers prefer one of the goods when the prices of the two goods are the same ones. The question we ask is how mergers influence the choices of the prices and qualities, and how it affects welfare.

We examine first the case where the quality of the product influences the costs of firms only by the fixed costs. This cost structure characterizes considerable industries with strong intensity of R&D for which an improvement of the quality of the products comes from investments which are used to develop new technologies. The variable costs are often very weak. One thinks in particular to software industries for which the manufacturing costs are very weak compared to the costs of engineering to develop new programs. One also thinks to biotechnology and telecommunication industries. Then we analyze the case where the variable costs depend on quality. This cost structure characterizes the industries of service and transportations.

We show that horizontal mergers involve the disappearance of one of qualities when the fixed costs depend on quality and that the variable costs are zero. The quality offered after merger lies between the qualities offered before merger by the two firms. We also show that when the variable costs are zero, a social planner chooses to produce only one quality on a level higher than the qualities chosen by the firms before and after merger.

When the average variable costs depend on quality, the merged firm produces two qualities. These qualities are identical to those produced by a social planner. We also find that: (i) the higher quality produced after merger is weaker than the higher quality produced before merger; and (ii) the lower quality produced after merger is higher than the lower quality produced before merger. Thus the qualities gap is weaker after merger than before merger.

For the two costs function, we examine the effects on total welfare of mergers. We show that the mergers always decrease the total welfare. We compare also the effect on welfare of prices adjustments alone with the joint effect on welfare of prices and qualities adjustments during mergers. We show that the loss of welfare due to the joint effect of prices and qualities adjustments is higher under fixed costs rather than variable costs.

Our methodology is close to that of Economides (1999) who studies the vertical integration of two firms which produce each a component of a system of two complementary goods. Economides (1999) assumes that the qualities of the components are similar and that only the fixed costs depend on quality. He shows that the vertical integration led to weaker prices, total profits of the integrated firm higher and a quality of the system higher than that which prevails in absence of integration. Our study is also close to Amacher and al. (2003) which compares the choices of a duopoly with those of a social planner but only for a quadratic cost function in quality. They do not examine the case of merger. Our analysis is more general. Indeed (i) we examine the cases where quality affects either the fixed costs or the average variable costs and we make the assumptions that these costs are increasing and convex functions in quality; and (ii) we examine the effect of mergers of duopoly on welfare by considering the joint effect on welfare of the adjustments of prices and qualities.

The paper is structured as follow: in section II, we characterize the consumer's choices, the demand functions, and the cost structures. In section III, we compare the equilibrium of each market structure for the case where only the fixed costs depend on quality. We analyze the case where the variable costs depend on quality in section IV. In section V, we present numerical applications and we examine the joint effect of adjustments of prices and qualities for a specific cost function. We conclude the paper in section VI.

II. The model

Each consumer buys either a unit of good, or nothing. A consumer of the type θ which buys a unit of a product of quality s at the price p receives a net surplus of:

$$U_\theta(s,p) = \theta s - p \text{, where } p \geq 0 .$$

The parameter θ is distributed on $[0,1]$ according to distribution function $F(\theta)$. If a consumer chooses not to buy, she receives her utility of reference which we standardize to zero.

We assume that there are two types of goods which are differentiated by their quality. p_h and s_h, are respectively the price and the quality of the good of higher quality and p_l and s_l, are respectively the price and the quality of the good of lower quality ($s_h > s_l$). The market is served by two independent firms, firm *h* and firm *l* which produce qualities respectively s_h and s_l[1]. The game proceeds in two stages. At the first stage the firms choose simultaneously their quality. At the second stage they choose simultaneously the prices.

The demand for each good depends on the prices and the qualities. We indicate by $\bar{\theta}$ the consumer indifferent between not buying at all and buying the good of lower quality, and $\tilde{\theta}$ the consumer indifferent between buying the good of lower quality and the good of higher quality.

To obtain the demand function for each good, we used the participation constraints (*CP*) and of the self-selection constraints (*CA*) of each consumer. The consumer who purchase the lower quality good satisfied the following constraints:

$$\theta s_l - p_l \geq 0 \quad , \qquad (CP\ l)$$

$$\theta s_l - p_l \geq \theta s_h - p_h \quad . \qquad (CA\ l)$$

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[1] When $s_h = s_l$, then the two goods are identical for the consumers and the two firms compete à la Bertrand. This competition implies a price equal to marginal cost and thus a null profit, or negative if the fixed costs are non negative. Therefore, we assume $s_h > s_l$ and that the firm *l* does not have an incentive to leapfrog firm *h*, i.e. to produce a quality s_l such that $s_l \geq s_h$.

While the one who purchase the higher quality good satisfies the following constraints:

$$\theta s_h - p_h \geq 0 \quad , \qquad\qquad (CP\ h)$$

$$\theta s_h - p_h \geq \theta s_l - p_l \quad . \qquad\qquad (CA\ h)$$

From (*CP l*), we obtain $\bar{\theta} = \frac{p_l}{s_l}$. We assume that the lower quality good is not dominated by the higher quality good, i.e. quality per unit of price of quality *l* is higher than that of quality *h*, $\frac{s_l}{p_l} \geq \frac{s_h}{p_h}$. Since the consumer indexed $\tilde{\theta}$ is indifferent between two qualities, we have $\tilde{\theta} = \frac{p_h - p_l}{s_h - s_l}$.

Consumer with $\theta \in [0, \bar{\theta}]$ buy nothing since (*CP l*) is not satisfied, those of which $\theta \in [\bar{\theta}, \tilde{\theta}]$ buy the lower quality good and those of which $\theta \in [\tilde{\theta}, 1]$ buy the higher quality good. If we note by $D_h(p_h, p_l, s_h, s_l)$ and $D_l(p_h, p_l, s_h, s_l)$ the respective demands of the higher quality good (*h*) and the lower quality good (*l*), then:

$$D_h(p_h, p_l, s_h, s_l) = 1 - F(\tilde{\theta}),$$

$$D_l(p_h, p_l, s_h, s_l) = F(\tilde{\theta}) - F(\bar{\theta}).$$

The general form of the total cost function of production is $C(q,s)$, where q and s represent respectively the production level and the quality level. We assume initially that:

$$C(q,s) = g(s) + q \text{, where } q = 0 \text{, } g'(s) > 0 \text{, } g''(s) > 0 \text{, et } g(0) = 0 .$$

In the second time we assume:

$$C(q,s) = qc(s) \text{, where } c'(s) > 0 \text{, } c''(s) > 0 \text{, and } c(0) = 0 .$$

III. Only the fixed costs depend on quality

III.1 Duopoly

The profits of firm *h* are:

$$\pi_h^D(p_h^D, p_l^D, s_h^D, s_l^D) = p_h^D[1 - F(\tilde{\theta}^D)] - g(s_h^D) \ . \qquad (1)$$

