Comments Welcome

The human development index as a criterion for optimal planning*

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Abstract

Planning strategies that maximize the Human Development Index (HDI) tend towards minimizing consumption and maximizing non-investment expenditures on education and health. This lopsided result arises because the income component in the HDI effectively double counts education and life expectancy. We argue that consumption (net of expenditures on education or health) is a better indicator of capabilities not already reflected in the education and life expectancy components of the index. When consumption is used instead of income in the index, optimal plans yield a balance between allocations for consumption, education and life expectancy. However, using the modified index may result in less equitable outcomes than those implied by using the HDI unless inequality aversion is incorporated into the index.

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1. Introduction

The Human Development Index (HDI) is a composite index published annually by the UN Human Development Report Office, since 1990, which is designed to measure "human well being" in different countries.[1] The index combines measures of life expectancy, school enrolment, literacy, and income to provide a broader-based measure of well-being and development than income alone. Since its publication, this index has become widely cited and is commonly used as a way of ranking the quality of life in different countries.

The impact of the HDI ranking on policy is reflected by the fact that some national governments have taken to announcing their HDI ranking and their aspirations for improving it. For example, in a recent speech, the President of India, Dr. Abdul Kalam, exhorts Indians to work together to improve India's current HDI rank of 127 to achieve a rank of 20; see Kalam (2005). The HDI is discussed in recent Indian budgets (e.g. Budget of India (2005)) and changes in India's ranking are covered by the media (e.g. Parsai (2006)). In announcing Canada's number one ranking in 1998, Prime Minister Jean Chrétien stated: "While the HDI tracks Canada's impressive achievements, it also tells us where we can improve." (Chrétien (1998)).

In this paper, we first consider the implications of using the HDI as a criterion for economic development plans. In particular, we examine the consequences of pursuing plans that maximize the HDI score for a given country. To do this, we construct an economic model where a planner chooses expenditures to maximize a well-defined objective function that includes the HDI index as a special case. We get two major results. First, the planner tends towards minimizing consumption and maximizing (non-investment) expenditures on education and health. We get this result despite the fact that the HDI includes an income index as one of its components. Second, the optimal plan tends to imply equitable outcomes even though inequality aversion is not explicitly modelled in the HDI. This latter result is arguably a surprisingly beneficial consequence of using the HDI that addresses the concern for equity expressed in the Human Development Reports and literature (e.g. Anand and Sen (2000)). In contrast,

[1] For a detailed description see http://hdr.undp.org/statistics/indices/.

the first result reveals what we consider to be a flaw with the HDI, but one that can be readily fixed.

The first result – that the planner tends towards minimizing consumption and maximizing expenditure on education and health – indicates that optimal allocations very heavily favour expenditures on education and health at the expense of consumption on other items. One reason for this lopsided result is that consumption does not enter the index (the objective function) or the production technology, but costs the planner through the resource constraint, so the optimal plan will set consumption to meet minimum consumption requirements. Another reason for this lopsided result is that the income component in the HDI plays mainly an indirect role in determining the allocations of funds between education and health. In our basic model, income can be decomposed into expenditures on consumption, education and health. As consumption is at its minimal level, the remaining expenditures are allocated to education and health. Education and health expenditures are valued directly in those components of the HDI and are also valued in the income component. The HDI is flawed because the income component "double counts" the value of allocating expenditures to education and health and because it does not effectively value consumption.

The income component of the HDI was originally justified as an indirect proxy of "command over resources to enjoy a decent standard of living" (Human Development Report 1990, p. 1). Anand and Sen (2000, p. 86) state:

> The use of `command over resources' in the HDI is strictly as a residual catch-all to reflect something of other basic capabilities not already incorporated in the measures of longevity and education. … For example, going hungry is a deprivation that is serious not just for its tendency to reduce longevity, but also for the suffering it directly causes. Similarly, resources needed for shelter and for being able to travel may be quite important in generating the corresponding capabilities.

