

# Optimal Monetary Policy and Price Stability over the Long Run<sup>\*†</sup>

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February 21, 2007

## Abstract

What is the role of monetary policy in an environment with aggregate risk, incomplete markets and long-term nominal bonds? In a two-period overlapping-generations model with aggregate uncertainty and nominal bonds, optimal monetary policy attains the ex-ante Pareto optimal allocation. This policy aims to stabilize the savings rate in the economy via the effect of expected inflation on real returns of nominal bonds. The equilibrium under optimal monetary policy is characterized by positive average inflation, positive correlation between expected inflation and income, inverse relationship between volatility of expected inflation and persistence of income, and nonstationary price level. These properties of the optimal policy have strong implications for the debate on inflation targeting and price-level targeting policy regimes. The optimal monetary policy has features of both inflation and price-level targeting. Welfare rankings of these targeting policies depend crucially on persistence of income.

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\*The views expressed in this paper are those of the authors. No responsibility for them should be attributed to the Bank of Canada.

†We would like to thank Robert Amano, Steve Ambler, Pierre Duguay, John Murray, Peter Ireland, Larry Jones and Sharon Kozicki for help and advise.

# 1 Introduction

What is the role of monetary policy in an environment with aggregate risk, incomplete markets and nominal bonds? We take a standard two-period overlapping-generations model with aggregate-endowment uncertainty and find that monetary policy can achieve the ex-ante Pareto optimal allocation. The optimal monetary policy that implements the first-best allocation stabilizes savings rates by affecting expected real returns on nominal bonds. It is characterized by positive average inflation, positive correlation between expected inflation and income, inverse relationship between volatility of expected inflation and income persistence, and a nonstationary price level.

The role for monetary policy stems from its effect on savings behavior under uncertainty. When faced with uninsurable endowment risk and a constant rate of return on savings, risk averse individuals try to self insure by varying their savings rates with income. When current income is higher than expected future income, individuals save larger fraction of income to move part of the current windfall to the future. When current income is lower than expected future income, individuals save less trying to take advantage of the anticipated income increases. Furthermore, due to precautionary reasons, on average they save more than would be optimal without uncertainty, or under complete insurance markets. Responsiveness of the savings rate to income fluctuations depends on persistence of income disturbances. When income fluctuations are permanent, there is less incentive to vary savings, than when fluctuations are transitory.

Optimal monetary policy aims to achieve constant savings rate associated with the Pareto optimal allocation in the economy. We emphasize four properties of such a policy. First, with nominal bonds as the only means of savings, monetary policy affects the rate of savings via its effect on expected inflation. Under aggregate uncertainty, risk-averse individuals have an incentive to save more than optimal for precautionary reasons. Positive average inflation serves as a tax on savings, which the monetary authority can use to discourage oversaving. Second, a positive correlation between expected inflation and income implies that when income is high (low) the expected tax on savings is higher (lower). This discourages individuals from varying savings rates to smooth consumption over time and thereby stabilizes the savings rate. The third property is a consequence of the fact that with higher income persistence individuals have less expected variation in income, and therefore less incentive to vary their savings rates. As a result optimal

expected inflation has to vary less to discourage consumption smoothing. In the limit, when the income process becomes a random walk, optimal expected inflation is constant. This implies the inverse relationship between volatility of expected inflation and the persistence of income. Finally, a nonstationary price level is the result of optimal expected inflation being independent of the current or past price levels, i.e. optimal monetary policy is completely forward-looking. It is the expected inflation that matters for savings rates, and not the price level. It is important to stress, that all of these four properties are part of the optimal response to aggregate uncertainty. Without uncertainty, the optimal expected inflation is always zero, and the optimal price level constant.

In our model the structure of the economy is publicly known and the optimal policy is simple to implement. In reality, incomplete information about preferences and about the ways economies work make it hard to know what is the optimal expected inflation at any given point in time. Moreover, as was shown by Kydland and Prescott (1977) monetary policy may suffer from an inflation bias due to a time inconsistency problem. For these reasons, and after the stagflation episodes of the 1970s-80s, unconstrained monetary policy was commonly perceived to be inferior to policies that are constrained by publicly observable targets, for example inflation targets. Starting in the 1990s, several central banks announced inflation targeting (IT) as their monetary policy framework.<sup>1</sup> More recently, researchers started to debate price-level targeting (PT) as an alternative to inflation targeting.<sup>2</sup> The main issue of the debate is whether a policy that stabilizes the price-level (possibly around a deterministic trend) is preferable to policies that stabilize the inflation rate. Under inflation targeting, monetary authority announces a desired level of annual inflation and conducts monetary policy in a way consistent with that objective.<sup>3</sup> Due to shocks and imperfect degree of control over

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<sup>1</sup>Ball and Sheridan (2003) provide a list of central banks that adopted inflation targeting, as well as timing details and performance evaluation for this policy change.

<sup>2</sup>There has been a series of policy oriented papers that discuss price-level targeting as an alternative to inflation targeting. To name just a few, Calstrom and Fuerst (2002), Dittmar Gavin and Kydland (1999), Dittmar and Gavin (2005), Vestin (2000), Gavin and Stockman (1991), Duguay (1994), Ortega and Rebei (2006), Svensson (1999), Smets (2003), Yetman (2005).

<sup>3</sup>As the Bank of Canada states: "monetary policy needs to aim at the 2 per cent target midpoint over the six to eight quarters that are required for monetary policy to have most of its effect. By consistently aiming at 2 per cent for the 12-month rate of inflation, monetary policy can enhance the predictability of average inflation over longer

prices, the actual inflation rate may deviate from the desired level. For this reason, policies aiming at stabilizing the *inflation level* do not necessarily imply a stationary *price level*. Specifically, to stabilize the level of inflation, monetary authority does not have to react to past deviations from the desired level of inflation. The price level may drift arbitrarily far away from any predetermined path, as the deviations accumulate over time. Conversely, to keep the price level close to the predetermined trend, the central bank must reverse deviations from the desired inflation level. Hence, the price level is non-stationary when transient deviations of the price level are not offset by the monetary authority, and it is (trend-) stationary when the price level (as opposed to inflation level) is controlled.

We analyze the welfare implications of IT and PT in our framework and find that the welfare rankings of inflation targeting and price level targeting depend crucially on income persistence. If income is close to a random walk, inflation targeting is close to the optimal monetary policy, as it implies stable expected inflation. On the other hand, if income fluctuations are transitory, price-level targeting is close to the optimal monetary policy since it implies a positive correlation between expected inflation and income. Specifically, when income is high, low realized price level calls for higher expected inflation to return price level back to the trend.

We summarize that the optimal monetary policy has features inherent to inflation targeting (nonstationary price level) and price-level targeting (reversion of price-level deviations from the trend). The balance of these features in optimal monetary policy depends on income persistence in the economy.

These results can be generalized to more complicated settings. In a natural extension of the benchmark model, we add a productive real asset, land, to the economy. Land is combined with the labor endowment of young individuals to produce consumption goods. The young are required to buy the land for money before they can produce. They borrow money from the old individuals via uncontingent nominal bonds. These nominal bonds dominate money in rates of return and can be thought of as mortgage contracts. In this richer model the same qualitative results are obtained under optimal and simple targeting regimes of monetary policy. Thus, our results are robust to introduction of extra assets.

The paper contributes to several areas of research in monetary economics.

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time horizons."

To our knowledge, this is the first paper to analyze monetary policy in a stochastic OLG environment. Perhaps surprisingly, previous research on monetary policy in OLG models focused exclusively on deterministic models. Suboptimality of inflation was one of the main findings of that literature.<sup>4</sup> In a recent paper, Akyol (2004) also finds positive optimal inflation in an environment with infinitely lived agents, who are subjects to uninsurable idiosyncratic endowment risk and borrowing constraints. With no aggregate uncertainty the price level in Akyol's model increases over time in a deterministic fashion. In our model, we provide full characterization of optimal monetary policy under aggregate uncertainty. The aggregate shocks create a rich environment for monetary policy analysis. Finally, our paper contributes to the debate on the merits of inflation stabilization versus price level stabilization. It shows that optimal policy combines some features of both inflation targeting (nonstationary price level) and price-level targeting (reversion of price-level deviations from the trend).

The paper proceeds as follows: Section 2 introduces and analyzes the model with fiat money as the only asset. Section 3 extends this model to include productive land and an additional interest bearing asset, and shows that the qualitative results do not change. Section 4 contains concluding remarks. Proofs and derivations are collected in the appendices.

## 2 OLG model with fiat money only

In this section, we focus on a classical two-period overlapping generations endowment economy in which fiat money is the only asset. The young individuals in this economy use money to save for the time when they are old. Monetary policy affects real returns on savings via its effect on inflation. Given asset market incompleteness, monetary policy has a potential to improve average welfare in the economy.<sup>5</sup>

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<sup>4</sup>See, for example, Wallace (1980) or Champ and Freeman (2001).

<sup>5</sup>Markets are incomplete for two reasons. First, overlapping-generations structure implies that newborn individuals cannot insure against the endowment risk. Second, young individuals, who save in the form of non-contingent asset, cannot fully insure against endowment risk when they are old.

## 2.1 The environment

There is a unit measure of identical individuals born in every period. Each generation lives for two periods. The young person born in period  $t$  is endowed with  $w_t$  units of a perishable consumption good in period  $t$  and zero units in period  $t + 1$ . The endowment  $w_t$  is random and represents the only source of uncertainty in the model. The log of the endowment follows a first-order autoregressive process:

$$\ln w_t = \rho \ln w_{t-1} + \varepsilon_t ,$$

where  $\varepsilon_t$  is i.i.d draws from zero-mean normal distribution.