The profits of firm *l* are:

$$\pi_l^D(p_h^D, p_l^D, s_h^D, s_l^D) = p_l^D[F(\tilde{\theta}^D) - F(\bar{\theta}^D)] - g(s_l^D) \ . \qquad (2)$$

p_h^D and p_l^D are the prices and $\tilde{\theta}^D = \frac{p_h^D - p_l^D}{s_h^D - s_l^D}$ and $\bar{\theta}^D = \frac{p_l^D}{s_l^D}$ are the parameters that index respectively the marginal consumers of the higher quality and the lower quality. The conditions of first order are:

$$1 - F(\tilde{\theta}^D) - \frac{p_h^D}{s_h^D - s_l^D} f(\tilde{\theta}^D) = 0, \qquad (3)$$

$$F(\tilde{\theta}^D) - F(\bar{\theta}^D) - \frac{p_l^D}{s_h^D - s_l^D} f(\tilde{\theta}^D) - \frac{p_l^D}{s_l^D} f(\bar{\theta}^D) = 0. \qquad (4)$$

For a uniform distribution of θ on $[0,1]$, that gives:

$$p_h^D = \frac{2s_h^D(s_h^D - s_l^D)}{4s_h^D - s_l^D} \ , \qquad (5)$$

$$p_l^D = \frac{s_l^D(s_h^D - s_l^D)}{4s_h^D - s_l^D} \ . \qquad (6)$$

By substituting (5) and (6) in (1) and (2), and maximizing with respect to qualities, we obtain:

$$g'(s_h^D) = \frac{2s_h^D(2s_h^D - s_l^D)}{(4s_h^D - s_l^D)^2} + \frac{6s_h^D(s_l^D)^2}{(4s_h^D - s_l^D)^3} \ , \qquad (7)$$

$$g'(s_l^D) = \frac{[s_h^D - s_l^D][4(s_h^D)^2 - 2(s_h^D)^2]}{(4s_h^D - s_l^D)^3} . \qquad (8)$$

III.2 Merged firm

The profits of the merged firm indexed by *m* are:

$$\pi^m(p_h^m, p_l^m, s_h^m, s_l^m) = p_h^m[1-F(\tilde{\theta}^m)] + p_l^m[F(\tilde{\theta}^m) - F(\bar{\theta}^m)] - g(s_h^m) - g(s_l^m), \quad (9)$$

Where p_h^m, and p_l^m are the prices, and $\tilde{\theta}^m = \frac{p_h^m - p_l^m}{s_h^m - s_l^m}$ et $\bar{\theta}^m = \frac{p_l^m}{s_l^m}$ are the parameters that index respectively the marginal consumers of the higher quality and the lower quality.

The maximization of (1.9) with respect to prices involves the following conditions of first order:

$$1 - F(\tilde{\theta}^m) - \frac{p_h^m}{s_h - s_l} f(\tilde{\theta}^m) + \frac{p_l^m}{s_h - s_l} f(\tilde{\theta}^m) = 0,$$

$$F(\tilde{\theta}^m) - F(\bar{\theta}^m) + \frac{p_h^m}{s_h - s_l} f(\tilde{\theta}^m) - \frac{p_l^m}{s_h - s_l} f(\tilde{\theta}^m) - \frac{p_l^m}{s_l} f(\bar{\theta}^m) = 0.$$

We can rewrite the first order conditions in the following:

$$\tilde{\theta}^m = \frac{1 - F(\tilde{\theta}^m)}{f(\tilde{\theta}^m)} \;, \quad (10)$$

$$\bar{\theta}^m = \frac{1 - F(\bar{\theta}^m)}{f(\bar{\theta}^m)}. \quad (11)$$

If we assume that $\frac{f(\theta)}{1-F(\theta)}$ who is the hazard rate[2] of the distribution of θ is the monotonous, then the opposite function of the hazard rate $\frac{1-F(\theta)}{f(\theta)}$ is also monotonous in θ. Consequently we can use (10) and (11) to compare $\tilde{\theta}^m$ with $\bar{\theta}^m$.

<u>Lemma 1</u>: *the merger reduces the number of qualities from two to one*.

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[2] To understand why this is called hasard rate, assume that one moves along the θ axis from 0 to 1 and eliminates types that are "passed by". Arriving at θ and moving by $d\theta$ to the right, one finds that the conditional probability that the consumer's type belongs to $[\theta, \theta + d\theta]$ and is thus eliminated is $\frac{f(\theta)}{1-F(\theta)} d\theta$. The hasard rate of many distributions, including the uniform, the normal, the Pareto, the logistic, the exponential, and any distribution with non decresing density is monotone. (Tirole, 1988)

Proof:

The conditions (10) and (11) involve $\tilde{\theta}^m = \bar{\theta}^m$. This implies that $F(\tilde{\theta}^m) = F(\bar{\theta}^m)$. Consequently the demand for the lower quality good which is $[F(\tilde{\theta}^m) - F(\bar{\theta}^m)]$ equals to zero.

Thus, the profit after merger writes as:

$$\pi^m(p^m, s^m) = p^m[1 - F(\bar{\theta}^m)] - g(s^m). \tag{12}$$

For a uniform distribution of θ on $[0,1]$, the price and the quality that maximize (12) are:

$$p^m = \frac{s^m}{2}, \tag{13}$$

$$g'(s^m) = \frac{1}{4}. \tag{14}$$

Now let's compare the qualities before and after merger.

<u>Lemma 2</u>: *quality after merger lies between qualities before merger*

Proof:

We show first that $s^m > s_l^D$ (I), then we show $s_h^D > s^m$ (II).

(I) - $s^m > s_l^D$ if and only if $g'(s^m) > g'(s_l^D)$. From (8) and (14), this condition is satisfied if:

$$\frac{1}{4} > \frac{[s_h^D - s_l^D][4(s_h^D)^2 - 2(s_h^D)^2]}{(4s_h^D - s_l^D)^3}$$

$$\Leftrightarrow [4s_h^D - s_l^D][16(s_h^D)^2 - 8s_h^D s_l^D + (s_l^D)^2] > [4(s_h^D - s_l^D)][4(s_h^D)^2 - 2(s_l^D)^2]$$

Or $4s_h^D - s_l^D > 4(s_h^D - s_l^D)$ and $[16(s_h^D)^2 - 8s_h^D s_l^D + (s_l^D)^2] > [4(s_h^D)^2 - 2(s_l^D)^2]$. This proves that:

$$s^m > s_l^D.$$

(II) - $s_h^D > s^m$ if and only if $g'(s_h^D) > g'(s^m)$. From (7) and (14), this condition is satisfied if:

$$\frac{2s_h^D(2s_h^D - s_l^D)}{(4s_h^D - s_l^D)^2} + \frac{6s_h^D(s_l^D)^2}{(4s_h^D - s_l^D)^3} > \frac{1}{4}$$

$$\Leftrightarrow 4s_h^D[16(s_h^D)^2 - 6s_h^D s_l^D + 8(s_l^D)^2] > [4s_h^D - s_l^D][16(s_h^D)^2 - 8s_h^D s_l^D + (s_l^D)^2]$$

Or $4s_h^D > 4s_h^D - s_l^D$, and $16(s_h^D)^2 - 6s_h^D s_l^D + 8(s_l^D)^2 > 16(s_h^D)^2 - 8s_h^D s_l^D + (s_l^D)^2$. This proves that: $s_h^D > s^m$.