In the later half of the paper we detail our critique of the income component in the HDI and argue for a modified HDI that replaces the income component with consumption component that is net of expenditures on education and heath. We do this for three reasons. First, we believe consumption expenditure is a better indirect indicator of command over resources and capabilities than income. Second, the income component leads to lopsided optimal plans that are flawed. Third, using a consumption indicator yields a direct trade-off between allocations for consumption, education and health and as a result a balance of expenditures for the three components.

The paper continues as follows. Section 2 develops the model. Section 3 solves the Planner's Problem, and Section 4 explores equity and taxation issues. Section 5 examines the role of income in the HDI index, and provides a critique of the income component. Section 6 develops a modified HDI index that we believe better captures the goals behind the index. Section 7 concludes. All proofs are found in the Appendix.

2. The model

We consider a static closed economy model, where a planner acts to maximize the following objective function, which nests the HDI:

$$I(w,W) = wI^{y}(y) + (1-w)[WI^{e}(e) + (1-W)I^{l}(l)] \tag{1}$$

Here, $I^{y}(y)$, $I^{e}(e)$, and $I^{l}(l)$ represent indexes of per capita income (*y*), educational attainment (*e*), and life expectancy (*l*) respectively. These are assumed to be differentiable, increasing and concave in their respective arguments. The parameters *w* and *W* are weights used when constructing the composite index, given in equation (1). The HDI is a special case of this index, where $w = 1/3$ and $W = 1/2$, so that each of the three component indexes are equally weighted.

Educational attainment is assumed to be a differentiable increasing function of expenditures on both education (*E*) and health (*H*). Thus:

$$e = e(E,H), \quad e_{E} > 0, \quad e_{H} \geq 0 \tag{2}$$

Similarly, life expectancy is differentiable and increasing in both of these arguments[2]:

$$l = l(E,H)\,,\quad l_E \geq 0\,,\quad l_H > 0\,,\quad l(0,0) > 0 \qquad (3)$$

To simplify the analysis, we are assuming that the economy in question has a level of per capita income high enough so that neither income nor consumption substantially affect life expectancy and educational attainment as measured in the HDI. This is formalized by the following assumption about individual consumption *c*:

$$c \geq c_{\min} \qquad (4)$$

where $c_{\min} > 0$ is a parameter which identifies the level of consumption beyond which no further increments in consumption will increase educational attainment or life expectancy.[3] Later we relax constraint (4) and show that the results become stronger.

In this simple static economy, we abstract away from capital and assume full utilization of labour. All individuals' work and the total number of workers in the economy is normalized to one unit. Given this, output per capita is determined by the following differentiable production technology:

$$y = f(e,l)\,,\quad f_e \geq 0\,,\quad f_l \geq 0 \qquad (5)$$

Here, education attainment affects output through human capital in the usual way. Also, increments in life expectancy increase the effective size of the labour force and thereby increase production. Increments in life expectancy can be thought to increase the effective size of the labour force in two ways: directly through increasing the possible amount of work time per individual over a life time and indirectly by indicating better health and hence greater productivity.

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[2] We have taken the short cut of specifying life expectancy (education) as a function of education (health) expenditure rather than education (health) attainment. Functions (3) and (4) can be shown to be consistent with the more general specification under minor restrictions. Assuming that minimum life expectancy is positive simplifies the analysis.

[3] This assumption is consistent with Anand and Ravallion's (1993) "capability expansion through social services". According to this explanation (also see Sen, 1981), the public provision of essential goods and services leads to improved social outcomes and income matters if it is used to finance suitable public services and alleviate poverty. For example, Anand and Ravallion find in a sample of 22 developing countries that after controlling for health expenditures and poverty (as measured by the proportion of population consuming less than one dollar per day in 1985 at PPP), life expectancy is not affected by consumption. Even the unconditional plot of income against life expectancy displays an income threshold (achieved by all developed countries) beyond which there is no discernable relationship (e.g. Deaton, 2003)). Anand and Ravallion contrast schools of thought on the importance of social services versus private consumption for human development.

