Participation and tacit collusion

PATRICK DE LAMIRANDE[a], JEAN-DANIEL GUIGOU[b]
and BRUNO LOVAT[c]

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Abstract

We use the model developed by Clayton and Jorgensen (2005) to analyze the effect of cross holding on collusion. Their model consists to a duopoly competing in quantity on a differentiated good market. Each firm sets the level of the equity position in each other's non-voting stock and the quantity to produce. We find a high level of collusion cannot be sustained very easily. However, when goods are perfect substitute, tacit collusion may more easily arise when firms plan to punish a deviation by playing a non-symmetric equilibrium. Consequently, antitrust laws can increase the likelihood of tacit collusion by limiting long position or limiting short position.

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[a]Department of Financial and Information Management, Shannon School of Business, Cape Breton University, 1250 Grand Lake Road, Sydney, Nova Scotia, B1P 6L2, Canada, e-mail: patrick_delamirande@capebretonu.ca.

[b]Université du Luxembourg, UMR CNRS.

[c]Université de Nancy 2, BETA UMR CNRS.

1 Introduction

Many papers study the impact of cross holding on firms profits. Reynolds and Snapp [6] and Farrell and Shapiro [2] demonstrate that positive cross holding decreases the total quantity produced and increases firms profits relatively to the no cross holding situation. Furthermore, Farrell and Shapiro study cases where increasing positive cross holding can be optimal. However, Flath [3] finds that this result depends on the assumption that firms already use positive cross holding. Assuming that the equity market is efficient, if the initial point is no cross holding, then it is never optimal to increase cross holding.[1] Moreover, Clayton and Jorgensen [1] demonstrate that positive cross holding by both firms is not a Nash-equilibrium. For example, one firm can increase its profits by choosing no cross holding when the other firm has positive cross holding.

In their paper Clayton and Jorgensen go further by solving the problem of cross holding when firms can use positive or negative cross holding. They demonstrate that firms face a prisoner dilemma. The unique Nash-equilibrium involves negative cross holding.This leaves firms with less profits than in the no cross holding situation. However, the authors do not discuss about the possibility of collusion under a cross holding framework. The objective of this paper is to explore the possibility of tacit collusion when cross holding (positive or negative) is possible.

Cross holding can influence the potential benefit to collude. Many authors suggests that cross holding facilitates collusion. In fact, the net effect of cross holding on tacit collusion is more ambiguous. On one side, deviation is more attractive in the short term since using negative cross holding gives a strategic advantage to the deviating firm. On the other side, punishment is more severe. Negative cross holding leaves firms with less profits. Consequently, the net effect

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[1]This result holds for the Cournot model. When firms compete in price, positive cross holding is optimal and increases profits.

can goes in either direction. [2]

In this paper, we use the model developed by Clayton and Jorgensen [1] to investigate the effect of cross holding on tacit collusion. In their model, two equity firms control two variables: the quantity to supply and the level of participation in the other firm equity. However, the two variables are not decided at the moment. At the first stage, firms decide the level of participation in the other firm equity. At the second stage, the quantity is set. By choosing cross holding in the first stage, equity firms can manipulate the objective function of managers. To study the possibility of tacit collusion, we introduce dynamic. We repeat infinitely the Clayton and Jorgensen's model.

Our main finding is that the effect of cross holding on tacit collusion is ambiguous. If firms collude at high level, then this collusion is not sustainable for average discount values. When firms use positive cross holding intensively, the gain from deviating are important. Consequently, even if the punishment is more important, it seems that the gain of deviation exceeds the loss from the punishment. Consequently, tacit collusion is difficult to maintain when positive cross holding is allowed. However, if positive cross holding is limited (by firms themselves or by antitrust agencies), then tacit collusion is more likely to be maintained. This result goes in the opposite direction of the traditional belief that cross holding can facilitate collusion.

The paper is divided as follow. In Section 2, we present the model and introduce our assumptions. We solve the shareholder problem in every step of tacit collusion in Section 3. Section 4 covers our results. In Section 5, we discuss about how the initial level of collusion is set. Section 6 concludes.

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[2]For more discussion on the effect of cross holding on collusion, see Malueg [5] and Gilo et al. [4].