<u>Lemma 3</u>: *the parameter of the marginal consumer who buys after merger is higher than the parameter of the marginal consumer who buys before merger, i.e.* $\overline{\theta}^m > \tilde{\theta}^D$.

Proof :

From (5) and (6), we deduce $\tilde{\theta}^D = \frac{2(s_h^D)^2 - 3s_h^D s_l^D + (s_l^D)^2}{(s_h^D - s_l^D)(4s_h^D - s_l^D)}$. From (13) we also deduce $\overline{\theta}^m = \frac{1}{2}$.

$$\overline{\theta}^m > \tilde{\theta}^D \Leftrightarrow \frac{1}{2} > \frac{2(s_h^D)^2 - 3s_h^D s_l^D + (s_l^D)^2}{(s_h^D - s_l^D)(4s_h^D - s_l^D)} \Leftrightarrow (s_h^D - s_l^D)(4s_h^D - s_l^D) > 4(s_h^D)^2 - 6s_h^D s_l^D + 2(s_l^D)^2$$

$\Leftrightarrow 4(s_h^D)^2 - 5s_h^D s_l^D + (s_l^D)^2 > 4(s_h^D)^2 - 6s_h^D s_l^D + 2(s_l^D)^2 \Leftrightarrow s_l^D(s_h^D - s_l^D) > 0$. This proves that:

$$\overline{\theta}^m > \tilde{\theta}^D.$$

III.3 Social planer

Since the only costs are fixed, the planner will never produce two qualities. The replacement of any quantity of lower quality by the same quantity of higher quality reduced the total cost and increases consumers' welfare.

The total welfare is:

$$BE_F = \int_{\overline{\theta}^{be}}^{1} (\theta s^{be} - p^{be}) dF(\theta) + p^{be}[1 - F(\overline{\theta}^{be})] - g(s^{be}). \qquad (15)$$

Where $\overline{\theta}^{be}$ is the parameter of the indifferent consumer between buying and not buying, and p^{be} and s^{be} are respectively the price and quality produced by the social planner.

For a uniform distribution of θ on $[0,1]$, BE_F is equal to:

$$BE_F = \frac{1-(\overline{\theta}^{be})^2}{2} s^{be} - g(s^{be}) \text{, where} \qquad \overline{\theta}^{be} s^{be} - p^{be} = 0 . \qquad (16)$$

Social planner maximizes BE_F under the constraint:

$$\pi(p^{be}, s^{be}) = p^{be}(1-\overline{\theta}^{be}) - g(s^{be}) = 0 . \qquad (17)$$

The condition (16) gives us:

$$p^{be} = \overline{\theta}^{be} s^{be} . \qquad (18)$$

By substituting (18) in (17), we obtain:

$$\overline{\theta}^{be} s^{be}(1-\overline{\theta}^{be}) - g(s^{be}) = 0 \ . \qquad (19)$$

While replacing $g(s^{be})$ by $\overline{\theta}^{be} s^{be}(1-\overline{\theta}^{be})$ in (16), we have:

$$BE_F = \frac{1-(\overline{\theta}^{be})^2}{2} s^{be} - \overline{\theta}^{be} s^{be}(1-\overline{\theta}^{be}) = s^{be}(1-\overline{\theta}^{be})(\frac{1+\overline{\theta}^{be}}{2} - \overline{\theta}^{be}) ,$$

$$BE_F = \frac{(1-\overline{\theta}^{be})^2}{2} s^{be} . \qquad (20)$$

The first order condition of the maximization of BE_F (20) with respect to quality is:

$$\frac{\partial BE_F}{\partial s^{be}} = \frac{(1-\overline{\theta}^{be})^2}{2} - 2s^{be} \frac{(1-\overline{\theta}^{be})}{2} \frac{\partial \overline{\theta}^{be}}{\partial s^{be}} = 0 . \qquad (21)$$

Starting from the condition (19) the theorem of the implicit functions enables us to write:

$$\frac{\partial \overline{\theta}^{be}}{\partial s^{be}} = \frac{g'(s^{be}) - \overline{\theta}^{be}(1-\overline{\theta}^{be})}{s^{be}(1-2\overline{\theta}^{be})} \ . \qquad (22)$$

While replacing (1.22) in (1.21), we obtain:

$$\frac{\partial BE_F}{\partial s^{be}} = \frac{(1-\overline{\theta}^{be})^2}{2} - 2s^{be} \frac{(1-\overline{\theta}^{be})}{2} \frac{g'(s^{be}) - \overline{\theta}^{be}(1-\overline{\theta}^{be})}{s^{be}(1-2\overline{\theta}^{be})} = 0,$$

$$\frac{\partial BE_F}{\partial s^{be}} = \frac{(1-\overline{\theta}^{be})}{2} - \frac{g'(s^{be}) - \overline{\theta}^{be}(1-\overline{\theta}^{be})}{(1-2\overline{\theta}^{be})} = 0.$$

This implies that:

$$g'(s^{be}) = \frac{1}{2}\left(1+\overline{\theta}^{be}\right). \qquad (23)$$

The condition (23) gives that quality is optimal when the marginal cost of production is equal to the appreciation of quality of the average purchaser of the product.

Proposition 1: *the quality chosen by the social planner is higher than the quality chosen after merger.*

Proof:

$s^{be} > s^m \Leftrightarrow g'(s^{be}) = \frac{1}{2} + \frac{\overline{\theta}^{be}}{2} > \frac{1}{4} = g'(s^m).$

III.4 Welfare effects under fixed costs

Proposition 1.2: *the merger decreases the consumer's welfare.*

Proof :

We know that, $p^m > p_h^D > p_l^D$, $s_h^D > s^m > s_l^D$ et $\overline{\theta}^m > \tilde{\theta}^D > \overline{\theta}^D$. We have $p^m > p_h^D > p_l^D$, $s_h^D > s^m > s_l^D$ and $\overline{\theta}^m > \tilde{\theta}^D > \overline{\theta}^D$. Figure 1.1 below gives the distribution of consumers before and after and merger.