Once produced, the single good in the economy can be allocated to three possible uses: aggregate consumption ($l \cdot c$), education expenditure (*E*), and health expenditure (*H*). Therefore, the economy must respect the aggregate constraint:

$$lc + E + H \leq y \tag{6}$$

Observe that consumption, *c*, is on items other than health and education and that we allow total consumption to be proportional to life expectancy.[4]

3. The Planner's Problem

Using equations 1-6, the planner's problem can be formulated as the programming problem (P1):

$$\underset{\{c,E,H\}}{Max}\, I(w,W) = wI^{y}(f(l(E,H),e(E,H))) + (1-w)\left(WI^{e}(e(E,H)) + (1-W)I^{l}(l(E,H))\right)$$

subject to: i) $l(E,H)c + E + H - f\left(l(E,H),e(E,H)\right) \leq 0$

ii) $-c \leq -c_{\min}$

Proposition 1. *Incremental reductions in consumption, c, towards $c_{\min}$ increase the HDI score. Maximizing the HDI requires setting consumption at the minimum level:* $c^{*} = c_{\min}$.

The intuition behind this result is quite straightforward. Consumption does not enter the objective function or the production technology, but costs the planner through the resource constraint. Thus, reductions in consumption that are optimally allocated to education and health expenditures will increase the HDI. The optimal plan will set consumption to its minimal allowed value.[5] Our formulation with a minimum

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[4] We only analyze situations where there is at least one feasible allocation (*c, E, H*) satisfying equations 2-6 and the non-negatively constraints $E \geq 0$ and $H \geq 0$. A sufficient condition for this is that $lc_{\min} \leq y$ when $E = H = 0$; i.e. minimum output can meet minimum consumption at minimum life expectancy.

[5] More elaborate models yield the same result. For example, our static formulation does not consider time allocations. We could, however, allow for the possibility that persons do not work beyond a retirement age, *R*, so that the amount of lifetime work is max[*l*,*R*]. When life expectancy exceeds the retirement age, $l > R$, this might lead to $f_l = 0$ at the margin (if there are no worker health benefits associated with greater life-expectancy). Still the objective function is increasing in *E* and *H* so that the resource constraint binds and the proposition obtains. If we use *y*/*l* instead of *y* in the objective

consumption requirement clearly reveals that all remaining resources are allocated to those expenditures, education and/or health, which increase the objective function.[6]

The fact that the minimum consumption constraint is binding at the optimum implies that in the absence of the consumption constraint that the planner would allocate even less to consumption and more to expenditures on education and health; i.e. if $c^* < c_{\min}$ then $E + H = y - lc^* > y - lc_{\min}$. Indeed, with the above specified functions, the planner's optimal choice would be $c^* = 0$ and $E + H = y$. This unrealistic corner solution arises only because we have excluded consumption from the education, health and production functions. But this exclusion was on the grounds that $c \geq c_{\min}$ is sufficiently high not to effect these functions. If consumption $c < c_{\min}$ has a sufficiently positive effect (on any of education, health, or production) then the optimal choice would be $c^* \in (0, c_{\min})$ in the absence of the constraint. Minimal output is allocated to consumption when consumption plays an instrumental role and is not valued directly.

4. Equity

Probably the most common concern with the HDI is that it only uses average per capita income and, therefore, is consistent with large income disparities within countries. Thus, two countries with the same average income would be scored the same by the index *ceteris paribus*, even though one country might have far more poor whose meagre 'command over resources' substantially inhibits their human development. For this reason that Anand and Sen (2000), Foster *et al* (2005) and others have argued that some sort of income inequality aversion should be built into the index explicitly. What seems to be missing in the literature is an analysis of how policies that promote human development as measured by the HDI affect inequality.

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function, the objective function would be increasing in y/l when $f_l l/f > 0$, i.e. the elasticity of output with respect to life expectancy is greater than 1. Similarly, if there is a trade off between time allocated to production and time allocated to education and health, equation (6) binds and Proposition 1 holds.

[6] While the optimal plan heavily emphasizes expenditures on education and health, strictly speaking it does not involve maximizing the combined expenditure $E + H$. Recall from the resource constraint that $E + H = f - lc_{\min}$ so that maximizing $E + H$ is the same as choosing E and H to maximize $f - lc_{\min}$ subject to the resource constraint. The planner's problem yields the same solution in two very special cases: (1) when $w \to 1$, so that the planner maximizes output, and l is unaffected by E and H, and (2) when l is at its upper bound so that all remaining expenditures go to education which is the only way to increase output.