The single asset in the economy is fiat money supplied by the government. In period 1 there is an initial old generation that has no endowment and holds  $M_1$  unites of the money stock.<sup>6</sup>

The timing of events is as follows. At the beginning of period  $t$  the old generation holds  $M_{t-1}$  units of fiat money which they acquired in the previous period. Before the current endowment  $w_t$  is realized, the government prints (or destroys) money in the amount of  $M_t - M_{t-1}$ , and distributes it evenly among the old individuals as a lump-sum transfer (or tax if negative)  $T_t = M_t - M_{t-1}$ . The assumption that monetary transfers occur before the realization of the current endowment, reflects limited ability of the government policy to react to current shocks in the economy, and implies an incomplete degree of control over the price level. After the realization of the current endowment,  $w_t$ , the young agents consume  $c_t^y$  units of their endowment. The remaining goods,  $(w_t - c_t^y)$ , they exchange for  $M_t^d$  units of money at the price  $P_t$ . Thus, a young person born in period  $t$ , solves the following problem:

$$\max u(c_t^y) + \beta E_t u(c_{t+1}^o) \tag{YP}$$

subject to

$$\begin{aligned} P_t c_t^y + M_t^d &\leq P_t w_t , \\ P_{t+1} c_{t+1}^o &\leq M_t^d + T_{t+1} , \end{aligned} \tag{1}$$

where  $c_{t+1}^o$  is her consumption when old,  $T_t$  is the monetary transfer from the government in period  $t$ ; and  $\beta$  is the discount factor. The expectations

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<sup>6</sup>In this model  $M$  can be thought of as the sum of two equivalent assets: fiat money and discount bonds. The fact that nominal interest rate on bonds is fixed at zero does not affect our results. We relax that assumption in the model of section 3, where nominal interest rate on bonds is endogeneous.

operator  $E_t$  is taken conditional on the realizations of endowments through the end of period  $t$ .

## 2.2 Monetary Equilibrium

Let  $\mu_t$  denote the growth of money supply in the economy in period  $t$ ,  $\mu_t = \frac{M_t}{M_{t-1}}$ , where  $\mu_1$  is fixed at 1 without a loss of generality. Monetary policy is defined as an infinite sequence of money growth rates,  $\{\mu_t\}_{t=1}^{\infty}$ .

**Definition 1** *Given a monetary policy  $\{\mu_t\}_{t=1}^{\infty}$ , a monetary equilibrium for this economy is a set of prices  $\{P_t\}_{t=1}^{\infty}$  and allocations  $\{c_t^y, c_t^o, M_t^d\}_{t=1}^{\infty}$ , such that for all  $t = 1, 2, 3, \dots$* <sup>7</sup>

- allocations  $c_t^y, c_{t+1}^o$  and  $M_t^d$  solve the generation  $t$ 's problem YP;
- and the goods and money market clear:

$$\begin{aligned} c_t^y + c_t^o &= w_t, \\ M_t^d &= M_t. \end{aligned}$$

In the next two subsections we, first, characterize the optimal allocation and the optimal monetary policy that implements it as a monetary equilibrium, and second, compare social welfare in equilibria with price-level and inflation targeting to that under the optimal monetary policy.

## 2.3 Optimal monetary policy

To find the optimal monetary policy, we start by defining the social welfare function and solve the social planner's problem for the optimal allocation. Then we ask whether this allocation can be implemented as a monetary equilibrium.

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<sup>7</sup>All of the variables in the definition are random variables conditional on realizations of endowments. We suppress state notation for the sake of notational simplicity.

### 2.3.1 Social planner's problem

We assume that the social planner treats all generations equally. Let the average (ex-post) utility over  $T$  periods be:

$$V_T = \frac{1}{T} \left[ \beta u(c_1^o) + \left[ \sum_{t=1}^{T-1} [u(c_t^y) + \beta u(c_{t+1}^o)] \right] + u(c_T^y) \right] \quad (2)$$

$$= \frac{1}{T} \sum_{t=1}^T [u(c_t^y) + \beta u(c_t^o)] . \quad (3)$$

We define the social welfare function as

$$\lim_{T \rightarrow \infty} \inf E [V_T] . \quad (4)$$

This welfare criterion treats all the generations equally, in a sense of attaching the same welfare weight to the expected utility of every generation.<sup>8</sup>

The social planner maximizes (4) subject to the resource constraint for all periods:

$$c_t^y + c_t^o \leq w_t, \text{ for } t = 1, 2, \dots, .$$

We show in Appendix A that the solution to this problem is the sequence of consumptions  $\{c_t^y, c_t^o\}_{t=1}^T$  such that:

$$\begin{aligned} u'(c_t^y) &= \beta u'(c_t^o) , \\ c_t^y + c_t^o &= w_t . \end{aligned}$$

In case of CRRA period utility,  $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ , the first-best allocation is:

$$c_t^y = \frac{1}{1 + \beta^{\frac{1}{\gamma}}} w_t , \quad (5)$$

$$c_t^o = \frac{\beta^{\frac{1}{\gamma}}}{1 + \beta^{\frac{1}{\gamma}}} w_t \quad (6)$$

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<sup>8</sup>In model simulations, we use

$$V_T = \frac{1}{T} \sum_{t=1}^T [u(c_t^y) + \beta u(c_t^o)] ,$$

to measure average welfare attained for any particular realization  $w^T$  of the endowment history. For  $T$  large this measure is close to our theoretical measure of social welfare.



for all  $t = 1, 2, \dots$ .

To conclude this subsection, we emphasize two reasons for using the undiscounted welfare function instead of the discounted one,

$$V_T = u(c_1^o) + \sum_{t=1}^T \beta^{t-1} [u(c_t^y) + \beta u(c_{t+1}^o)].$$

The later criterion implies that the optimal consumption allocation is  $c_t^y = c_t^o = \frac{1}{2}w_t$ . First, the consumption allocation (5)-(6) implied by our welfare function, maximizes the unconditional expected utility of any given generation (except the initial old). Hence, it is the unique ex-ante optimal allocation. Second, without endowment uncertainty, this allocation is implementable as a market equilibrium with constant money stock. In the next section we show that, with uncertainty, monetary policy can improve welfare by redistributing income between generations. With the chosen undiscounted welfare criterion the amount of redistribution required to achieve the optimal allocation will be minimal. In contrast, under the discounted social welfare function monetary policy would have to perform much more in terms of redistribution, even when there is no uncertainty.

### 2.3.2 Implementing the optimal allocation as a monetary equilibrium

Suppose the first-best allocation can be implemented in a monetary equilibrium. Any monetary equilibrium must satisfy the following first order conditions

$$u'(w_t - \frac{M_t^d}{P_t}) = \beta E_t \left[ u'(\frac{M_t^d + T_{t+1}}{P_{t+1}}) \frac{P_t}{P_{t+1}} \right] \quad (7)$$

$$\frac{P_t}{P_{t+1}} = \frac{M_t}{M_{t+1}} \frac{w_{t+1} - c_{t+1}^y}{w_t - c_t^y}. \quad (8)$$

These conditions, with the first-best allocation (5)-(6), imply the expression for money growth

$$\frac{M_{t+1}}{M_t} = E_t \left[ \left( \frac{w_{t+1}}{w_t} \right)^{1-\gamma} \right],$$

and for prices

$$\frac{P_{t+1}}{P_t} = \frac{M_{t+1}}{M_t} \frac{w_t}{w_{t+1}} = E_t \left[ \left( \frac{w_{t+1}}{w_t} \right)^{1-\gamma} \right] \frac{w_t}{w_{t+1}}.$$

Given the assumption of log normality of the the endowment process, we obtain

$$m_{t+1} - m_t = \frac{(1 - \gamma)^2 \sigma^2}{2} + (1 - \rho)(\gamma - 1)\omega_t \quad (9)$$

$$p_{t+1} - p_t = \frac{(1 - \gamma)^2 \sigma^2}{2} + \gamma(1 - \rho)\omega_t - \varepsilon_{t+1} , \quad (10)$$

where  $m_t \equiv \ln M_t$ ,  $p_t \equiv \ln P_t$ ,  $\omega_t \equiv \ln w_t$  and  $\sigma$  is the standard deviation of innovations to the endowment process.

Equations (9)-(10) fully characterize the dynamics of the money stock and price level in equilibrium that implements the optimal consumption allocation. We summarize four main features of price level dynamics under the optimal monetary policy:

1. The average inflation under the optimal policy is positive,  $\bar{\pi} = \frac{(1-\gamma)^2 \sigma^2}{2}$ , and increasing with the size of uncertainty, as long as  $\gamma \neq 1$ .<sup>9</sup>
2. The expected inflation is positively correlated with current endowment

$$E_t [p_{t+1} - p_t] = \bar{\pi} + \gamma(1 - \rho)\omega_t. \quad (11)$$

3. The variance of the expected inflation is decreasing in the persistence of the endowment process,  $\rho$ , if  $\rho < 1$ . If the endowment follows a random walk,  $\rho = 1$ , then the optimal expected inflation is constant:  $E_t [p_{t+1} - p_t] = \bar{\pi}$ .

4. The log price level under the optimal policy is non-stationary.<sup>10</sup>

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<sup>9</sup>It might appear from (9), that our conclusion on positive inflation depends on the choice of log-normal distribution of endowment. It is easy to show, however, that  $E \left[ \frac{M_{t+1}}{M_t} \right] = E \left[ \left( \frac{w_{t+1}}{w_t} \right)^{1-\gamma} \right] > 1$  for any stationary distribution of endowment as well as for the random walk (with no drift) case.

<sup>10</sup>Suppose the money stock  $m_t$  is a stationary time series. Then it must be that  $var(m_{t+1}) = var(m_t)$ . Equation (9) then implies:  $0 = (1 - \rho)^2(\gamma - 1)^2 \frac{\sigma^2}{1 - \rho^2} + 2cov(m_t, (1 - \rho)(\gamma - 1)\omega_t)$ . Expanding  $m_t$  and  $\omega_t$ , it is easy to see that the covariance term on the right-hand side is non-negative, leading to a contradiction. Thus, the money stock  $m_t$  cannot be stationary. Further, since  $p_t = m_t - \log x_t$ , and since  $x_t \in [0, w_t]$ , the log of the price level must also be nonstationary.

To understand these properties of optimal policy, recall from equations (5)-(6) that the first-best allocation calls for a constant savings rate,  $\frac{\beta^{\frac{1}{\gamma}}}{1+\beta^{\frac{1}{\gamma}}}$ . In a monetary equilibrium savings rate depends on expected return to money  $E_t[p_t - p_{t+1}]$ , which is negative of expected inflation. Expected inflation is the monetary policy tool that is used to stabilize equilibrium savings rate at the optimal level. The first three properties of the optimal monetary policy describe how expected inflation must be set to achieve the first-best allocation. The last property (nonstationary price level) is an outcome of optimal monetary policy. Let us look at each of the properties:

Property 1 is due to asset market incompleteness implying that individuals cannot perfectly insure themselves against endowment risk. In the face of uncertainty about future income, risk-averse individuals have an incentive to self-insure by smoothing consumption across time. Without positive trend inflation, they tend to save more than optimal for precautionary reasons, as in Aiyagari (1994). Positive average inflation serves as a tax on savings, which discourages oversaving.