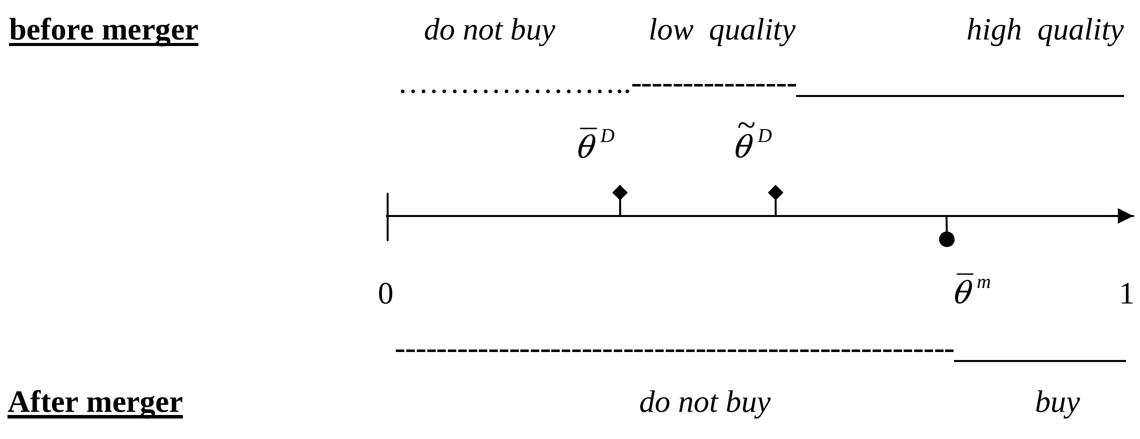

Figure 1. Distribution of consumers under fixed costs**.**

A consumer with $\theta \leq \overline{\theta}^D$ is indifferent between the two regimes since it doesn't buys before and after merger. A consumer with $\theta \in (\overline{\theta}^D, \overline{\theta}^m)$ prefers to buy before merger because she carries out a positive surplus, whereas after merger she does not buy. A consumer with $\theta \geq \overline{\theta}^m$ can buy before and after merger. However, she prefers to buy before merger because she buys a product of better quality at a weaker price.

IV. The variable costs depend on quality

IV.1 Duopoly

The profits are:

$$\pi_h^D(p_h^D, p_l^D, s_h^D, s_l^D) = [p_h^D - c(s_h^D)][1 - F(\tilde{\theta}^D)], \tag{24}$$

$$\pi_l^D(p_h^D, p_l^D, s_h^D, s_l^D) = [p_l^D - c(s_l^D)][F(\tilde{\theta}^D) - F(\bar{\theta}^D)], \tag{25}$$

$\tilde{\theta}^D$ and $\bar{\theta}^D$ are defined as in section I.1. The optimal values of $\tilde{\theta}^D$ and $\bar{\theta}^D$ satisfy the conditions:

$$1 - F(\tilde{\theta}^D) - \frac{p_h^D - c(s_h^D)}{s_h^D - s_l^D} f(\tilde{\theta}^D) = 0, \tag{26}$$

$$F(\tilde{\theta}^D) - F(\bar{\theta}^D) - \frac{p_l^D - c(s_l^D)}{s_h^D - s_l^D} f(\tilde{\theta}^D) - \frac{p_l^D - c(s_l^D)}{s_l^D} f(\bar{\theta}^D) = 0\,. \tag{27}$$

The first order conditions of the maximization with respect to qualities are:

$$c'(s_h^D)\left[1 - F(\tilde{\theta}^D)\right] = -[p_h^D - c(s_h^D)]\, f(\tilde{\theta}^D)\frac{\partial \tilde{\theta}^D}{\partial s_h^D}, \tag{28}$$

$$c'(s_l^D)\left[F(\tilde{\theta}^D) - F(\tilde{\theta}^D)\right] = -[p_l^D - c(s_l^D)]\left[f(\tilde{\theta}^D)\frac{\partial \tilde{\theta}^D}{\partial s_l^D} - f(\bar{\theta}^D)\frac{\partial \bar{\theta}^D}{\partial s_l^D}\right]. \tag{29}$$

The left hand side of (28) represents the marginal cost of increasing the quality of the higher quality. Indeed, it's the unit cost of an increase in s_h^D represented by $c'(s_h^D)$ times the number of unit sold of s_h^D represented by $1 - F(\tilde{\theta}^D)$. The right hand side of (28) represents the marginal revenue of increasing the higher quality.

The condition (29) is interpreted in the same way. It indicates the equality between the marginal cost and the marginal revenue of increasing the quality of the lower quality.

IV.2 Merged firm

The profit after merger is:

$$\pi^m(p_h^m, p_l^m, s_h^m, s_l^m) = [p_h^m - c(s_h^m)][1 - F(\tilde{\theta}^m)] + [p_l^m - c(s_l^m)][F(\tilde{\theta}^m) - F(\bar{\theta}^m)].$$

The first order conditions with respect to prices are:

$$1 - F(\tilde{\theta}^m) - \frac{p_h^m - c(s_h^m)}{s_h^m - s_l^m} f(\tilde{\theta}^m) + \frac{p_l^m - c(s_l^m)}{s_h^m - s_l^m} f(\tilde{\theta}^m) = 0, \qquad (30)$$

$$F(\tilde{\theta}^m) - F(\bar{\theta}^m) + \frac{p_h^m - c(s_h^m)}{s_h^m - s_l^m} f(\tilde{\theta}^m) - \frac{p_l^m - c(s_l^m)}{s_h^m - s_l^m} f(\tilde{\theta}^m) - \frac{p_l^m - c(s_l^m)}{s_l^m} f(\bar{\theta}^m) = 0. \qquad (31)$$

The first order conditions with respect to qualities are:

$$c'(s_h^m)\left[1 - F(\tilde{\theta}^m)\right] = -[p_h^m - c(s_h^m)] f(\tilde{\theta}^m) \frac{\partial \tilde{\theta}^m}{\partial s_h^m} + [p_l^m - c(s_l^m)] f(\tilde{\theta}^m) \frac{\partial \tilde{\theta}^m}{\partial s_h^m}, \qquad (32)$$

$$c'(s_l^m)\left[F(\tilde{\theta}^m) - F(\tilde{\theta}^m)\right] = -[p_l^m - c(s_l^m)]\left[f(\tilde{\theta}^m)\frac{\partial \tilde{\theta}^m}{\partial s_l^m} - f(\bar{\theta}^m)\frac{\partial \bar{\theta}^m}{\partial s_l^m}\right] - [p_h^m - c(s_h^m)] f(\tilde{\theta}^m)\frac{\partial \tilde{\theta}^m}{\partial s_l^m}. \qquad (33)$$

The conditions (32) and (33) give for the higher and lower qualities respectively, the equality between the marginal cost and the marginal revenue of increasing quality. When we compare the effect of increasing the quality of higher quality before merger given by (28) to the one after merger given by (32), we note that the marginal revenue before merger is higher than the one after merger. Thus, the higher quality before merger is higher than the one after merger ($s_h^D > s_h^m$). On the other, when we compare the effect of increasing the quality of lower quality before merger given by (29) to that after merger given by (33), we observe that the marginal revenue before merger is low than to the one after merger. The lower quality before merger is lower than to the one after merger ($s_l^D < s_l^m$). The dispersion of qualities before merger is thus higher than the one after merger.