Our analysis has the surprising implication that policies that maximize the HDI score should dramatically reduce consumption inequality, *ceteris paribus.* The optimal policy according to Proposition 1 requires $c^* = c_{\min}$. This implies the optimal plan is egalitarian, at least with respect to consumption, even though no inequality aversion appears explicitly in the HDI itself. Though income inequality may still remain in a market economy, consumption inequality is what matters since a government following the optimal plan with access to non-distortionary taxation would tax away all disposable income leaving $c^* = c_{\min}$.[7]

To make the argument formally, we model individuals and derive how the planner would allocate expenditures for them. Suppose there are $i = 1, 2, \ldots N$ individuals in the economy and each has a corresponding consumption constraint

$$c_i \geq c_{\min}(i) \qquad (4')$$

Further, education and life expectancy expenditures and attainments might be distinguished by individual:

$$e_i = e_i(E_i, H_i, E, H) \qquad (2')$$

$$l_i = l_i(E_i, H_i, E, H) \qquad (3')$$

where these functions are non-decreasing in all their arguments (and strictly increasing in at least one argument) and include aggregate values

$$E = \frac{1}{N}\sum_{i=1}^{N} E_i \text{ and } H = \frac{1}{N}\sum_{i=1}^{N} H_i$$

to capture any external effects. The planner only cares about individual allocations insofar as they improve average output, education and life expectancy:

[7] In practice, the government may only have distortionary tax instruments, in which case it we would have a second-best problem. In order to maintain a high level of income, the government would have to set taxes in a way that leaves those with higher incomes greater disposable income. We have chosen to not fleshing out the second-best problem as this would involve specifying a detailed microstructure to the problem and the particular results would depend on the particular microstructure used. Second-best problems in taxation are well known to limit the ability of government to implement allocations.

$$y=f(e,l)\ ,\ e=\frac{1}{N}\sum_{i=1}^{N}e_i\ ,\ \text{and}\ \ l=\frac{1}{N}\sum_{i=1}^{N}l_i\ .$$

The following proposition considers homogenous individuals. In our model, individuals are homogenous if they have the same minimum consumption needs, $c_{min}(i) = c_{min}$, and their education and life expectancy functions are of the same form, $e_i = e(E_i, H_i, E, H)$ and $l_i = l(E_i, H_i, E, H)$.

Proposition 2. *Consider an economy of homogenous individuals.*

(a) *Maximizing the HDI requires setting* $c_i^* = c_{\min}$ *for all individuals so that the society is egalitarian with respect to consumption.*

(b) *Incremental reductions in average consumption increase the HDI score but need not be consumption inequality reducing.*

(c) *Maximizing the HDI yields completely egalitarian outcomes for consumption, education and life expectancy, when the education and life expectancy attainment functions are strictly concave.*

Cases 2(a) and 2(b) are generalizations of Proposition 1. However, there is an important proviso in 2(b): incremental reductions in average consumption may not reduce inequality. This follows simply from the fact that a reduction in average consumption can be achieved by reducing the consumption of a subset of individuals. The scope for dispersion in individual consumption narrows as average consumption approaches the minimum. Proposition 2(c) shows that the society is completely egalitarian when there are diminishing returns to individual expenditures on education and health.[8] Overall, Proposition 2 indicates that, if governments use the existing HDI as an objective function to devise their plans, then this leads to equitable outcomes – through the implied emphasis on maximizing funding to education and health.