According to Property 2, a positive correlation between expected inflation and income implies a high (low) expected tax on savings when income is high (low). This discourages individuals from varying savings rates to smooth consumption over time and thereby stabilizes the savings rate.

Property 3 implies that with higher income persistence individuals have less expected variation in income, and therefore less incentive to vary their savings rates. As a result, optimal expected inflation has to vary less to discourage consumption smoothing. In the limit, when the income process becomes a random walk, optimal expected inflation is constant.

Finally, Property 4, a nonstationary price level, is the result of optimal expected inflation being independent of current or past price levels, or equivalently, optimal monetary policy being completely forward-looking. Under optimal policy, it is the expected inflation that matters for savings rates, and not the price level.

Note that without uncertainty, the optimal expected inflation is always zero, and the optimal price level is constant. Hence, this paper emphasizes the role of monetary policy in affecting the savings behavior of risk-averse individuals under uncertainty and market incompleteness. In Section 2.4, we will see that welfare implications of other monetary policy regimes depend on how well they can approximate the four properties of optimal monetary policy.

## 2.4 Evaluating targeting regimes

In the previous section, we derived the unrestricted optimal monetary policy. The government knows preferences of individuals, observes realized shocks and fully commits to the optimal policy. In reality, such an optimal policy is hard to implement because of fundamental uncertainty about the structure of the economy. Moreover, it was shown by Kydland and Prescott (1977), that monetary authority may suffer from a time-inconsistency problem. For example, the high inflation episodes of the 1970s and subsequent recessions of the 1980s are often blamed on monetary policy mistakes and on inflationary bias due to wrong incentives of policymakers. As a result, an unrestricted monetary policy was widely perceived to be inferior to monetary policy regimes targeting some publicly observable monetary goals (e.g. money growth rates, exchange rate or inflation rate). So the attention of monetary policy theorists shifted from unrestricted optimal policies to those constrained by observable economic variables. Such policies often take a form of targeting rules, whereby a policy variable (e.g. nominal interest rate) is mostly determined by the expected path of the inflation rate or the output gap relative to the respective target level. The focus of monetary-policy research has been on what is the best policy rule (i.e. constraint) that should be imposed on the monetary authority. In particular, this topic is often framed in terms of a loss function that should be "delegated" to the central bank.

In this section we contribute to that debate by characterizing monetary equilibria with constrained monetary policies. We consider a class of competing monetary regimes that specify monetary policy indirectly, via its effect on the expected path of the price level: inflation targeting (IT) and price-level targeting (PT). As equations (9)-(10) show, due to the timing restriction, monetary policy cannot fully control the price level and innovations to endowment act as shocks to the price level. Hence, both policy regimes target only the expected level of inflation or price level, and allow deviations from the targets due to unexpected price shocks. In this subsection, we first introduce a parametrized class of constrained monetary policies that nest IT and PT. Next, we characterize the equilibrium of our OLG economy under the imposed constraints. Finally, we compare dynamics and welfare under IT and PT to those under the unrestricted optimal policy.

Suppose that the government sets the money growth rate so that in equilibrium the expected price level evolves according to the following dynamic

constraint

$$E_t [p_{t+1} - z_{t+1}] = \lambda(p_t - z_t) \quad (12)$$

where  $z_t = p_1 + \bar{\pi}t$  is a deterministic trend of the log price level and  $\bar{\pi}$  is the average inflation rate. Since we focus on the dynamic properties of equilibria under the constraint (12), we assume that the average inflation rate is at the optimal level,  $\bar{\pi} = \frac{(1-\gamma)^2\sigma^2}{2}$ .

The adjustment coefficient  $\lambda \in [0, 1]$  determines how fast the price level returns to the trend. In particular,  $\lambda = 0$  corresponds to the strict price level targeting,  $\lambda = 1$  corresponds to the strict inflation targeting, while any  $\lambda \in (0, 1)$  correspond to gradual price level targeting where deviations from the trend are reversed more slowly than under the strict price level targeting.<sup>11</sup>

For convenience let's refer to the set of monetary policies that satisfy the constraint (12) for any value of  $\lambda$  as  $PT(\lambda)$  rules. In general, this restricted set of monetary policies will not include the (unrestricted) optimal monetary policy. Thus, the welfare attained under the best monetary policy in the restricted set, will be lower than under the optimal monetary policy.

Appendix C shows that targeting policy rules (12) are equivalent to a monetary authority choosing the price level path that minimizes the "loss function" delegated to it by the government. For example, a monetary authority that minimizes the loss function

$$L = E \sum_{t=1}^{\infty} \kappa^t (\pi_t - \bar{\pi})^2$$

where  $0 < \kappa < 1$ , pursues the strict inflation targeting described by (12) with  $\lambda = 1$ . If the loss function is

$$L = E \sum_{t=1}^{\infty} \kappa^t (p_t - z_t)^2$$

then the monetary authority follows the strict price-level targeting given by (12) with  $\lambda = 0$ .

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<sup>11</sup>We also consider an alternative class of monetary policy regimes:  $E_t [p_{t+1} - p_t] = (1 - \phi)\bar{\pi} + \phi(p_t - p_{t-1})$ , where  $\phi \in [0, 1]$ . For  $\phi > 0$ , this regime corresponds to gradual inflation targeting, whereby deviations of inflation from  $\bar{\pi}$  are gradually eliminated. It turns out that strict inflation targeting ( $\phi = 0$ ) welfare dominates every gradual inflation targeting ( $\phi > 0$ ). Hence, we can focus our analysis on strict inflation targeting rules only.

In our model we focus entirely on the savings decisions and do not explicitly model labor-leisure or capital investment decisions. As a result, there is no inflation-output tradeoff, and no output gap terms in the loss functions. Incorporating the labor-leisure and investment decisions would be an interesting extension.

### 2.4.1 Characterizing equilibria under targeting policy regimes

Define  $x_t = \frac{M_t^d}{P_t}$ , and  $h_t = M_t \exp(-z_t)$ . We can rewrite the first-order conditions (7)-(8) as

$$\exp(\bar{\pi})x_t(w_t - x_t)^{-\gamma} = \beta \frac{h_t}{h_{t+1}} E_t [x_{t+1}^{1-\gamma}] ,$$

$$\frac{P_t}{P_{t+1}} = \frac{h_t}{h_{t+1}} \frac{x_{t+1}}{x_t} \exp(-\bar{\pi}) . \quad (13)$$

The appendix B shows that equation (13) and policy rule (12) imply another dynamic equation

$$h_{t+1} = \left( \frac{h_t}{x_t} \right)^\lambda \exp(E_t [\ln x_{t+1}]) . \quad (14)$$

Thus, to solve for a monetary equilibrium, we need to find sequences  $\{x_t, h_t\}$  that satisfy the following two dynamic equations:

$$\exp(\bar{\pi}) \left[ \frac{x_t}{h_t} \right]^{1-\lambda} (w_t - x_t)^{-\gamma} = \beta \exp(-E_t [\ln x_{t+1}]) E_t [x_{t+1}^{1-\gamma}] \quad (15)$$

$$h_{t+1} = \left( \frac{h_t}{x_t} \right)^\lambda \exp(E_t [\ln x_{t+1}]) . \quad (16)$$

The state of the economy in period  $t$  can be fully summarized by  $s_t = (\omega_t, h_t)$ . To solve the system (15)-(16), we find functions  $x_t = x(\omega_t, h_t)$  and  $h_{t+1} = h(\omega_t, h_t)$  that satisfy the two dynamic equations.

### 2.4.2 Welfare comparisons of targeting regimes

What constrained policy (parametrized by  $\lambda$ ) in the class of targeting regimes (12) (i.e.  $PT(\lambda)$ ) maximizes the unconditional expected utility attained in a stationary equilibrium? It is instructive to first consider the log utility case ( $\gamma = 1$ ). For this special case it is easy to verify from equations (15) and (16))

that the optimal monetary policy that implements the first-best allocation is consistent with  $PT(\lambda)$  if  $\lambda = \rho$ . Indeed, the expected inflation under the optimal policy satisfies (11), which for this special case can be written as

$$E_t [p_{t+1} - p_t] = \gamma(1 - \lambda^*)\omega_t .$$

Hence, in the log utility case the price-level targeting regime  $PT(\lambda)$  with  $\lambda^* = \rho$  attains the first best allocation. It implies a constant money stock (see equation (9)), zero average inflation, and a stationary price level.

For  $\gamma \neq 1$ , there is also a special case for which an analytical solution exists. If endowment is a random walk,  $\rho = 1$ , then the unrestricted optimal monetary policy is consistent with  $PT(\lambda)$  if  $\lambda^* = \rho = 1$ . In this case, expected inflation is constant

$$E_t [p_{t+1} - p_t] = \frac{(1 - \gamma)^2 \sigma^2}{2} = \bar{\pi} ,$$

so inflation targeting attains the first best allocation.

For other values of  $\gamma$  and  $\rho$ , the welfare comparisons have to be done numerically. The welfare across equilibria are compared to the welfare in equilibrium under the optimal policy:

$$EV^* = \frac{1}{1 - \gamma} \left( \frac{1}{1 + \beta^{\frac{1}{\gamma}}} \right)^{-\gamma} \exp \left( \frac{(1 - \gamma)^2 \sigma^2}{2(1 - \rho^2)} \right) .$$

To calculate welfare under targeting (12), we simulate the economy over 10,000 periods, and calculate the average utility of all the generations in the simulated sample,  $V_T = \frac{1}{T} \sum_{t=1}^T [u(c_t^y) + \beta u(c_t^o)]$ . We compare the value of the average utility,  $V_T$ , with  $EV^*$  by calculating the percentage by which consumption of every individual in every period has to be raised to reach the same level of average welfare as under the optimal allocation. Specifically, let  $\delta$  be a real number such that

$$\delta^{1-\gamma} V_T = \frac{1}{T} \sum_{t=1}^T \left[ \frac{(\delta c_t^y)^{1-\gamma}}{1 - \gamma} + \beta \frac{(\delta c_t^o)^{1-\gamma}}{1 - \gamma} \right] = EV^* .$$

We report welfare loss as lifetime-consumption-equivalent compensation  $Comp.\% = (\delta - 1) * 100\%$ .