2   Model

Consider two all-equity firms (equity firms) which own shares of two firms (producing firms). The two producing firms involve on the same market and produce a differentiated good. Each equity firm owns all shares of one producing firm. However, it is possible for equity firms to buy a position in the other producing firm. The position can be a long position (positive) or a short position (negative). We assume that the position hold for one period. Positioning is chosen before taking decisions on production.

Firms are involved in an infinite horizon noncooperative game. At each time period, there are two stages. At the first stage, equity firms choose simultaneously the position in the other producing firm that maximizes their equity value. Equity firms decide a different position at the beginning of each period. At the second stage, managers of producing firms observe the position of both equity firms and set the quantity maximizing the equity value of their shareholders. Since equity firms can buy a position in the other producing firm, maximizing the equity value of the producing firm does not tie to maximizing the equity value of equity firms. Consequently, managers face an externality in their decision. In this paper, as in Clayton and Jorgensen [1], we assume that managers know about this externality and maximize the equity value of their shareholders.

We denote the position of firm i in firm j's equity at period t by θ_i^t. The value and the cost of position in firm j at period t is given respectively by PV_j^t and PC_j^t. We can write the equity firm's problem as follows.

$$\max_{\{\theta_i^t \in [-1,1]\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \delta^t \left[\pi_i \left(q_i^t, q_j^t \right) + \theta_i^t \left(PV_j^t - PC_j^t \right) \right]$$

where δ be the discount factor and π_i is the profit function of producing firm i.

We have already assumed that positions hold for one period. Consequently, the benefit from position is given by the difference between the current equity value of the producing firm minus the equity value of the producing firm at the beginning of the next period evaluated at the current period. Since the equity value of firm is the sum of present and discounted future profits, we can write the value of position in firm j at time τ as

$$
\begin{aligned}
PV_j^\tau &= EV_j^\tau - \delta EV_j^{\tau+1} \\
&= \sum_{t=\tau}^{\infty} \delta^{(t-\tau)} \pi_j \left(q_j^t, q_i^t \right) - \sum_{t=\tau+1}^{\infty} \delta^{(t-\tau)} \pi_j \left(q_j^t, q_i^t \right) \\
&= \pi_j \left(q_j^\tau, q_i^\tau \right)
\end{aligned}
$$

As Clayton and Jorgensen [1], we assume that equity positions are publicly disclosed and the equity market is competitive. Consequently, the cost of position equals the value of position. This means that equity firm profits are equal only to

$$
\max_{\{\theta_i^t \in [-1,1]\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \delta^t \left[\pi_i \left(q_i^t, q_j^t \right) \right] \tag{1}
$$

We see that equity firm profits do not depend directly on their decision on position. However, by using position, you can influence manager's decisions. In fact, managers consider the cost of position as a fixed cost. Since managers set quantities after equity firms decide positions, they consider the cost of those positions as constant and do not consider this cost in their decision. Therefore, they take into account the value of the position when they will take decisions about production. Explicitly, producing firm i manager want to maximize

$$
\max_{\{q_i^t\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \delta^t \left[\pi_i \left(q_i^t, q_j^t \right) + \theta_i^t \pi_j \left(q_j^t, q_i^t \right) \right] \tag{2}
$$

As any multiple stage problem, we solve our problem using backward in-

duction. We start by analyzing the manager's problem. Also, we assume that managers can not collude. This assumption can be explained by assuming that managers change at each period.

Throughout this paper, we assume that inverse demands are constant over time and are given by:

$$P_i^t = a - q_i^t - \beta q_j^t \qquad i = 1, 2 \quad j \neq i$$

with $\beta \in [0, 1]$.[3] Production costs are represented by a linear function with $c < 1$ as the marginal cost.

Solving the manager problem for both producing firms, we find that firm i's quantity at any period

$$q_i^* \left(\theta_i^t, \theta_j^t \right) = \frac{\left(2 - \left(1 + \theta_i^t \right) \beta \right) (a - c)}{4 - \left(1 + \theta_i^t \right) \left(1 + \theta_j^t \right) \beta^2} \qquad i = 1, 2 \quad j \neq i \qquad (3)$$

If we introduce 3 into 1, then we find

$$\max_{\{\theta_i^t \in [-1,1]\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \delta^t \left[\frac{\left(2 - \theta_j^t \theta_i^t \beta^2 - \theta_i^t \beta^2 + \theta_i^t \beta - \beta \right)}{2 - \left(1 + \theta_i^t \right) \beta} q_i^* \left(\theta_i^t, \theta_j^t \right)^2 \right] \qquad (4)$$