Observe that our complete egalitarian result is due to assumptions on functional forms and does not impose horizontal equity. Of course, deviations from egalitarianism

[8] The analysis assumes that education and health are to an extent rivalrous. If they are considered pure public goods, $e_i = e(E,H)$ and $l_i = l(E,H)$, egalitarian outcomes arise without assume diminishing returns. If education and health facilities are equally accessible to everyone in the economy, perhaps because of their public good nature, then maximizing the HDI implies equality of treatment though not necessarily outcomes among heterogeneous individuals.

would be optimal to the extent that agents are heterogeneous. However, since the index only incorporates averages, optimal plans lead to equality of treatment in the sense that the planner doesn't care about the identity of individuals except for identifying their consumption, education and health needs. Recall that the minimum consumption was motivated as being sufficiently high such that consumption did not impact the education, health or production functions. If the constraint were relaxed (as described in Section 3), the optimal plan would allocate consumption instrumentally by equating and individual's marginal benefit of the consumption (in terms of the increase in education, health and production) to the marginal resource cost.

5. A Critique of the Role of Income

The income index $I^y(y)$ in the HDI was originally justified as an indirect proxy of "command over resources to enjoy a decent standard of living" (Human Development Report 1990, p. 1). Here we argue that income is a poor proxy for this purpose.

The emphasis on education and health expenditures in optimal plans naturally leads us to consider what role income plays in the HDI. In the optimal plan, given that c^*= $c_{\min}$, the remaining problem of how to allocate resources to *E* and *H* is affected by $I^y(y)$ only because of the effects of *E* and *H* on production, *indirectly* through life expectancy $l(E,H)$ and education $e(E,H)$. By way of contrast, both $l(E,H)$ and $e(E,H)$ have *direct* impacts on the indexes $I^e(e)$, and $I^l(l)$ respectively. This reasoning is formalized in the following proposition.

Proposition 3.

a) *If $f_l = f_e = 0$ then the weight w on the income index $I^y(y)$ in the HDI plays no role in determining the optimal plan.*

b) *If $f_l > 0$ or $f_e > 0$ then the weight w on the income index $I^y(y)$ in the HDI affects only the trade-off between expenditures on education E and health H.*

In case *3a)* changing *w* has no affect on the optimal plan. The key parameter is *W*, the relative weight on education versus health. In the HDI, the education and income indexes have the same weight, *W* = (1 - *W*) = ½. However, this does not imply that expenditures are equal on education and health. First, the resource constraint $E+H = f - lc_{\min}$ reveals that increasing in life expectancy have the cost of overall increasing consumption. This feature discourages expenditures that enhance life expectancy compared to education. Second, the education and health indexes are not necessarily symmetric nor are the achievement functions $e(E, H)$ and $l(E, H)$. In case *3b)* the particular way that changing w affects the trade-off between E and H is complicated and ambiguous (as it depends on all partial derivatives of the functions).

Another way to see the role of income is to examine how increases in output are apportioned. In the basic model $y = l \cdot c_{\min} + E + H$ and income growth does not affect the individual rate of consumption, c. Rather, $\Delta y = \Delta(E+H) + \Delta l \cdot c_{\min}$, so that output growth is correlated with expenditures $E+H$ and longevity l. Thus, the income component leads to the direct double counting of life expectancy. As the income component counts the inputs that increase $e(E, H)$ and $l(E, H)$, it indirectly double counts education and life expectancy.

Investment

The simple model we have examined so far is static and does not include capital expenditures. In lieu of developing a full-blown dynamic model, we now briefly consider an extension where a fraction of current output is required for production. We think of this as investment infrastructure S in the planning problem. The production function now includes S:

$$y = f(e, l, S) \qquad (5')$$

To simplify the analysis we assume that f is a strictly concave function and also separable in S. As expenditures on S must come from national income:

$$lc + E + H + S \leq y \qquad (6')$$

It is quite evident that Propositions 1 and 2 would be unaffected by this change since, as before, consumption enters the planner's problem only as a cost. All other non-investment income would be allocated to education and health. The result on equality of treatment would be unchanged. However, the role of income now changes as it is directly affected by investment and doesn't just affect the trade-off between E and H as described in Proposition 3. In the extended model $y = lc_{\min}+E+H+S$. Now the output component of the index, y, is related to S, a variable that does not enter the either the education or life expectancy components of the HDI. The income affects the level of optimal investment, S^*, as described in the following proposition.