From our experiments with various sets of parameters, we find that the discount rate,  $\beta$ , and the coefficient of relative risk aversion,  $\gamma$ , have little

effect on the magnitude of welfare losses and on relative welfare rankings. We pick standard values of these two parameters,  $\beta = 0.96^{30}$  and  $\gamma = 1.5$  (see Table 1). Standard deviation of innovations to endowment,  $\sigma$ , does affect the level of welfare losses. Smaller values of  $\sigma$  imply smaller welfare differences between various policies. In particular, when  $\sigma = 0$ , all of the policies we consider are identical. However, the standard deviation of innovations to endowment has no effect on relative welfare rankings of the targeting regimes. We set  $\sigma = 0.08$ .<sup>12</sup> As we noted earlier, income persistence,  $\rho$ , is crucial for welfare rankings of the alternative policy regimes, as well as for welfare loss magnitudes. For that reason, we report welfare losses for a range of persistence values.

Table 1: Parameter values for model simulations)

Parameter	Values
Discount factor, $\beta$	$0.96^{30}$
Relative risk aversion, $\gamma$	1.5
Standard deviation of innovations to endowment, $\sigma$	0.08
Persistence of the endowment process, $\rho$	0.1, 0.5, 0.9.

Figure 1 presents the welfare losses across equilibria with various values of  $\lambda$ . We repeat the experiment for three different values of the serial correlation of the endowment process:  $\rho = 0.1, 0.5$  and  $0.9$ , keeping other parameters fixed. The welfare loss is U-shaped, with the minimum located close, but to the left of  $\lambda = \rho$ . At the minimum, the welfare loss is very small, less than 0.02% of consumption, so the best policy in the class of targeting regimes (12) is nearly optimal. For policy rules different from the best one, welfare losses grow faster as persistence of income decreases. In particular, this implies that under strict inflation targeting, welfare losses are disproportionately higher than under price level targeting. For example, if  $\rho = 0.1$ , the welfare loss under the strict IT is 0.62%, whereas if  $\rho = 0.9$ , the welfare loss from following the strict PT is much smaller at 0.08%.

Why does the best policy regime correspond to the transition coefficient  $\lambda$  close to the persistence of endowment  $\rho$ ? As we demonstrated before,

<sup>12</sup>Our estimates of  $\sigma$  from GDP data ranged from 0.02 to 0.16 depending on how we detrended data, and what we assumed about the stationarity of income process. We picked roughly the middle point of this range.



a monetary policy provides high average welfare if it is able to stabilize the savings rate at the optimal level. The policy instrument of monetary policy is the expected return on fiat money  $E_t [p_t - p_{t+1}]$ , or equivalently, the expected inflation rate  $E_t \pi_{t+1} = E_t [p_{t+1} - p_t]$ . So a monetary policy yields high welfare if it is successful in replicating the path of the expected inflation under the optimal policy :

$$E_t [\pi_{t+1}^*] = \bar{\pi} + \gamma(1 - \rho)\omega_t . \quad (17)$$

In the special case of log utility  $\gamma = 1$  and  $\lambda = \rho$ , the equilibrium expected inflation is given by

$$E_t [\pi_{t+1}] = \bar{\pi} + \gamma(1 - \lambda)\omega_t . \quad (18)$$

For risk aversion other than 1, if  $\lambda \approx \rho$ , the equilibrium expected inflation is approximately

$$E_t [\pi_{t+1}] \approx \bar{\pi} + \gamma(1 - \lambda)\omega_t . \quad (19)$$

From (17)-(19) it follows that for  $\lambda$  close to  $\rho$ , the path of expected inflation under the constraint (12) is going to be close to the optimal expected inflation. This has simple policy implications for the targeting horizon: deviations of the price level from the trend should be as persistent as income fluctuations.

We conclude that strict IT is closer to the (unrestricted) optimum if income fluctuations are highly persistent; and strict PT is closer to the optimum if income fluctuations are transitory. To understand the intuition behind this result we examine how well each of the policy regimes replicates the four properties of the optimal monetary policy, discussed in Section 2.3.2.

To focus on welfare differences due to dynamic effects of monetary policy on savings under uncertainty, we assume that IT and PT have the same trend inflation,  $\bar{\pi}$ , as the optimal policy. This implies that without uncertainty, IT and PT would be equivalent not only to each other, but also to the optimal policy.

Under aggregate uncertainty, income persistence is crucial for shaping dynamic properties of monetary policy (Properties 2 and 3 in Section 2.3.2). If income is a random walk,  $\rho = 1$ , then optimal monetary policy requires constant expected inflation, which corresponds to inflation targeting. Thus, when the income process is highly persistent, strict IT is close to the optimum in terms of expected inflation dynamics, while strict PT implies too much

variation in expected inflation. On the other hand, if income fluctuations are transitory, the optimal monetary policy requires a positive correlation between expected inflation and income, which corresponds to price-level targeting. Indeed, when income is high, low realized price level calls for higher expected inflation to return price level back to trend. Conversely, under inflation targeting, the expected inflation is constant, and thus its correlation with income is zero. Thus, when income persistence is low, strict PT is nearly optimal in terms of inflation dynamics, while strict IT implies too little variation in expected inflation.

The last property of optimal policy is a nonstationary price level. Inflation targeting satisfies that property, while price-level targeting does not. Hence, typically, optimal monetary policy has elements in common with both price-level and inflation targeting: it reverts temporary deviations of the price level from the trend but does not return the price level all the way to the predetermined trend as implied by price-level targeting. As we learned from model simulations, depending on income persistence, the balance between these two features of optimal policy changes, so that the implied price level dynamics is mimicked better by the price- or inflation targeting policy regime.

Given the importance of income persistence: what the empirical evidence on persistence of aggregate income? The empirical literature is mostly debating on whether income is a trend stationary variable or a non-stationary variable with drift. Campbell and Mankiw (1987) examine persistence of U.S. real GNP by looking at the long-run impulse response in an estimated ARIMA model, and conclude that shocks to GNP are largely permanent. However, they caution that it appears impossible to reject the view that output reverts to the trend after twenty years.

Cochrane (1988) questions that finding using a non-parametric variance test. He finds little persistence in real U.S. GNP, and asserts that conventional methods of estimating persistence are misleading, because they are trying to estimate a long-run impulse response from short-run dynamics. He concludes that the existence or size of a random walk component in GNP is not a precisely measured "stylized fact" that any reasonable model must reproduce, but cautions that standard errors are large. Cochrane elaborates that it is unavoidable, since one needs long-run data to estimate long-run persistence of income, but there are inherently few nonoverlapping long runs available.

Perron (1989) also examines persistence of U.S. GNP series in the same dataset as Campbell and Mankiw (1987), and finds that traditional unit root

tests cannot reject a unit root hypothesis if the true process is stationary around a trend which contains a one-time break. He develops a test which allows for one time break in trend and finds that fluctuations are stationary around a deterministic trend function. The only shocks which have had persistent effects are the 1929 crash and the 1973 oil price shock. Perron also cautions that the rejection of the unit root is conditional on treating one-time breaks in trend as exogeneous events that are not part of the data generating process. He further notes that even if these breaks are indeed part of the data generating process, apparently they arrive extremely rarely. "Whichever view one adopts cannot be decided by data alone. Nevertheless, the picture under any of these views is basically the same: shocks had little, if any, persistence effect over a long horizon. Only ... the Great Depression and the oil price shock significantly altered the long run behavior of the series."

Finally Serletis (1992) replicates Perron's test in Canadian 1870-1985 real GNP and concludes that the unit root hypothesis could be rejected if allowance is made for the possibility of a one-time break of the trend function during the Great Depression.

Given these somewhat inconclusive results, we chose to report our findings for various values of the persistence parameter. The main finding of this section is theoretical: In a two period OLG model with aggregate uncertainty and noncontingent nominal assets, if income is trend stationary, strict price-level targeting dominates strict inflation targeting and is close to the optimal policy in welfare terms. If income is nonstationary, then inflation targeting dominates price-level targeting and is close to the optimal policy. Nonetheless, we interpret the evidence of a unit root in aggregate income as rather weak. Given that, our results suggest that targeting rules that put significant weight on reverting temporary price-level deviations from the trend are likely to do better.

Overall, our results and intuition for optimal policy and targeting rules come from the analysis of savings decisions under uncertainty. For that reason, we feel that our findings are general enough to be applicable to alternative environments, in which nominal assets play an important role for savings. Certainly, our model imposes strong assumptions. In particular, there are no alternative assets except fiat money, and the nominal return on money is fixed (at unity). Under these assumptions changes in expected inflation translate one for one into changes in real interest rates. If the economy had alternative assets in addition to money, with endogeneously determined nominal interest rates, then changes in inflation could have smaller effects on

real interest rates. In the next section, we present a richer model in which there are two assets besides money, and nominal interest rates are determined endogeneously. We find that our results on optimal policy and on the PT and IT welfare ranking apply with little change.

### **3 OLG economy with land, nominal bonds and money**

In order to keep things tractable, the model in the previous section has strong assumption that there are no alternative assets, except fiat money. In this section, we present a richer model, in which there is productive land and nominal bonds in addition to money. Nominal bonds are different from money, and dominate it in the rate of return. We generate demand for money by assuming that young agents have to use money to buy land. The young borrow money from the old by issuing noncontingent nominal bonds. We assume that the trade of bonds and the purchase of land take place before the realization of the current productivity shock. One can think of these arrangements as mortgage contracts. The young take a mortgage, buy land, learn their productivity shock, produce and then repay the bonds. The assumption of money being necessary to buy land, can be motivated by some underlying credit market intermediation technology, in which money serves as a medium of exchange. In this section, we first present the environment and characterize the monetary equilibrium. Then, we derive the optimal monetary policy which implements the first-best allocation, using the same welfare criterion as before. Finally, we will apply the model to compare welfare implications of PT and IT monetary policy regimes. Derivations will be relegated to the appendix. Despite the model being richer and more complex than the previous one, the results and intuition will remain essentially unchanged.