For the intuition of this result let us note that before merger, to reduce the intensity of the competition, both firms increase their difference in quality to differentiate their product. While after merger, a weak dispersion of qualities does not imply an intensification of competition in price since the merged firm takes into account this externality in its choices of prices and qualities. We illustrate this result in appendices for a specific cost function (appendices 2 and 3).

For a uniform distribution of θ on $[0,1]$, the prices that maximize the merged firm's profit are:

$$p_h^m = \frac{s_h^m + c(s_h^m)}{2}, \tag{34}$$

$$p_l^m = \frac{s_l^m + c(s_l^m)}{2}. \tag{35}$$

Consequently:

$$\tilde{\theta}^m = \frac{1}{2}\left[1 + \frac{c(s_h^m) - c(s_l^m)}{s_h^m - s_l^m}\right], \text{ and } \bar{\theta}^m = \frac{1}{2}\left[1 + \frac{c(s_l^m)}{s_l^m}\right].$$

The maximization of the profit with respect to qualities gives:

$$c'(s_h^m) = \tilde{\theta}^m = \frac{1}{2}\left[1 + \frac{c(s_h^m) - c(s_l^m)}{s_h^m - s_l^m}\right], \tag{36}$$

$$c'(s_l^m) = \tilde{\theta}^m + \bar{\theta}^m - 1 = \frac{1}{2}\left[\frac{c(s_h^m) - c(s_l^m)}{s_h^m - s_l^m} + \frac{c(s_l^m)}{s_l^m}\right]. \tag{37}$$

The merged firm is interested in marginal consumers $\tilde{\theta}^m$ and $\bar{\theta}^m$. It offers s_h^m, such as the marginal cost of production of this quality is equal to the reserve price of the marginal consumers of this quality: $c'(s_h^m) = \tilde{\theta}^m$. The condition (37) can be rewritten as $c'(s_l^m) = \bar{\theta}^m - (1 - \tilde{\theta}^m)$. The merged firm offers, such as the marginal cost of production of this quality is lower than the reserve price: $c'(s_l^m) < \bar{\theta}^m$.

IV.3 Social planer

For a uniform distribution of θ on $[0,1]$, the welfare function (BE_V) is:

$$\int_{\bar{\theta}^{be}}^{\tilde{\theta}^{be}} (\theta s_l^{be} - p_l^{be}) d\theta + \int_{\tilde{\theta}^{be}}^{1} (\theta s_h^{be} - p_h^{be}) d\theta + [p_l^{be} - c(s_l^{be})](\tilde{\theta}^{be} - \bar{\theta}^{be}) + [p_h^{be} - c(s_h^{be})](1 - \tilde{\theta}^{be}).$$

We can rewrite in the following way:

$$BE_V = \frac{(\tilde{\theta}^2 - \bar{\theta}^2)}{2} s_l + \frac{(1-\tilde{\theta}^2)}{2} s_h - (1-\tilde{\theta})c(s_h) - (\tilde{\theta} - \bar{\theta})c(s_l).$$

The welfare is maximized when:

$$p_h^{be} = c(s_h), \tag{38}$$

$$p_l^{be} = c(s_l). \tag{39}$$

Qualities maximize the welfare when:

$$c'(s_h^{be}) = \frac{1+\tilde{\theta}^{be}}{2} = \frac{1+\dfrac{(p_h^{be} - p_l^{be})}{(s_h^{be} - s_l^{be})}}{2} = \frac{1}{2}\left[1 + \frac{c(s_h^{be}) - c(s_l^{be})}{s_h^{be} - s_l^{be}}\right], \tag{40}$$

$$c'(s_l^{be}) = \frac{(\tilde{\theta}^{be} + \bar{\theta}^{be})}{2} = \frac{1}{2}\left[\frac{c(s_h^{be}) - c(s_l^{be})}{s_h^{be} - s_l^{be}} + \frac{c(s_l^{be})}{s_l^{be}}\right]. \tag{41}$$

The conditions (40) and (41) show that qualities are optimal when the marginal costs of quality are equal to the appreciation of quality of the average purchasers of the products. Let us notice that first order conditions of quality choices (36) and (37) of the merged firm and those (40) and (41) of the social planner are similaires. We thus deduce proposition 3.

<u>Proposition 3</u>: *the qualities produced by the merged firm are identical to those of the social planner.*

Crampes and Hollander (1995) show that in markets with vertically differentiated goods and totally covered a duopoly producing two qualities of a good led to a strong discrepancy between these qualities compared to the social optimum. We show that a duopoly which produces two qualities of a good led to a strong discrepancy between these qualities compared to a merged firm. Since we showed that the merged firm and the social planner choose same qualities, we get the same the result as Crampes and Hollander (1995) under the assumption of partial cover, i.e. when there are consumers who do not buy[3].

[3] Crampes and Hollander (1995) assume that the market is totaly covered.

IV.4 Welfare effects under variable costs

<u>Proposition 4</u>: *the merger decreases the consumer's welfare.*

***Proof** :*

Figure 2 gives the distribution of consumers before and after merger when $\tilde{\theta}^m > \tilde{\theta}^D > \bar{\theta}^m$.

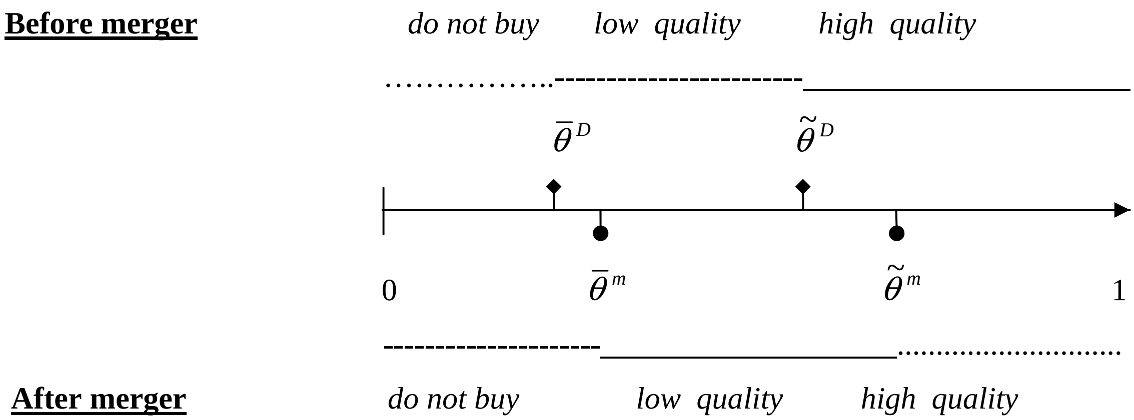

Figure 2. Distribution of consumers under variable costs.