Proposition 4. *Increasing in the weight on income, w, in the HDI, increases the level of optimal investment, S^*, and income, y^*, and reduces the measure of attainment on education and life expectancy, $WI^e(e^*)+(1-W)I^l(l^*)$, found in the HDI. Consumption is unaffected, $c^* = c_{\min}$* .

With investment in the model there is a direct trade off between income and attainment on education and life expectancy. This trade off does not involve consumption, which is fixed in the optimal plan. The increase in investment increases income but reduces expenditures on education and health. This reveals another weakness of the HDI. The extra investment is wasted in the sense there is a reduction in the part of the index that excludes output and there is no increase in the rate of consumption. With the income component, some investment takes place for increasing output for its own sake rather for valued ends.

6. Considerations in Modifying the HDI

This section first examines a simple modification of the income component of the HDI that isn't subject to the critique of "double counting". We then examine whether this modified component is a good proxy for a "command over resources to enjoy a decent standard of living". Finally, we argue that this criterion is best meet by modifying the HDI by replacing the income component with a consumption component. However, the consumption component has to be carefully selected if it is to be a good proxy.

Recall that double counting arose because the optimal plan resulted in increases in the income component being matched by increases in education and health expenditures.

As these expenditures are the inputs into the education and life expectancy indexes, the income component indirectly emphasized education and life expectancy. A simple way to avoid the double counting is to replace income with "net income", income less expenditures on education and health; i.e. y-(E+H).

In the basic model, net income is the same as total consumption, $lc=y-(E+H)$. When either lc or c consumption is used instead of income in the objective function, Proposition 1 no longer obtains. By putting more structure on the problem (e.g. concave objective function and convex constraints), it is straightforward to find an internal solution $c > c_{\min}$ where the planner trades off expenditures on education and health for more net income.

In the extended model with investment, net income is no longer the same as consumption, $lc + S = y - (E+H)$. Using consumption in an appropriately modified planner's problem, can yield an internal solution like above. In contrast, using net income in the index yields optimal plans that yield less consumption, possibly at the corner solution $c = c_{\min}$. When net income is in the index, the planner chooses investment over consumption because investment is an input to production. Accordingly, increases in net income largely proxy investment.

The choice of which is a better replacement for income comes down to whether the index is meant to reflect means or ends. Infrastructure is a means to an end, as it is valuable for its instrumental value in production. Consumption is a valued end in itself. As the other two components of the index, education and life-expectancy are meant to proxy things that are valuable in themselves rather than for their instrumental value in production, it follows that consumption is the better replacement.

Whereas using average consumption in a modified index can yield internal solutions, it can also yield outcomes that are less equitable. This is because when optimal average consumption is above the minimum, $c^* > c_{\min}$, there is room for dispersion of income. For example, it is possible for half of individuals to be at the minimum and the other half to be at $c^* + (c^* - c_{\min})$. This possibility is not possible under the HDI criterion as optimal plans require average consumption to $c^* = c_{\min}$ forcing all individual consumption to also be at the same level, $c_{\min}$. This analysis contrasts inequality under different optimal plans. The story is different when the reduction in consumption must be incremental and limited to changes well away from the optima. Then the scope for inequality would be the same. Ideally, a modified HDI index should incorporate distribution information (e.g. Gini coefficient).

Considerations in choosing a consumption proxy for 'command over resources'

Our analysis indicates that the consumption variable we want excludes expenditures on education, health and capital expenditures. In our model, the feasibility constraint implies $lc = y - E - H - S$. Rearranging, $lc + E + H = y - S$, we have uses for resources on the left hand side and sources on the right hand side. The planner would chose investment, S, to maximize available resources, $y - S$ for division among consumption, education and health expenditures.

Our model is static and the feasibility constraint can be interpreted as a lifetime constraint, where lc is lifetime consumption and c is the rate of consumption per unit time. A question arises which is the better indicator. We choose c because lc includes life expectancy, which is a component index in the HDI. Counting l in years, c would be average yearly per capita income over a lifetime and accordingly is best thought of as "permanent income". Similarly, both E/l and H/l should be thought of a permanent expenditures expressed on an annual basis. Our variable S is best though of as the capital stock and S^*/l^* as the investment to maintain the optimal capital stock S^* in a steady state. The static optimal plan is akin to the golden-rule steady state analysis.