#### **3.1 The environment**

There is a unit measure of agents born every period. All individuals of the same generation are identical in all respects. Every generation lives for two periods. Further, each period is subdivided into two subperiods: morning and afternoon. In the morning of period  $t$ , the current productivity shock  $A_t$  has not been observed yet. The young generation has nothing except the

endowment of labor. The old own the entire stock of land  $L$  plus the entire stock of money  $M_{t-1}$ . The government prints (destroys) new money in the amount  $M_t - M_{t-1}$ , and allocates it equally among the old with a lump-sum transfer (tax). The young borrow some amount of money from the old to finance their land purchases. The old lend the money and then sell their land for the money. The morning subperiod ends.

In the afternoon of period  $t$  the productivity shock  $A_t$  is realized. The young combine their labor endowment ( $N = 1$ ) and the purchased land in a Cobb-Douglas production function to produce output,  $Y_t = A_t L^\alpha N^{1-\alpha} = A_t L^\alpha$ . They consume part of their output  $c_t^y$ , and use the remainder to retire their debt and to purchase money  $M_t^d$  for the next period, when they will be old. The evening subperiod ends.

We assume that the log of the productivity shock  $a_t = \ln A_t$  follows a first-order autoregressive process:

$$a_t = \rho a_{t-1} + \varepsilon_t$$

where  $\varepsilon_t$  is distributed as  $N(0, \sigma^2)$ . It is the only source of uncertainty in the model. In this section it is important to keep track of timing of various events. For that reason we introduce state space notation and show the timing of events in the figure 3. We will use  $s^t$  to represent the history of the economy up to period  $t$ , and  $s_t$  to represent the current state of the economy. The history evolves as  $s^{t+1} = (s^t, s_{t+1})$ . Observe that with this notation,  $M(s^{t-1})$  represents  $M_t$ , since we assumed that  $M_t$  is determined in the morning of period  $t$ , that is before  $A_t = A(s^t)$  is realized.

The problem of the young is the following:

$$\max \sum \Pr(s^t | s^{t-1}) u(c^y(s^t)) + \beta \sum \Pr(s^{t+1} | s^t) u(c^o(s^{t+1}))$$

subject to constraints

- morning of period  $t$ , before  $A_t$  is realized

$$q(s^{t-1})L^d(s^{t-1}) \leq d(s^{t-1})B^d(s^{t-1})$$

where  $q(s^{t-1})$  is the morning nominal price of land,  $d(s^{t-1})$  is the discount factor on money borrowed in the morning, and  $B^d(s^{t-1})$  is the nominal value of goods to be returned to the old in the afternoon in repayment of the debt;

- afternoon of period  $t$ , after  $A_t$  is realized

$$c^y(s^t) + \frac{M^d(s^t)}{P(s^t)} \leq A_t (L^d(s^{t-1}))^\alpha - \frac{B^d(s^{t-1})}{P(s^t)}$$

- morning of period  $t + 1$ , before  $A_{t+1}$  is realized

$$d(s^t)B^s(s^t) \leq M^d(s^t) + T(s^t)$$

- afternoon of period  $t + 1$ , after  $A_{t+1}$  is realized

$$c^o(s^{t+1}) \leq \frac{B^s(s^t)}{P(s^{t+1})} + \frac{q(s^t)L^s(s^t) + (M^d(s^t) + T(s^t) - d(s^t)B^s(s^t))}{P(s^{t+1})}$$

where  $T(s^t) = M(s^t) - M(s^{t-1})$ .

In equilibrium, of course, it must be the case that

$$\begin{aligned} L^s(s^t) &= L^d(s^{t-1}) = L^d(s^t) = L \\ B^s(s^t) &= B^d(s^t). \end{aligned}$$

It remains to specify a monetary policy to complete the description of the model. Let us again start from the optimal monetary policy.

### 3.2 Optimal monetary policy

We maintain the same social welfare criterion as in the section 2.3.1. In appendix E we solve the social planner's problem for the first best allocations and then derive the optimal monetary policy that implements it. The optimal monetary policy implies the following price-level dynamics:

$$p(s^{t+1}) - p(s^t) = \ln \left( \frac{1}{d(s^t)} \right) + \frac{(1 - \gamma)^2 \sigma^2}{2} + \gamma(1 - \rho)a_t - \varepsilon_{t+1}. \quad (20)$$

Observe that, except for the first term on the right hand side, this is exactly the same formula as in the money only economy. The discount rate  $d(s^t)$  is a constant in three special cases: first, when shocks to productivity are i.i.d. ( $\rho = 0$ ), second, when shocks to productivity follow a random walk ( $\rho = 1$ ), and third, when utility is logarithmic ( $\gamma = 1$ ). Outside of these special cases, the discount rate does vary with productivity, but fluctuations are small. So we can state essentially the same results as in the money only economy:

1. The average inflation under the optimal policy is positive,  $\bar{\pi} = E \left[ \ln \left( \frac{1}{d(s^t)} \right) \right] + \frac{(1-\gamma)^2 \sigma^2}{2}$ .<sup>13</sup>
2. The expected inflation is positively correlated with current income

$$E_t [p_{t+1} - p_t] = \ln \left( \frac{1}{d(s^t)} \right) + \frac{(1-\gamma)^2 \sigma^2}{2} + \gamma(1-\rho)a_t.$$

3. The variability of expected inflation is decreasing in persistence of endowment process,  $\rho$ . In particular, when  $\rho = 1$ , the optimal expected inflation is constant:  $E_t [p_{t+1} - p_t] = \bar{\pi}$ .
4. The optimal price level is nonstationary.

Let us now see if our results for PT/IT monetary policy regimes also carry over to this model.

### 3.3 Inflation targeting and price level targeting policy regimes

As in the money only economy we apply the model to compare welfare in monetary equilibria with inflation and price level targeting monetary policy rules. As before we refer to these constrained monetary policies as  $PT(\lambda)$  rules. In the appendix D we show how to solve the model and outline the computation procedure.

What constant value of  $\lambda$  maximizes the unconditional expected utility  $E [u(c^y(s)) + \beta u(c^o(s'))]$ , attained in a monetary equilibrium? There are again the same special cases, for which we can solve the model analytically. First, look at the log utility case ( $\gamma = 1$ ). For this special case it is easy to (guess and) verify from equations (39) - (41) that the optimal policy implementing the first-best allocation is consistent with  $PT(\lambda)$  if  $\lambda = \rho$ .

Similarly, if endowment is a random walk,  $\rho = 1$ , then  $PT(\lambda)$  with  $\lambda = \rho = 1$  is consistent with the optimal policy implementing the first-best allocations.

For other values of parameters we have to resort to simulations. We need to pick parameters. The model is homogeneous in  $L$ , so without loss of generality, we can normalize it to unity<sup>14</sup>. The advantage of doing this is that

<sup>13</sup>Note that that  $\bar{\pi}$  is not the same as before, but it is still the optimal long-run inflation.

<sup>14</sup>LTCE welfare losses are independent of  $L$ .

income is now equal to productivity  $A_t$ . As a result we can use the same parameters for the stochastic process of productivity, as the ones for endowment in the money only economy. So, we use the same set of parameters as before, except for one additional, the income share of land. The share of land  $\alpha$  in the production function is taken as the share of structures in GDP reported in Krussel et al. (2000). Table 2 lists all the parameters.

Table 2: Parameter values for model simulations, land economy)

Parameter	Values
Discount factor, $\beta$	0.96 <sup>30</sup>
Relative risk aversion, $\gamma$	1.5
Standard deviation of innovations to endowment, $\sigma$	0.08
Persistence of endowment process, $\rho$	0.1, 0.5, 0.9
Income share of land, $\alpha$	0.12

Just like before, we simulated the economy with different values of  $\lambda$ , over many periods, and for each rule we found the average utility of all the generations in the simulated sample,  $V_T = \frac{1}{T} \sum_{t=1}^T [u(c_t^y) + \beta u(c_t^o)]$ . We compared the value of the average utility,  $V_T$ , with the maximum expected utility  $EV^*$  computed at the first-best allocations:

$$c_t^{y*} = \frac{1}{1 + \beta^{\frac{1}{\gamma}}} A_t L^\alpha$$

$$c_t^{o*} = \frac{\beta^{\frac{1}{\gamma}}}{1 + \beta^{\frac{1}{\gamma}}} A_t L^\alpha.$$

With the optimal allocation, and  $L = 1$  the unconditional expected utility is

$$EV^* = \frac{1}{1 - \gamma} \left( \frac{1}{1 + \beta^{\frac{1}{\gamma}}} \right)^{-\gamma} \exp \left( \frac{(1 - \gamma)^2 \sigma^2}{2(1 - \rho^2)} \right)$$

just like in the money only model. As a welfare metric we use the same consumption equivalent compensation measure as before. Results of these simulations are presented in the Figure 2. It presents the welfare losses across equilibria with various values of  $\lambda$ . As is evident from the Figure 2 results are very similar to those from the money only economy: income



persistence determines whether PT or IT is closer to the optimum, and the best targeting regime has  $\lambda \approx \rho$ . Thus our results transfer with almost no change to this richer environment with additional assets.

## 4 Conclusions

What is the role of monetary policy in the environment with aggregate risk, incomplete markets and long-term nominal bonds? In a two-period overlapping-generations model with aggregate uncertainty and nominal bonds, optimal monetary policy attains the ex-ante Pareto optimal allocation. This policy is characterized by positive average inflation, positive correlation between expected inflation and income, inverse relationship between volatility of expected inflation and persistence of income, and a nonstationary price level. The model sheds light on the debate over the advantages of price level targeting compared to inflation targeting regimes. Persistence of income is the crucial parameter. If income is a random walk, inflation targeting is the optimal monetary policy, while price-level targeting is not. When income persistence is low price-level targeting dominates inflation targeting and is close to the first best in welfare terms.