3 Shareholder problem

Now, we have to consider the equity firm problem. Taking into account that managers will maximize the equity value of equity firms, shareholders must choose the position solving 1. Since we face a multiperiod game between same players, they could have incentives to collude together. If equity firms want to collude, then they must gain from it and have incentives to do not deviate from the collusion agreement. Three factors determine if collusion can be sustainable:

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[3]By assuming that demand functions are constant over, we assume that a and β are constant. Also, we restrict our attention to the case where goods are substitute.

profits from collusion, the potential gain from deviation before detection and profits during the punishment phase. If we assume that the detection take one period and the punishment is playing a Nash-equilibrium forever, then the minimum discount such that collusion can be maintained is given by

$$\delta^* = \frac{\pi_i^d - \pi_i^c}{\pi_i^d - \pi_i^p} \tag{5}$$

where π_i^d is profits gained by firm i when firm j plays collusion and firm i deviates, π_i^c is profits from collusion and π_i^p is profits when firm i is punished. To determine if collusion can be sustained, then we must calculate profits in the three cases.

3.1 Collusion

If there is no constrain on the level of position, then equity firms will set their position such that their joint profits are maximized. Since each period are identical, we can use 3 to solve the equity firm problem.

$$\max_{\theta_1, \theta_2 \in [-1,1]} \quad \pi_1 \left(q_1^* \left(\theta_1, \theta_2 \right), q_2^* \left(\theta_2, \theta_1 \right) \right) + \pi_2 \left(q_2^* \left(\theta_2, \theta_1 \right), q_1^* \left(\theta_1, \theta_2 \right) \right) \tag{6}$$

Let θ^c be the solution to 6. Explicitly, it is easy to show that the optimal value for θ^c is 1 which represents the case where equity firms buy 100% of position in the other firm. However, this strategy can be blocked by antitrust agency. Consequently, to test what is the impact of such constrain on collusion, let θ^c vary between 0 (the case where no long position is permitted) and 1 (the case where there is no constrain). Calculating equity firm profits when firms collude with a level of position equals to θ^c, given by $\pi^c \left(\theta^c \right)$, we find

$$\pi_i^c \left(\theta^c \right) = \frac{\left(1 + \theta^c \beta \right) \left(a - c \right)^2}{\left(2 + \beta + \theta^c \beta \right)^2} \tag{7}$$

If there is no constrain on long position, then firms will choose $\theta^c = 1$ which maximize their joint profits.

3.2 Deviation

To find the best deviation, we use the reaction function for the firm i when firm j plays θ^c. To find the reaction function, we must solve the equity firm problem 1. Since $|\theta_i| \leq 1$, we find that

$$
\begin{aligned}
\theta_i^* \left(\theta^c\right) &= -\frac{(1+\theta^c)(2-\beta)\beta}{4-\beta(1+\theta^c)(2+\beta)} \quad \text{if } \theta^c < \frac{1-\beta}{\beta} \\
\theta_i^* \left(\theta^c\right) &= -1 \quad \text{if } \theta^c \geq \frac{1-\beta}{\beta}
\end{aligned} \tag{8}
$$

Let $\pi_i^d \left(\theta^c\right)$ be the firm i profits when firm j plays θ^c and firm i deviates. Then, equity firm i profits when deviating from collusion is

$$
\begin{aligned}
\pi_i^d \left(\theta^c\right) &= \frac{(2-\beta)^2(a-c)^2}{8(2-\theta^c\beta^2-\beta^2)} \quad \text{if } \theta^c < \frac{1-\beta}{\beta} \\
\pi_i^d \left(\theta^c\right) &= \frac{\left(2-2\beta+\beta^2+\theta^c\beta^2\right)(a-c)^2}{8} \quad \text{if } \theta^c \geq \frac{1-\beta}{\beta}
\end{aligned} \tag{9}
$$

3.3 Punishment

Now, we must discuss about the consequence of deviating. In other words, we must know what would be the strategy after the deviation. The common assumption used in papers on tacit collusion is that players return to a Nash-equilibrium when one cartel member deviates.