A consumer with $\theta \le \bar{\theta}^D$ is indifferent between the two market structures since it buys neither before and nor after merger. A consumer with $\theta \in [\bar{\theta}^D, \bar{\theta}^m]$ buys the lower quality before merger and does not buy after merger. A consumer with $\theta \in [\bar{\theta}^m, \tilde{\theta}^D]$ can buy the lower quality before and after merger. She prefers to buy the lower quality before merger if and only if $\theta s_l^D - p_l^D \ge \theta s_l^m - p_l^m$. Or $\theta s_l^D - p_l^D \ge \theta s_l^m - p_l^m$ if and only if $\theta \ge \dfrac{p_l^D - p_l^m}{s_l^D - s_l^m}$.

We will prove that $\theta \ge \dfrac{(p_l^D - p_l^m)}{(s_l^D - s_l^m)}$. We know already that $\bar{\theta}^m = \dfrac{p_l^m}{s_l^m} > \dfrac{p_l^D}{s_l^D} = \bar{\theta}^D$. This implies that $\dfrac{p_l^m}{s_l^m} > \dfrac{p_l^D - p_l^m}{s_l^D - s_l^m}$. Since we are in the case $\theta \in (\bar{\theta}^m, \tilde{\theta}^D)$, then:

$$\theta \ge \frac{p_l^m}{s_l^m} \ge \frac{(p_l^D - p_l^m)}{(s_l^D - s_l^m)}$$

A consumer with $\theta \in [\tilde{\theta}^{D}, \tilde{\theta}^{m}]$ can buy as well the higher quality before merger than the lower quality after merger. But like we let us know already that: (i) before merger, this consumer prefers the higher quality than the lower quality; and (ii) this consumer prefers the lower quality after merger than the lower quality before merger. We thus deduce that this consumer prefers the higher quality before merger than the lower quality after merger. A consumer with $\theta \geq \tilde{\theta}^{m}$ strictly prefers the situation before merger to the situation after merger because she buys the higher quality before merger at a price per unit of quality which is lower than the price per unit of quality after merger $\left(\frac{p_h^D}{s_h^D} < \frac{p_h^m}{s_h^m} \right)$.

The same analysis applies when $\tilde{\theta}^{D} > \tilde{\theta}^{m} > \overline{\theta}^{m}$ and we obtain easily the same conclusions as previously.

V. Numerical applications

V.1 Fixed costs

We assume now that the cost function has the following form:

$$g(s_i) = \frac{s_i^k}{a} \quad , \quad i = h, l \, , k \in [2, k*] \, .$$

For the numerical simulations we take $a = 10$, $k* = 5$ and θ uniformly distributed on $[0,1]$.

The equilibrium prices, qualities and parameters of marginal consumers before merger and after merger are respectively consigned in Tables 1 and 2 (see appendices 5 and 6)[4].

We observe that the level of welfare depends on the consideration of the joint effect of adjustments of prices and quality after merger. The effects of adjustments of prices are known and generally well taken into account. Indeed, if mergers do not lead considerable reduction costs

[4] The results obtained are identical to those of Motta (1993) where the equilibrium prices and qualities of duopoly are given both under fixed and variable costs.

and efficiency gains, then the rise of prices after merger reduces the total welfare. The last columns of Tables 1 and 2 show that the reduction of consumer's welfare is higher than the additional profits obtained by the merged firm.

On the other hand, the effects on total welfare of adjustments of quality are less foreseeable. When the fixed costs are assumed to be recoverable, Table 4 indicates that the reduction of the welfare with adjustments of quality is higher than that without adjustments of quality (see appendix 8). This result is explained by the fact that merger reduces both the level and the number of quality when only the fixed costs depend on quality. By adjusting the quality, this decreases the level of quality whereas the price per unit of quality does not change. By taking account the adjustments of quality, we observe a reduction of consumer's surplus and a rise of firm's profit. The adjustments of quality have a negative effect on total welfare since the reduction of consumer's surplus is more considerable than the rise of firm's profits.

Thus, the joint adjustment of prices and qualities during horizontal mergers when only the costs depend on quality increases the loss of welfare compared to the adjustment of the prices alone. By taking into account the two adjustments after merger, the welfare should drop more compared to the situation where qualities are assumed to be exogenous or given.

V.2 Variable costs

We assume:

$$C(q_i, s_i) = q_i c(s_i) = q_i \frac{s_i^k}{a},\ i = h, l, k \in [2, k^*].$$

For the numerical simulations we take $a = 10$, $k^* = 5$ and θ uniformly distributed on $[0,1]$.

The equilibrium prices, qualities and parameters of marginal consumers before merger and after merger are respectively consigned in Tables 5 and 6 (see appendices 9 and 10).

The analysis of the welfare in Table 7 indicates that the fall of welfare with the joint adjustment of prices and qualities is weaker than that with adjustment of the prices alone (appendix 11).

Merger preserves the number of varieties of products and increases the level of lower quality. Thus the adjustment of qualities increases lower quality. Consequently the price per unit of lower quality is decreased. We observe a rise of the consumer's surplus and a rise of the firms'profits after the adjustments. The adjustments of quality have a positive effect on total welfare.

Thus the joint adjustment of prices and qualities during horizontal mergers when the variable costs depend on quality thus decreases the loss of welfare compared to the adjustment of the prices alone. In this case, considering the adjustments of quality reduces less the welfare comparatively to a situation where qualities are assumed to be exogenous or given. Remark that this result is contrary to the case where only the fixed costs depend on quality.

VI. Conclusion

We analyzed the horizontal mergers of duopoly vertically differentiated under two assumptions about the cost functions. Initially, we assumed that only the fixed costs depend on quality; in the second time we assumed that the variable costs depend on quality. In the first case a merger involves the elimination of one of qualities, the reduction of the market coverage, the raising of prices and the reduction of the welfare. In the second case, after merger the number of qualities is preserved and selected qualities are identical to qualities of a social planner. We also showed that the merger leads to a reduction of the discrepancy between the two levels of quality. However, after merger the firm maintains prices much higher than those of the duopoly. What causes to reduce the total welfare.