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## A The solution of the social planner's problem

Suppose, for a given history of endowment realizations  $w^T = \{w_1, w_2, \dots, w_T\}$ , we are solving the following problem:

$$\begin{aligned}
 & \max V_T \\
 & \text{subject to} \quad : \\
 & \quad c_t^y + c_t^o \leq w_t, \text{ for all } t = 1, 2, \dots, T.
 \end{aligned} \tag{21}$$

The solution of this problem is  $\{c_t^y, c_t^o\}_{t=1}^T$  such that:

$$\begin{aligned}
 u'(c_t^y) &= \beta u'(c_t^o) \\
 c_t^y + c_t^o &= w_t.
 \end{aligned}$$

It is a pair of consumption functions  $c_t^y = c^{y*}(w_t)$ , and  $c_t^o = c^{o*}(w_t)$ . Given  $w_t$  they are independent of  $T$  and of the realized endowment history  $w^T$ .

Let

$$V_T^* = \frac{1}{T} \sum_{t=1}^T [u(c^{y*}(w_t)) + \beta u(c^{o*}(w_t))].$$

Let  $\{c_t^y, c_t^o\}_{t=1}^T$  be any other sequence of consumptions that satisfies (21) in each period  $t$ , and let  $V_T$  be the corresponding average ex-post utility as defined in (2). Then  $V_T^* \geq V_T$ , since for all  $t = 1, 2, \dots, T$  we have

$$u(c^{y*}(w_t)) + \beta u(c^{o*}(w_t)) \geq u(c_t^y) + \beta u(c_t^o). \quad (22)$$

Taking expectation of  $V_T^* - V_T$  with respect to realizations of  $w^T$  we have:

$$E[V_T^*] - E[V_T] \geq 0.$$

Taking the liminf with respect to  $T$  we have

$$\liminf_{T \rightarrow \infty} (E[V_T^*] - E[V_T]) \geq 0.$$

Since the sequence  $\{c_t^y, c_t^o\}_{t=1}^T$  was arbitrary, the stationary policy  $c^{y*}(w), c^{o*}(w)$  attains the maximum of the expected average utility  $E[V_T]$  for all  $T$ .

## B Derivation of the equation (14)

Let  $x_t = \frac{M_t}{P_t}$  be the real money balances in period  $t$ . With this notation, the real return on money is

$$\frac{P_t}{P_{t+1}} = \frac{M_t}{M_{t+1}} \frac{x_{t+1}}{x_t}. \quad (23)$$

Let us rewrite the equation (23) in log terms

$$p_t - p_{t+1} = m_t - m_{t+1} + \ln x_{t+1} - \ln x_t. \quad (24)$$

We concentrate on  $PT(\lambda)$  monetary policy rules that satisfy the dynamic equation

$$p_t - z_t = \lambda(p_{t-1} - z_{t-1}) - v_t, \quad (25)$$

where  $v_t$  is a zero-mean, serially-uncorrelated stochastic process, that is orthogonal to the information set available at the end of period  $t$ .

We first need to find the money supply rule that will be consistent with (25). The equation (25) implies

$$p_t - p_{t+1} = \lambda(p_{t-1} - p_t) - (1 - \lambda)\bar{\pi} - v_t + v_{t+1}. \quad (26)$$

Let  $\xi_t = \ln x_t$ . From equations (25) and (26) it follows that the monetary policy rule that is consistent with  $PT(\lambda)$  must satisfy:

$$m_{t+1} - m_t = \lambda(p_t - p_{t-1}) + (1 - \lambda)\bar{\pi} + \xi_{t+1} - \xi_t + v_t - v_{t+1}.$$

Taking conditional expectation  $E_t$  on both sides, we obtain the money supply rule:

$$m_{t+1} - m_t = \lambda(p_t - p_{t-1}) + (1 - \lambda)\bar{\pi} + E_t [\xi_{t+1}] - \xi_t + v_t, \quad (27)$$

which implies that  $v_{t+1} = \xi_{t+1} - E_t [\xi_{t+1}]$ , and  $E_t [v_{t+1}] = 0$ . Thus,  $v_{t+1}$  is indeed an innovation, and is, therefore, orthogonal to all the variables known as of period  $t$ :

$$E_t [\Omega_t v_{t+1}] = \Omega_t E_t [v_{t+1}] = 0.$$

Further, since  $E(v_t v_{t+1}) = E[E_t(v_t v_{t+1})]$  by the law of iterated expectations, and since  $E_t(v_t v_{t+1}) = v_t E_t(v_{t+1}) = 0$ , it follows that  $E(v_t v_{t+1}) = 0$ . Thus, the error process  $v_t$  is serially uncorrelated. Thus, the government must change the money supply according to (27) to guarantee that the price level follows the dynamic equation (25) in each period.

We are ready to derive the equation (14). From  $PT(\lambda)$  rule (25) and the fact that  $P_t = \frac{M_t}{x_t}$  it follows that

$$\frac{M_{t+1}}{x_{t+1} Z_{t+1}} = \left( \frac{M_t}{x_t Z_t} \right)^\lambda \exp(-v_{t+1}).$$

Which implies

$$\begin{aligned} \frac{M_{t+1}}{Z_{t+1}} &= \left( \frac{M_t}{Z_t} \right)^\lambda \frac{x_{t+1}}{x_t^\lambda} \exp(-\xi_{t+1} + E_t [\xi_{t+1}]) \\ &= \left( \frac{M_t}{Z_t} \right)^\lambda \frac{x_{t+1}}{x_t^\lambda} \exp(-\ln x_{t+1}) \exp(E_t [\ln x_{t+1}]) \\ &= \left( \frac{M_t}{Z_t} \right)^\lambda \frac{\exp(E_t [\ln x_{t+1}])}{x_t^\lambda}. \end{aligned}$$

Denoting  $h_t = \frac{M_t}{Z_t}$  we obtain the desired equation (14)

$$h_{t+1} = \left(\frac{h_t}{x_t}\right)^\lambda \exp(E_t[\ln x_{t+1}]).$$

## C Equivalence of loss functions and strict targeting rules

Suppose the monetary authority minimizes the loss function

$$L = E \sum_{t=1}^{\infty} \kappa^t (\pi_t - \bar{\pi})^2 \quad (28)$$

where  $\kappa$  is an arbitrary constant in  $(0, 1)$ , subject to  $\{\pi_t\}_{t=1}^{\infty}$  being consistent with a monetary equilibrium. The constraint says that in choosing its monetary policy the monetary authority has to respect the first-order conditions of individuals. Let's ignore the constraint for a moment, and see what the solution of an unconstrained problem is

$$\min L = E \sum_{t=1}^{\infty} \kappa^t (\pi_t - \bar{\pi})^2.$$

Due to our timing assumption the monetary authority has control of the next period expected inflation only, and not of the realized inflation. So we can rewrite the problem as

$$\min L = E \sum_{t=1}^{\infty} \kappa^t (E_{t-1}\pi_t + v_t - \bar{\pi})^2, \quad (29)$$

where  $v_t$  is part of inflation in period  $t$  that is not under the control of the monetary authority. The first order condition of the unconstrained problem is

$$2\kappa^t E_{t-1}(E_{t-1}\pi_t + v_t - \bar{\pi}) = 0$$

or equivalently,

$$E_{t-1}(\pi_t - \bar{\pi}) = 0$$

If the monetary authority sets  $E_{t-1}\pi_t = \bar{\pi}$  every period, then the first order conditions of the unconstrained problem are satisfied. Now under the strict

IT the monetary authority is required to have  $E_{t-1}\pi_t = \bar{\pi}$ . In any monetary equilibrium with this constraint imposed, the first order conditions of individuals hold as well. It follows then, that any strict IT regime monetary equilibrium, satisfies the necessary and sufficient conditions of the constrained problem with the loss function (28).

Now, to show the converse, suppose there is a solution of the constrained problem with the loss function (28) such that  $E_{t-1}\pi_t \neq \bar{\pi}$  in at least one period with positive probability. That means that the constraint set must be binding, and the value of the loss function must be greater than in the unconstrained problem. In which case we get a contradiction, because any strict IT regime equilibrium minimizes the unconstrained loss function, and thus must do better than our assumed solution. This proves the equivalence.

For the strict PT regimes and the loss function  $L = E \sum_{t=1}^{\infty} \kappa^t (p_t - z_t)^2$  ther logic is identical.

## D Solving the model with land, nominal bonds and money

We can state the problem of the young born in period t as follows:

$$\begin{aligned} \max \sum \Pr(s^t|s^{t-1})u & \left( A_t (L^d(s^{t-1}))^\alpha - \frac{q(s^{t-1})L^d(s^{t-1})}{d(s^{t-1})P(s^t)} - \frac{M^d(s^t)}{P(s^t)} \right) \\ + \beta \sum \Pr(s^{t+1}|s^t)u & \left( \frac{B^s(s^t)}{P(s^{t+1})} + \frac{q(s^t)L^d(s^{t-1}) + (M^d(s^t) + T(s^t) - d(s^t)B^s(s^t))}{P(s^{t+1})} \right), \end{aligned}$$

subject to the constraint

$$d(s^t)B^s(s^t) \leq M^d(s^t) + T(s^t). \quad (30)$$

The first-order conditions of the problem are:

$$0 = \sum \Pr(s^t|s^{t-1}) \left[ \begin{aligned} & u' \left( A_t L^\alpha - \frac{q(s^{t-1})L}{d(s^{t-1})P(s^t)} - \frac{M^d(s^t)}{P(s^t)} \right) \left( \alpha A_t L^{\alpha-1} - \frac{q(s^{t-1})}{d(s^{t-1})P(s^t)} \right) + \\ & \beta \sum \Pr(s^{t+1}|s^t) \left\{ u' \left( \frac{B^s(s^t)}{P(s^{t+1})} + \frac{q(s^t)L + (M^d(s^t) + T(s^t) - d(s^t)B^s(s^t))}{P(s^{t+1})} \right) \frac{q(s^t)}{P(s^{t+1})} \right\} \end{aligned} \right] \quad (31)$$

$$\begin{aligned}
0 &= \xi(s^t) - u' \left( A_t L^\alpha - \frac{q(s^{t-1})L}{d(s^{t-1})P(s^t)} - \frac{M^d(s^t)}{P(s^t)} \right) \frac{1}{P(s^t)} + & (32) \\
&\quad \beta \sum \Pr(s^{t+1}|s^t) \left[ u' \left( \frac{B^s(s^t)}{P(s^{t+1})} + \frac{q(s^t)L + (M^d(s^t) + T(s^t) - d(s^t)B^s(s^t))}{P(s^{t+1})} \right) \frac{1}{P(s^{t+1})} \right] \\
&\quad \beta \sum \Pr(s^{t+1}|s^t) \left[ u' \left( \frac{B^s(s^t)}{P(s^{t+1})} + \frac{q(s^t)L + (M^d(s^t) + T(s^t) - d(s^t)B^s(s^t))}{P(s^{t+1})} \right) \frac{1 - d(s^t)}{P(s^{t+1})} \right] \\
&= \xi(s^t)d(s^t), & (33)
\end{aligned}$$

where  $\xi(s^t) \geq 0$  is the Lagrange multiplier on the constraint (30).