To find the Nash-equilibrium, we use equity firm reaction functions. In their paper, Clayton and Jorgensen (2005) found the Nash-equilibrium for the equity firm problem when $\beta \in (0,1)$. The optimal position and profits at the

Nash-equilibrium are respectively

$$\theta_i^p = -\frac{\beta}{2+\beta} \tag{10}$$

$$\pi_i^p = \frac{\left(4-\beta^2\right)}{(1+\beta)}\frac{(a-c)^2}{16} \tag{11}$$

The other cases, are not covered by Clayton and Jorgensen. The first case is $\beta = 0$, i.e. goods are independent. This case is trivial. firm i profits are unrelated to the other firm. Consequently, cross-holding cannot increase (or decrease) profits. The other case is more interesting. When $\beta = 1$, there is no more a unique Nash-equilibrium. Now, we faces a continuum of Nash-equilibrium given by $\theta_i^p = \frac{\theta_j^p+1}{3\theta_j^p-1}$. Consequently, firms can use punishment to diminish the benefit to deviate. By choosing a θ_j^p close to -1, firm j diminishes profits during punishment and increases the possibility to maintain collusion. In that sense, negative position can improve the probability of tacit collusion. Furthermore, the strategy to punishment severely deviation is credible. Since a hard punishment is a Nash-equilibrium, the credibility of that punishment is high.

If firm j decides to punish by choosing θ_j^p, firm i's profits equal

$$\pi_i^p = \frac{(a-c)^2}{8\left(1-\theta_j^p\right)} \tag{12}$$

4 Tacit collusion

The net impact on cross-holding on tacit collusion depends on the magnitude of the cross-holding effects on profits during the collusion phase, the deviation phase and the punishment phase. We have found above profits during those three phases. By introducing 7, 9, 11 and 12 in 5, we are now able to compute the minimum discount rate such that collusion can be maintained. For the case

$\beta \in (0, 1)$, we obtain

$$
\begin{aligned}
\delta^* &= \frac{2(\beta+1)(\beta+\beta\theta^c+2\theta^c)}{(2-\beta)(2+\beta+\beta\theta^c)^2} \quad \text{if } \theta^c < \frac{1-\beta}{\beta} \\
\delta^* &= \frac{2(\beta+1)(\theta^c+1)\left((\beta\theta^c)^2+2\theta^c+2\theta^c\beta^2+2\beta\theta^c+2\beta+\beta^2-2\right)}{(2\beta+2\beta\theta^c+2\theta^c-1)(2+\beta+\beta\theta^c)^2} \quad \text{if } \theta^c \geq \frac{1-\beta}{\beta}
\end{aligned}
\tag{13}
$$

If we set θ^c to different values (0, 0.1, 0.2, 0.5, 1), we are able to draw the minimum threshold for collusion. The following graph shows the minimum threshold for collusion for different value of the level of collusion in function of the differentiation factor β.
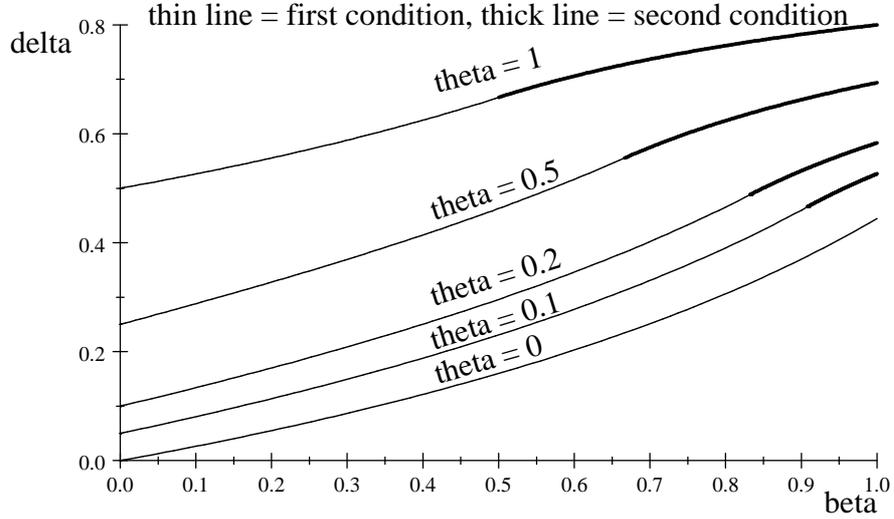

Figure 1: Threshold $\beta < 1$

Many things appears on the previous graph. First, it is clear that, if the differentiation diminishes (β approaches 1), then collusion is harder to maintain. It seems that, with β close to 1, the potential gain from deviation exceeds the effect of the punishment. Even if the benefit from collusion is more important in the case where goods are close substitute, the short term gain from deviation is high enough to diminish the possibility of tacit collusion. Second, when the

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initial level of collusion increases (θ^c increases), tacit collusion is more difficult to maintain. If the initial level of collusion is higher, then the potential gain from deviation is greater than the potential loss from the punishment. Consequently, firms have more incentive to deviate.