Our formulation does not allow for running down the capital stock to increase consumption without a reduction in output. As a practical matter this alerts us that using current consumption in a modified index leaves open a short-term temptation to increase consumption today at the expense of the future. Any modified index that uses current consumption data should also makes some provision for investment that maintains or enhances future consumption.

Another issue is that our allocation model did not distinguish between public and private sectors. Were we to model a simple market economy with a government sector, we would find that consumption expenditures whether they were in the private or public sectors should be aggregated together and physical capital in so far as it is productivity enhancing should also be aggregated independent of sector.[9]

7. Conclusion

The HDI is a widely cited statistic that is commonly used as a measure of well-being in different countries. Here, we have examined some of the implications that follow if government planners decide to use maximization of the HDI as a criterion for optimal plans. We have found that, if they do so, planners will tend to heavily emphasize expenditures on education and health by lowering consumption. This eventually leads the economy towards a more egalitarian allocation – even though inequality aversion does not appear explicitly in the HDI itself. Another interesting feature of the optimal plan is that the income component in the HDI only plays a direct role for guiding investment, but the outcome is excessive investment.

To generate more reasonable optimal plans we consider modifications of the HDI index. We argue for a modified HDI that replaces the income component with a permanent consumption component that excludes expenditures on education and health. The modified index captures direct trade-offs between allocations for consumption, education and health, and the optimal plan yields a balance of expenditures on the three components. We also argue that using our consumption component is a better proxy for command over resources and capabilities in the spirit of the human development approach.

Our analysis falls short both theoretically and empirically in a number of ways. Our simple static normative analysis ignores capital accumulation and growth issues and does not look at the positive issues around incentive and participation constraints. Empirically, we have yet to develop our modified HDI index and examine its properties. We intend to pursue this research in the future.

[9] A benevolent optimizing government with access nondistortionary taxation would publicly provide goods (rivalrous or nonrivalrous) up to the point where the marginal value of the funds was the same in both private and public sectors. A large part of government expenditures is often on the military. It is not clear if these expenditures, capital or services, should be included in a proxy for 'command over resources' to enhance basic human development capabilities.

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APPENDIX

Proof to Proposition 1. The Lagrangian for problem (P1) is:

$$L_{HDI} = wI^{y}(f(l(E,H),e(E,H))) + (1-w)\left(WI^{e}(e(E,H)) + (1-W)I^{l}(l(E,H))\right)$$
$$+\lambda_1\left(f(l(E,H),e(E,H)) - E - H - l(E,H)c\right) + \lambda_2\left(c - c_{\min}\right) \qquad (7)$$

Among the Kuhn-Tucker conditions for are the following:

$$c(\lambda_2 - \lambda_1 l(E,H)) = 0 \qquad (8)$$

$$\lambda_2(c - c_{\min}) = 0 \qquad (9)$$

$$\lambda_1 \geq 0, \quad \lambda_2 \geq 0 \qquad (10)$$

We now show that at an optimum $\lambda_2 > 0$. Suppose not. Then, by (10), $\lambda_2 = 0$. By (8), since $l(E,H) > 0$ and $c > 0$, this implies that $\lambda_1 = 0$. Since the objective function is strictly increasing in *E* and *H*, the resource constraint (6) binds, and so $\lambda_1 > 0$. This is a contradiction. Thus, $\lambda_2 > 0$. By (9), this then implies that $c = c_{\min}$. It remains to check the constraint qualification when constraint (ii) in problem (P1) is not binding. If (i) is also not binding the constraint qualification is trivially satisfied. If (i) is binding then the gradient vector to (i) is $\nabla g_1(c, E, H) = (l, l_E c + 1 - f_l\, l_E - f_e\, e_E, l_H c + 1 - f_l\, l_H - f_e\, e_H)$ for $c > c_{\min}$. When there is only one vector, it is linear dependent only if $\nabla g_1(c, E, H) = (0, 0, 0)$. The assumption in (3), $l(0,0) > 0$, rules this out. Even without this assumption, the gradient must be linearly independent. A necessary condition for linear dependence is $l_E c + 1 - f_l\, l_E - f_e\, e_E = 0$. But this condition cannot correspond to local maximum because decreasing *c* to increase E and/or H along the constraint increases the objective function. Hence, the constraint qualification is satisfied and the Kuhn-Tucker conditions are necessary for a maximum. Thus, at an optimum $c^* = c_{\min}$.