From the last first-order condition it follows that  $d(s^t) \leq 1$ . It must be strictly less than 1 in equilibrium, and this implies that old agents will lend all of their money stock, while young agents will borrow just enough to buy land. So, in equilibrium we must have

$$\begin{aligned}
M^d(s^t) &= M(s^{t-1}) \\
q(s^t)L &= d(s^t)B^s(s^t) = M(s^{t-1}) + T(s^t) = M(s^t) & (34)
\end{aligned}$$

Taking into account the equilibrium conditions (34), we can rewrite the first order conditions (31)-(33) as follows

$$\begin{aligned}
0 &= \sum \Pr(s^t|s^{t-1}) \left[ u' \left( A_t L^\alpha - \frac{M(s^{t-1})}{d(s^{t-1})P(s^t)} - \frac{M(s^{t-1})}{P(s^t)} \right) \left( \alpha A_t L^{\alpha-1} - \frac{M(s^{t-1})/L}{d(s^{t-1})P(s^t)} \right) \right. \\
&\quad \left. + \beta \sum \Pr(s^{t+1}|s^t) \left\{ u' \left( \frac{M(s^t)}{d(s^t)P(s^{t+1})} + \frac{M(s^t)}{P(s^{t+1})} \right) \frac{M(s^t)/L}{P(s^{t+1})} \right\} \right] \\
0 &= \xi(s^t) - u' \left( A_t L^\alpha - \frac{M(s^{t-1})}{d(s^{t-1})P(s^t)} - \frac{M(s^{t-1})}{P(s^t)} \right) \frac{1}{P(s^t)} \\
&\quad + \beta \sum \Pr(s^{t+1}|s^t) \left[ u' \left( \frac{M(s^t)}{d(s^t)P(s^{t+1})} + \frac{M(s^t)}{P(s^{t+1})} \right) \frac{1}{P(s^{t+1})} \right] \\
\xi(s^t)d(s^t) &= \beta \sum \Pr(s^{t+1}|s^t) \left[ u' \left( \frac{M(s^t)}{d(s^t)P(s^{t+1})} + \frac{M(s^t)}{P(s^{t+1})} \right) \frac{1 - d(s^t)}{P(s^{t+1})} \right].
\end{aligned}$$

From the last equation find  $\xi(s^t)$

$$\xi(s^t) = \frac{1 - d(s^t)}{d(s^t)} \beta \sum \Pr(s^{t+1}|s^t) \left[ u' \left( \frac{M(s^t)}{d(s^t)P(s^{t+1})} + \frac{M(s^t)}{P(s^{t+1})} \right) \frac{1}{P(s^{t+1})} \right], \quad (35)$$

and then substitute the expression for  $\xi(s^t)$  into the previous two equations

$$0 = \sum \Pr(s^t|s^{t-1}) \left[ u' \left( A_t L^\alpha - \frac{M(s^{t-1})}{d(s^{t-1})P(s^t)} - \frac{M(s^{t-1})}{P(s^t)} \right) \left( \alpha A_t L^{\alpha-1} - \frac{M(s^{t-1})/L}{d(s^{t-1})P(s^t)} \right) + \frac{\xi(s^t)d(s^t)M(s^t)}{(1-d(s^t))L} \right]$$

$$\frac{\xi(s^t)}{1 - d(s^t)} = u' \left( A_t L^\alpha - \frac{M(s^{t-1})}{d(s^{t-1})P(s^t)} - \frac{M(s^{t-1})}{P(s^t)} \right) \frac{1}{P(s^t)}$$

Eliminating  $\xi(s^t)$  we obtain

$$0 = \sum \Pr(s^t|s^{t-1}) \left[ u' \left( A_t L^\alpha - \frac{M(s^{t-1})}{d(s^{t-1})P(s^t)} - \frac{M(s^{t-1})}{P(s^t)} \right) \times \left( \alpha A_t L^{\alpha-1} - \frac{M(s^{t-1})}{d(s^{t-1})P(s^t)L} + \frac{d(s^t)M(s^t)}{P(s^t)L} \right) \right].$$

So, we derived two dynamic equations

$$\beta \sum \Pr(s^{t+1}|s^t) \left[ u' \left( \frac{M(s^t)}{d(s^t)P(s^{t+1})} + \frac{M(s^t)}{P(s^{t+1})} \right) \frac{P(s^t)}{P(s^{t+1})} \right] \quad (36)$$

$$= u' \left( A_t L^\alpha - \frac{M(s^{t-1})}{d(s^{t-1})P(s^t)} - \frac{M(s^{t-1})}{P(s^t)} \right) d(s^t)$$

$$0 = \sum \Pr(s^t|s^{t-1}) \left[ u' \left( A_t L^\alpha - \frac{M(s^{t-1})}{d(s^{t-1})P(s^t)} - \frac{M(s^{t-1})}{P(s^t)} \right) \times \left( \alpha A_t L^{\alpha-1} - \frac{M(s^{t-1})}{d(s^{t-1})P(s^t)L} + \frac{d(s^t)M(s^t)}{P(s^t)L} \right) \right]. \quad (37)$$

In addition the money market clearing condition has to be satisfied

$$\frac{M(s^{t-1})}{P(s^t)} = \frac{M^d(s^t)}{P(s^t)} = A_t L^\alpha - \frac{B^d(s^{t-1})}{P(s^t)} - c^y(s^t)$$

$$= A_t L^\alpha - \frac{M(s^{t-1})}{d(s^{t-1})P(s^t)} - c^y(s^t),$$

and hence,

$$\frac{M(s^{t-1})}{P(s^t)} = \frac{A_t L^\alpha - c^y(s^t)}{1 + \frac{1}{d(s^{t-1})}}.$$



Let  $x(s^t) = \frac{M(s^{t-1})}{P(s^t)} = \frac{A_t L^\alpha - c^y(s^t)}{1 + \frac{1}{d(s^{t-1})}}$ . The money market clearing conditions, implies the law of motion for the price level

$$\frac{P(s^t)}{P(s^{t+1})} = \frac{M(s^{t-1})}{M(s^t)} \frac{x(s^{t+1})}{x(s^t)}. \quad (38)$$

Substituting that into the dynamic equations (36) and (37) we get

$$\begin{aligned} & \beta \sum \Pr(s^{t+1}|s^t) \left[ u' \left( \frac{x(s^{t+1})}{d(s^t)} + x(s^{t+1}) \right) \frac{M(s^{t-1})}{M(s^t)} \frac{x(s^{t+1})}{x(s^t)} \right] \\ &= u' \left( A_t L^\alpha - \frac{x(s^t)}{d(s^{t-1})} - x(s^t) \right) d(s^t) \\ 0 &= \sum \Pr(s^t|s^{t-1}) \left[ \begin{aligned} & u' \left( A_t L^\alpha - \frac{x(s^t)}{d(s^{t-1})} - x(s^t) \right) \\ & \times \left( \alpha A_t L^\alpha - \frac{x(s^t)}{d(s^{t-1})} + d(s^t) x(s^t) \frac{M(s^t)}{M(s^{t-1})} \right) \end{aligned} \right]. \end{aligned}$$

To close the system, we need to determine how  $\frac{M(s^{t-1})}{M(s^t)}$  changes over time. Let  $h(s^{t-1}) = \frac{M(s^{t-1})}{Z_t}$ . The appendix B shows that

$$h(s^t) = \left( \frac{h(s^{t-1})}{x(s^t)} \right)^\lambda \exp\left(\sum \Pr(s^{t+1}|s^t) [\ln x(s^{t+1})]\right).$$

We end up with a system of  $2S + 1$  dynamic equations (39)-(41) that must be solved to compute the monetary equilibrium

$$\begin{aligned} & u' \left( A_t L^\alpha - \frac{x(s^t)}{d(s^{t-1})} - x(s^t) \right) d(s^t) \exp(\bar{\pi}) \quad (39) \\ &= \beta \sum \Pr(s^{t+1}|s^t) \left[ u' \left( \frac{x(s^{t+1})}{d(s^t)} + x(s^{t+1}) \right) \frac{h(s^{t-1})}{h(s^t)} \frac{x(s^{t+1})}{x(s^t)} \right], \text{ for all } s_t \\ 0 &= \sum \Pr(s^t|s^{t-1}) \left[ \begin{aligned} & u' \left( A_t L^\alpha - \frac{x(s^t)}{d(s^{t-1})} - x(s^t) \right) \\ & \times \left( \alpha A_t L^\alpha - \frac{x(s^t)}{d(s^{t-1})} + d(s^t) x(s^t) \frac{h(s^t)}{h(s^{t-1})} \exp(\bar{\pi}) \right) \end{aligned} \right] \quad (40) \end{aligned}$$

$$h(s^t) = \left( \frac{h(s^{t-1})}{x(s^t)} \right)^\lambda \exp\left(\sum \Pr(s^{t+1}|s^t) [\ln x(s^{t+1})]\right), \text{ for all } s_t. \quad (41)$$

where  $x(s^t) = \frac{M(s^{t-1})}{P(s^t)}$  and  $h(s^t) = \frac{M(s^t)}{Z_{t+1}}$ .

Suppose we know  $x(s^{t+1})$ ,  $d(s^t)$ . We can solve the above  $2S + 1$  equations for  $2S + 1$  unknown functions:  $d(A_{t-1}, h(s^{t-1}))$ ,  $x(A_t, A_{t-1}, h(s^{t-1}))$ , and  $h'(A_t, A_{t-1}, h(s^{t-1}))$ . Note that in case of i.i.d. productivity shocks these three functions do not depend on  $A_{t-1}$ . This is because the only way  $A_{t-1}$  enters the equations above, is through conditional probabilities  $\Pr(s^t | s^{t-1})$ . With i.i.d. shocks those are independent of  $s^{t-1}$ , and we are looking for  $d(h(s^{t-1}))$ ,  $x(A_t, h(s^{t-1}))$ , and  $h'(A_t, h(s^{t-1}))$ .