For the case of perfect substitutes ($\beta = 1$), the threshold depends on two variables: the level of the cross-holding during collusion (θ^c) and the level of cross-holding during the punishment phase (θ_j^p). Explicitly, we have

$$\delta^* = \frac{\left(1 - \theta_j^p\right)\left(6\theta^c + (\theta^c)^2 + 1\right)(1 + \theta^c)}{\left(\theta^c - \theta^c\theta_j^p - \theta_j^p\right)\left(3 + \theta^c\right)^2} \quad \text{with } \theta_j^p \in (-1, 0) \qquad (14)$$

As we do for the case of imperfect substitute, we draw the threshold in function the punishment for different values of θ^c.
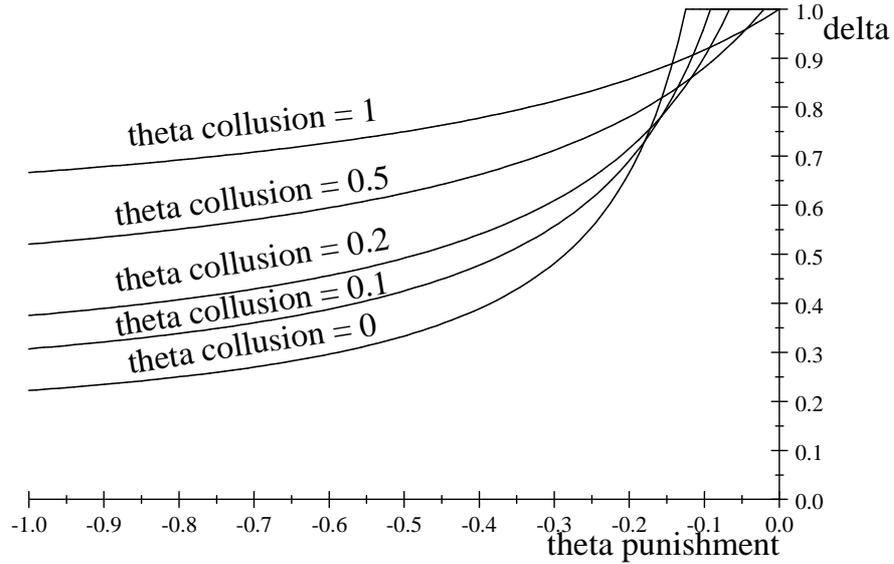

Figure 2: Threshold $\beta = 1$

From the previous graph, three things appear. First, when punishment

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is not strong, the threshold which can maintain tacit collusion is close to 1. Furthermore, in the case of a low level of collusion, if the punishment is not strong enough, tacit collusion can not be maintained (δ^* equals 1). Second, as in the case of imprefect substitute, when the level of collusion increases, the benefit from deviation increases also and tacit collusion is more difficult to maintain. Third, the effect of the level of punishment is important, specially when the level of collusion is low. If punishment is high, then a low level of collusion is easier to maintain than a high level collusion. This result is not obvious. It seems that, if the initial level of collusion is high, then the punishment can hardly persuade one firm to do not deviate. On the other side, if the initial level of collusion is relatively low, firms can threat with credibility to punishment deviation strongly and maintain tacit collusion. Consequently, it appears that, if firms want to maintain tacit collusion easily, then they must collude at a low level by choosing a low θ^c.