The evaluations of welfare effects of a merger generally do not take into account the adjustments of quality which follow a merger. Our results indicate that not taking account of adjustments of quality over-estimates or underestimates the effects of welfare according to the costs structure of industry. If merger takes place in an industry whose costs are primarily fixed like industries with strong intensity of R&D (telecoms, biotechnologies), not to hold account of the adjustments of quality underestimates the loss of welfare after a merger. On the other hand if merger take place in an industry where in fact the variable costs depend on quality (services and transportations) not taking account of adjustments of quality over-estimates the loss of welfare.

That suggests that mergers are too often accepted in the first case and are too often rejected into the second. In practice, the qualities adjustements are neglected when one makes the examination of the effects of welfare of a merger. Under the terms of certain Laws of competition like the Canadian Law, all the profits of welfare can constitute a defence according to article 96 of the Competition Law. Our results show that it is also important to take account of the adjustments of qualities in the analysis of mergers to get all the efficiency gains.

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Appendices

Appendix 1

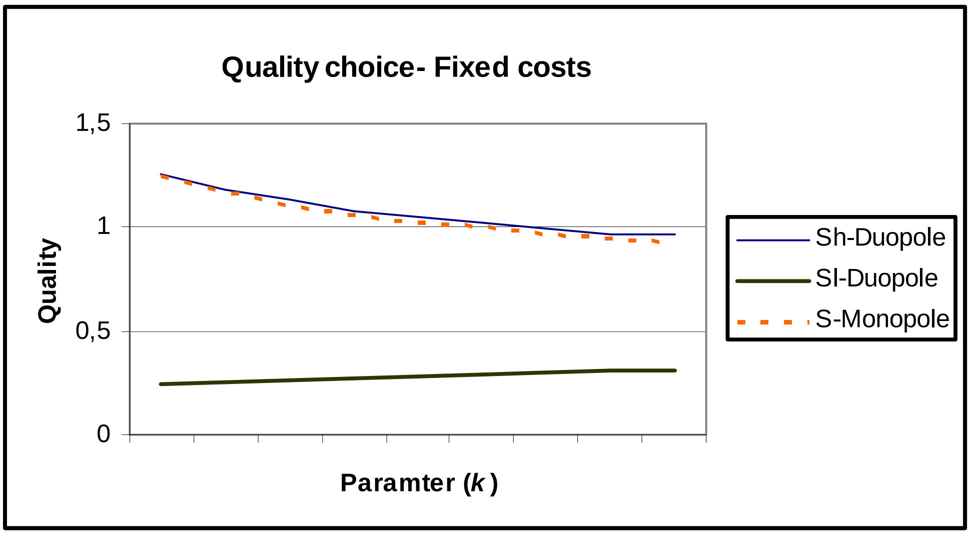

Figure 1.3. Optimal quality choice under fixed costs.

Appendix 2

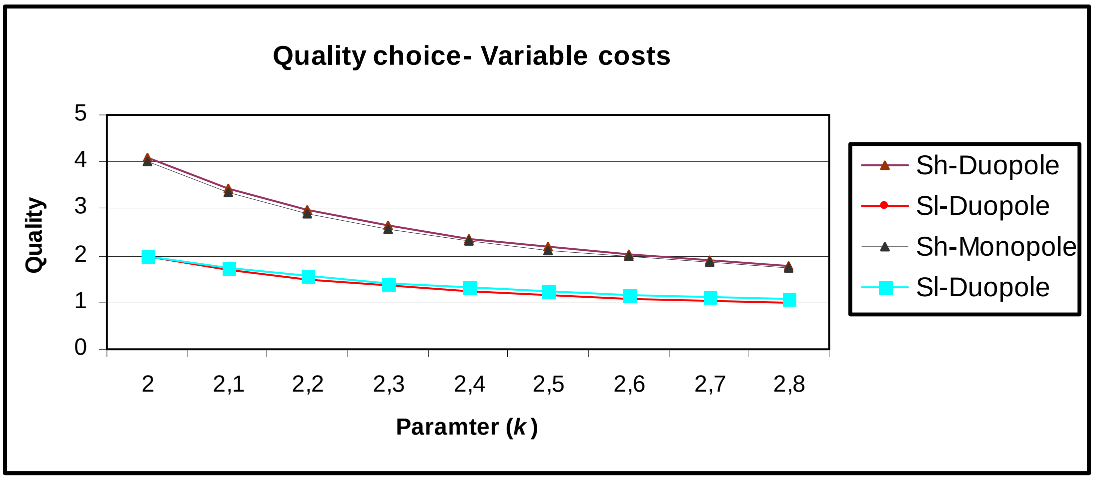

Figure 1.4 : Optimal quality choice under variable costs.

Appendix 3

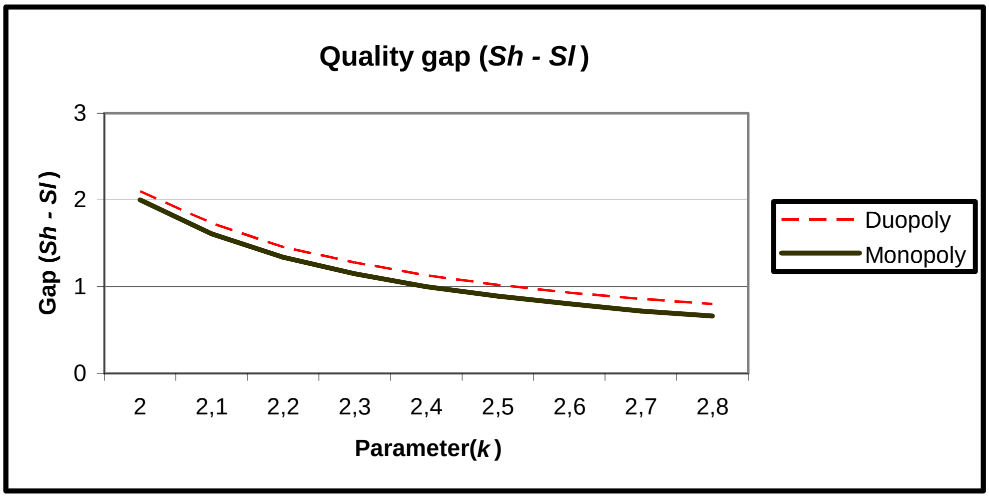

Figure 1.5 : Quality dispersion

Appendix 4

Table 1: *Qualities, prices, preference parameters, profits, surplus, and welfare before merger under fixed costs.*

k	s_h^D	s_l^D	$s_h^D - s_l^D$	p_h^D	p_l^D	$\tilde{\theta}^D$	$\bar{\theta}^m$	π^D	$BE^D - AAQ$
2	1,26	0,241	1,019	0,53509	0,05117	0,47489	0,2123	0,12985	0,345096
2,1	1,18	0,252	0,928	0,49017	0,05234	0,4718	0,2077	0,12563	0,334184
2,2	1,13	0,263	0,867	0,46028	0,05356	0,46911	0,2037	0,12243	0,328003
2,3	1,08	0,272	0,808	0,43115	0,05429	0,4664	0,1996	0,12017	0,322328
2,4	1,05	0,282	0,768	0,41164	0,05528	0,46401	0,196	0,11823	0,319689
2,5	1,02	0,291	0,729	0,39249	0,05599	0,4616	0,1924	0,11675	0,317305
2,6	0,99	0,298	0,692	0,37416	0,05631	0,45931	0,189	0,11581	0,314969
2,7	0,97	0,306	0,664	0,36043	0,05685	0,45719	0,1858	0,11488	0,314131
2,8	0,96	0,313	0,648	0,35272	0,05744	0,45568	0,1835	0,1143	0,314625

$BE^D - AAQ$: Welfare before merger with adjustment of qualities.