Suppose policy limits the level to which consumption can be reduced to some level c_p, where $c_p > c_{\min}$. Then the planner's problem yields a Lagrangian that is the same as (7) except for c_p replacing $c_{\min}$. As above $\lambda_2 > 0$. Thus, the marginal value of restricting the policy level c_p further is $\frac{dL_{HDI}}{dc_p} = -\lambda_2 < 0$. It follows that reductions in c_p increase the Lagrangian, which is just the constrained optimal *HDI* score. ■

Proof to Proposition 2. The Lagrangian for this problem is:

$$\boldsymbol{L_{HDI}} = wI^y(f(l,e)) + (1-w)\left(WI^e(e) + (1-W)I^l(l)\right)$$
$$+\lambda_1\left(f(l,e) - E - H - lc\right) + \sum_{i=1}^{N} \lambda_{2i}\left(c_i - c_{\min}\right) \qquad (7')$$

Among the Kuhn-Tucker conditions for are the following:

$$c_i\left(\lambda_{2i} - \lambda_1 l / N\right) = 0 \qquad (8')$$

$$\lambda_{2i}\left(c_i - c_{\min}\right) = 0 \qquad (9')$$

$$\lambda_1 \geq 0, \quad \lambda_{2i} \geq 0 \qquad (10)$$

a) This is a generalization of the proof to Proposition 1 and is identical except for having to show that $\lambda_{2i} = \lambda_2 > 0$ for all *i* at an optimum.

b) The incremental result with respect to average consumption is also a generalization Proposition 1 but with individual values of $c_{pi} > c_{\min}$. The possibility of increasing inequality is demonstrated in the text.

c) Given the strict concavity of the attainment functions e_i and l_i, it follows that the Lagrange multipliers corresponding to E_i and H_i must be the same across individuals which requires equal allocations and outcomes.

Proof to Proposition 3.

a) Consider an alternative objective function where only the indexes $I^e(e)$ and $I^l(l)$ have weight: $I(\hat{W}) = \hat{W}I^e(e(E,H)) + (1-\hat{W})I^l(l(E,H))$. When $f_l = f_e = 0$ then $I(w,W)$ is simply an affine transformation of $I(W)$.

b) By Proposition 1, $c = c_{\min}$. Problem P1 is therefore equivalent to the following problem, P2, which determines the choices of *E* and *H*:

$$\underset{\{E,H\}}{Max}\, I(w,W,c_{\min}) = wI^{y}(f(l(E,H),e(E,H))) + (1-w)\left(WI^{e}(e(E,H)) + (1-W)I^{l}(l(E,H))\right)$$

subject to: $l(E,H)c_{\min} + E + H - f\left(l(E,H),e(E,H)\right) \le 0$ ■

Proof to Proposition 4. The optimal condition for infrastructure is

$$w\frac{\partial I^{y}}{\partial y} f_{S} = \lambda_{1}(1 - f_{S}) \quad ,$$

where λ_1 is the Lagrange multiplier on income. This first order condition requires that $f_S < 1$ when $w > 0$. In contrast, when income has zero weight in the index, $w = 0$, then $f_S = 1$. The optimum infrastructure for this later problem, where $w = 0$, is $S^*(0)$. As *f* is a strictly concave function separable in *S*, the difference $S^*(w) - S^*(0) > 0$ is increasing in *w* for $w > 0$. Conversely, the weighted level of attainment on education and life expectancy, $WI^{e}(e^{*}) + (1-W)I^{l}(l^{*})$, is at its maximum when $w = 0$ and is declining in *w*.