Now, it could be interesting to compare our result with the threshold obtained in the classic Cournot model. If two firms competing in quantity collude, they would maintain tacit collusion if

$$\delta \geq \frac{(\beta + 2)^2}{8 + 8\beta + \beta^2}$$

Let δ^C be the minimum threshold such that tacit collusion can be maintained in the two-firms Cournot model, i.e. $\delta^C = \frac{(\beta+2)^2}{8+8\beta+\beta^2}$. Graphically, we obtain

Now, by comparing this threshold with thresholds when cross-holding is possible, we are able to confirm that cross-holding affects the possibility of tacit collusion. When the initial level of collusion is not high, cross-holding decreases the threshold such that tacit collusion can be maintained. This is true no matter what is the value of β. Consequently, it appears that the degree of differentiation is not the most important variable determining if tacit collusion can be
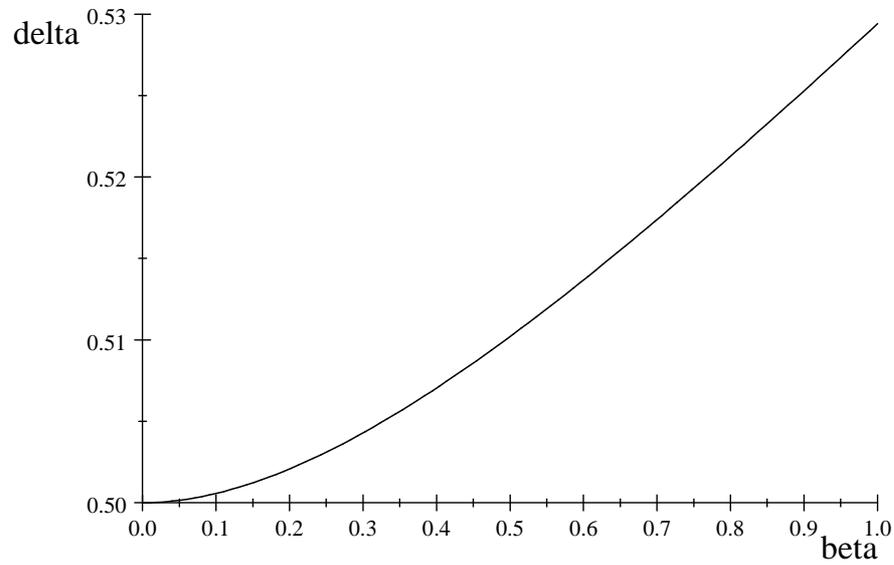
Figure 3: Threshold under the two-firms Cournot model

maintained. Initial level of collusion affects greatly the probability of collusion. In the case of perfect substitute, the impact of the level of punishment is an important but not the most important variable. The initial level of collusion is still the variable with the most impact on collusion.

5   Discussion on the initial level of collusion

What determine the initial level of collusion? Suppose initially that firms can set θ^c freely. If they collude, they will earn the monopoly profit by choosing $\theta^c = 1$. However, this value cannot be sustained very easily. The benefit from deviation is too high relatively to the punishment. Consequently, to maintain collusion, they have to set θ^c lower than 1. But the problem by setting a value lower than 1 is that leaves the impression of potential loss for firms. They would be better off by setting a higher θ^c. So the final situation could be the one where

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firms begin with a low level of collusion, increase this level after some periods of successful collusion and break the collusion since the gain from deviation becomes more important. At the end, we could see a cycle in the collusion process.

Another possibility is the implication of an antitrust agency. Since positive cross-holding has a negative effect on competition, antitrust agencies could put in place some restrictions on cross-holding. They can limit the maximum of positive cross-holding ($\bar{\theta}$). Consequently, if two firms want to collude, they cannot set θ higher than $\bar{\theta}$. By doing that, antitrust agencies could facilitate collusion since they eliminate the possibility to increase cross-holding above the limit allowing tacit collusion. As a consequence, the cycle of collusion and no collusion can be broken and collusion can be maintained indefinitely.

6 Conclusion

To be added

References

[1] M. J. Clayton and B. N. Jorgensen. Optimal cross holding with externalities and strategic interactions. *Journal of Business*, pages 1505–1522, 2005.

[2] J. Farrell and C. Shapiro. Asset ownership and market structure in oligopoly. *RAND Journal of Economics*, pages 275–292, 1990.

[3] D. Flath. When is it rational for firms to acquire silent interests in rivals? *International Journal of Industrial Organization*, pages 573–583, 1991.

[4] D. Gilo, Y. Moshe, and Y. Spiegel. Partial cross ownership and tacit collusion. *RAND Journal of Economics*, pages 81–99, 2006.

[5] D. A. Malueg. Collusive behavior and partial ownership of rivals. *International Journal of Industrial Organization*, pages 27–34, 1992.

[6] R. J. Reynolds and B. R. Snapp. The competitive effects of partial equity interests and joint ventures. *International Journal of Industrial Organization*, pages 141–153, 1986.