CS^D : Consumer's surplus before merger.

Appendix 5

Table 2: *Qualities, prices, preference parameters, profits, surplus, and welfare after merger under fixed costs.*

k	s^m	p^m	$\bar{\theta}^m$	π^m	$BE^m - AAQ$
2	1,25	0,625	0,5	0,15625	0,3125
2,1	1,17	0,585	0,5	0,15344382	0,29969382
2,2	1,11	0,555	0,5	0,15169134	0,29044134
2,3	1,06	0,53	0,5	0,1506586	0,2831586
2,4	1,02	0,51	0,5	0,15013262	0,27763262
2,5	1	0,5	0,5	0,15	0,275
2,6	0,97	0,485	0,5	0,15011393	0,27136393
2,7	0,95	0,475	0,5	0,15043297	0,26918297
2,8	0,93	0,465	0,5	0,15088833	0,26713833

$BE^m - AAQ$: Welfare after merger with adjustment of qualitie.

CS^m : Consumer’s surplus after merger.

Appendix 6

Table 3: *Variation of the welfare after merger without adjustment of qualities when the fixed costs are irremediable*

k	*ΔBE- SAQ*
2	*0,03716*
2,1	*0,03878*
2,2	*0,0404*
2,3	*0,0417*
2,4	*0,04315*
2,5	*0,04445*
2,6	*0,04543*
2,7	*0,04657*
2,8	*0,04758*

ΔBE- SAQ: Variation of welfare after merger without adjustment of qualities (the variations are negative).

Appendix 7

Table 4: *Variation of the welfare after merger with and without adjustment of qualities when the fixed costs are recoverable*

k	$\Delta BE - SAQ$	$\Delta BE - AAQ$
2	0,03135598	0,03259598
2,1	0,03324778	0,03449017
2,2	0,03510233	0,03756134
2,3	0,03669239	0,03916943
2,4	0,0383614	0,042056
2,5	0,03988044	0,04230519
2,6	0,04113968	0,043605
2,7	0,04248578	0,04494771
2,8	0,04370972	0,04748709

$\Delta BE - SAQ$: Variation of welfare after merger without adjustment of qualities (the variations are negative).

$\Delta BE - AAQ$: Variation of welfare after merger with adjustment of qualities (the variations are negative).

Appendix 8

Table 5: *Qualities, prices, preference parameters, profits, surplus, and welfare after merger under variable costs.*

k	s_h^D	s_l^D	$s_h^D - s_l^D$	p_h^D	p_l^D	$\tilde{\theta}^D$	$\bar{\theta}^D$	π^D	$BE^D - AAQ$
2	4,09	1,99	2,1	2,2604	0,7479	0,72022	0,375827	0,28557	0,755406
2,1	3,43	1,7	1,73	1,8338	0,6068	0,70925	0,356952	0,252667	0,677275
2,2	2,96	1,5	1,46	1,529	0,5094	0,69833	0,339608	0,228074	0,621596
2,3	2,63	1,35	1,28	1,3217	0,4389	0,68967	0,325134	0,210582	0,579642
2,4	2,37	1,24	1,13	1,1545	0,3858	0,68027	0,31114	0,196075	0,54804
2,5	2,18	1,16	1,02	1,0347	0,3478	0,67349	0,299789	0,184537	0,523277
2,6	2,02	1,09	0,93	0,9333	0,3144	0,6655	0,288398	0,175422	0,503428
2,7	1,89	1,03	0,86	0,852	0,2863	0,65781	0,277985	0,168315	0,487138
2,8	1,79	0,99	0,8	0,7886	0,2667	0,65238	0,26938	0,161576	0,474146

$BE^D - AAQ$: Welfare before merger with adjustment of qualities.

CS^D : Consumer's surplus before merger.

Appendix 9

Table 6: *Qualities, prices, preference parameters, profits, surplus, and welfare after merger under variable costs.*

k	s_h^m	s_l^m	$s_h^m - s_l^m$	p_h^m	p_l^m	$\tilde{\theta}^m$	$\bar{\theta}^m$	π^m	$BE^m - AAQ$
2	4	2	2	2,8	1,2	0,8	0,6	0,4	0,6
2,1	3,34	1,73	1,61	2,2993	1,0231	0,79267	0,591374	0,358076	0,53711468
2,2	2,89	1,55	1,34	1,9614	0,9061	0,78748	0,5846	0,327985	0,4919782
2,3	2,56	1,41	1,15	1,7144	0,8152	0,78194	0,578155	0,305596	0,45839368
2,4	2,31	1,31	1	1,5279	0,7506	0,77735	0,572971	0,288457	0,43268621
2,5	2,12	1,23	0,89	1,3872	0,6989	0,77337	0,568207	0,275038	0,4125564
2,6	1,97	1,17	0,8	1,2765	0,6602	0,77032	0,564279	0,26433	0,39649549
2,7	1,84	1,12	0,72	1,1794	0,6279	0,76598	0,560623	0,255649	0,38347283
2,8	1,74	1,08	0,66	1,1058	0,602	0,76327	0,557429	0,248526	0,37278919

$BE^m - AAQ$: Welfare after merger with adjustment of qualitie.

CS^m : Consumer’s surplus after merger.

Appendix 10

Table 7: *Variation of the welfare after merger with and without adjustment of qualities under variable costs.*

k	$\Delta BE - SAQ$	$\Delta BE - AAQ$
2	0,15560122	0,15540636
2,1	0,14043941	0,14016045
2,2	0,12992005	0,12961824
2,3	0,12168844	0,12124837
2,4	0,11581494	0,11535418
2,5	0,1112868	0,1107207
2,6	0,10754967	0,10693239
2,7	0,10435262	0,10366537
2,8	0,10212676	0,10135692

$\Delta BE - SAQ$: Variation of welfare after merger without adjustment of qualities (the variations are negative).

$\Delta BE - AAQ$: Variation of welfare after merger with adjustment of qualities (the variations are negative).