In CES utility case the dynamic equations can be simplified somewhat

$$\begin{aligned} & \left( \frac{A_t L^\alpha - \left( \frac{1}{d(s^{t-1})} + 1 \right) x(s^t)}{\frac{1}{d(s^t)} + 1} \right)^{-\gamma} d(s^t) x(s^t) \exp(\bar{\pi}) \\ &= \beta \frac{h(s^{t-1})}{h(s^t)} \sum \Pr(s^{t+1} | s^t) [x(s^{t+1})]^{1-\gamma}, \text{ for all } s_t \\ 0 &= \sum \Pr(s^t | s^{t-1}) \left[ \begin{aligned} & \left( A_t L^\alpha - \left( \frac{1}{d(s^{t-1})} + 1 \right) x(s^t) \right)^{-\gamma} \\ & \times \left( \alpha A_t L^\alpha - \left( \frac{1}{d(s^{t-1})} - d(s^t) \frac{h(s^t)}{h(s^{t-1})} \exp(\bar{\pi}) \right) x(s^t) \right) \end{aligned} \right] \\ h(s^t) &= \left( \frac{h(s^{t-1})}{x(s^t)} \right)^\lambda \exp\left( \sum \Pr(s^{t+1} | s^t) [\ln x(s^{t+1})] \right), \text{ for all } s_t. \end{aligned}$$

## E Derivation of the optimal inflation

The same welfare criterion as in the money only economy results in the following first-best allocations

$$\begin{aligned} c_t^{y*} &= \frac{1}{1 + \beta^{\frac{1}{\gamma}}} A_t L^\alpha \\ c_t^{o*} &= \frac{\beta^{\frac{1}{\gamma}}}{1 + \beta^{\frac{1}{\gamma}}} A_t L^\alpha. \end{aligned}$$

The first-order conditions (31) (33) with the optimal allocations become:

$$0 = \sum \Pr(s^t | s^{t-1}) \left[ \begin{aligned} & \left( \frac{1}{1 + \beta^{\frac{1}{\gamma}}} A_t L^\alpha \right)^{-\gamma} \left( \alpha A_t L^{\alpha-1} - \frac{M(s^{t-1})/L}{d(s^{t-1})P(s^t)} \right) + \\ & + \beta \sum \Pr(s^{t+1} | s^t) \left[ \left( \frac{\beta^{\frac{1}{\gamma}}}{1 + \beta^{\frac{1}{\gamma}}} A_{t+1} L^\alpha \right)^{-\gamma} \frac{M(s^t)/L}{P(s^{t+1})} \right] \end{aligned} \right]$$

$$\begin{aligned}
0 &= \xi(s^t) - \left( \frac{1}{1 + \beta^{\frac{1}{\gamma}}} A_t L^\alpha \right)^{-\gamma} \frac{1}{P(s^t)} \\
&\quad + \beta \sum \Pr(s^{t+1}|s^t) \left[ \left( \frac{\beta^{\frac{1}{\gamma}}}{1 + \beta^{\frac{1}{\gamma}}} A_{t+1} L^\alpha \right)^{-\gamma} \frac{1}{P(s^{t+1})} \right] \\
\frac{\xi(s^t)d(s^t)}{1 - d(s^t)} &= \beta \sum \Pr(s^{t+1}|s^t) \left[ \left( \frac{\beta^{\frac{1}{\gamma}}}{1 + \beta^{\frac{1}{\gamma}}} A_{t+1} L^\alpha \right)^{-\gamma} \frac{1}{P(s^{t+1})} \right].
\end{aligned}$$

After simplifications, we obtain

$$\begin{aligned}
0 &= \sum \Pr(s^t|s^{t-1}) \left[ A_t^{-\gamma} \left( \alpha A_t L^\alpha - \frac{M(s^{t-1})}{d(s^{t-1})P(s^t)} \right) + \sum \Pr(s^{t+1}|s^t) \left\{ A_{t+1}^{-\gamma} \frac{M(s^t)}{P(s^{t+1})} \right\} \right] \\
d(s^t) &= \sum \Pr(s^{t+1}|s^t) \left[ \left( \frac{A_{t+1}}{A_t} \right)^{-\gamma} \frac{P(s^t)}{P(s^{t+1})} \right]
\end{aligned}$$

From the money market clearing conditions

$$P(s^t) = \frac{M(s^{t-1})}{x(s^t)},$$

where the optimal  $x(s^t)$  is given by

$$x(s^t) = \frac{\beta^{\frac{1}{\gamma}}}{\left(1 + \beta^{\frac{1}{\gamma}}\right) \left(\frac{1}{d(s^{t-1})} + 1\right)} A_t L^\alpha.$$

Substituting these values of  $P(s^t)$ , and  $x(s^t)$ , we obtain

$$0 = \sum \Pr(s^t|s^{t-1}) \left[ \begin{aligned} &A_t^{-\gamma} \left( \alpha A_t L^\alpha - \frac{\beta^{\frac{1}{\gamma}} A_t L^\alpha}{\left(1 + \beta^{\frac{1}{\gamma}}\right) \left(\frac{1}{d(s^{t-1})} + 1\right) d(s^{t-1})} \right) \\ &+ \sum \Pr(s^{t+1}|s^t) \left\{ A_{t+1}^{-\gamma} \frac{\beta^{\frac{1}{\gamma}}}{\left(1 + \beta^{\frac{1}{\gamma}}\right) \left(\frac{1}{d(s^t)} + 1\right)} A_{t+1} L^\alpha \right\} \end{aligned} \right]$$

$$d(s^t) = \sum \Pr(s^{t+1}|s^t) \left[ \left( \frac{A_{t+1}}{A_t} \right)^{-\gamma} \frac{M(s^{t-1}) \frac{\beta^{\frac{1}{\gamma}}}{(1+\beta^{\frac{1}{\gamma}}) \left( \frac{1}{d(s^t)} + 1 \right)}}{M(s^t) \frac{\beta^{\frac{1}{\gamma}}}{(1+\beta^{\frac{1}{\gamma}}) \left( \frac{1}{d(s^{t-1})} + 1 \right)}} A_{t+1} L^\alpha \right].$$

Which can be further simplified to

$$0 = \sum \Pr(s^t|s^{t-1}) \left[ A_t^{1-\gamma} \left( \alpha - \frac{\beta^{\frac{1}{\gamma}}}{(1+\beta^{\frac{1}{\gamma}}) \left( \frac{1}{d(s^{t-1})} + 1 \right) d(s^{t-1})} \right) + \frac{\beta^{\frac{1}{\gamma}}}{(1+\beta^{\frac{1}{\gamma}}) \left( \frac{1}{d(s^t)} + 1 \right)} \sum \Pr(s^{t+1}|s^t) [A_{t+1}^{1-\gamma}] \right]$$

$$d(s^t) = \sum \Pr(s^{t+1}|s^t) \left[ \left( \frac{A_{t+1}}{A_t} \right)^{1-\gamma} \frac{M(s^{t-1}) \frac{1}{d(s^{t-1})} + 1}{M(s^t) \frac{1}{d(s^t)} + 1} \right].$$

For brevity let's use the expectation operator

$$\left( 1 - \frac{1}{\frac{1}{d(s^{t-1})} + 1} - \frac{\alpha \left( 1 + \beta^{\frac{1}{\gamma}} \right)}{\beta^{\frac{1}{\gamma}}} \right) E_{t-1} [A_t^{1-\gamma}] = E_{t-1} \left[ \frac{1}{\frac{1}{d(s^t)} + 1} E_t [A_{t+1}^{1-\gamma}] \right] \quad (42)$$

$$\frac{M(s^t)}{M(s^{t-1})} = \frac{1}{d(s^t)} \frac{\frac{1}{d(s^{t-1})} + 1}{\frac{1}{d(s^t)} + 1} E_t \left[ \left( \frac{A_{t+1}}{A_t} \right)^{1-\gamma} \right]. \quad (43)$$

Let  $\Psi(s^t) = \frac{1}{\frac{1}{d(s^t)} + 1} E_t [A_{t+1}^{1-\gamma}]$ , then we can rewrite (42) as

$$\left( 1 - \frac{\alpha \left( 1 + \beta^{\frac{1}{\gamma}} \right)}{\beta^{\frac{1}{\gamma}}} \right) E_{t-1} [A_t^{1-\gamma}] = \Psi(s^{t-1}) + E_{t-1} [\Psi(s^t)]. \quad (44)$$

Let us concentrate on stationary (first-best) equilibria, i.e let  $\Psi(s^t) = \Psi(s_t)$ . The last equation is a functional equation which determines  $\Psi(A_t)$ . Once  $\Psi(A_t)$  is known, we can use (43) to solve for the optimal monetary policy.

The optimal inflation is

$$\frac{P(s^{t+1})}{P(s^t)} = \frac{M(s^t)}{M(s^{t-1})} \frac{x(s^t)}{x(s^{t+1})} \quad (45)$$

$$= \frac{1}{d(s^t)} E_t \left[ \left( \frac{A_{t+1}}{A_t} \right)^{1-\gamma} \right] \frac{A_t}{A_{t+1}}. \quad (46)$$

Taking logs on both sides, and using the property of log-normal distribution we obtain:

$$p(s^{t+1}) - p(s^t) = \ln \left( \frac{1}{d(s^t)} \right) + \frac{(1-\gamma)^2 \sigma^2}{2} + \gamma(1-\rho)a_t - \varepsilon_{t+1}. \quad (47)$$

It is easy to see from the equation (42) that the discount rate  $d(s^t)$  is a constant in three special cases: first, when shocks to productivity are i.i.d. ( $\rho = 0$ ), second, when shocks to productivity are random walk ( $\rho = 1$ ), and third, when utility is logarithmic ( $\gamma = 1$ ). Let us prove this for an i.i.d. case.

When productivity shocks are i.i.d., the equation (42) simplifies to

$$1 - \frac{\alpha \left( 1 + \beta^{\frac{1}{\gamma}} \right)}{\beta^{\frac{1}{\gamma}}} = \frac{1}{\frac{1}{d(s^{t-1})} + 1} + E_{t-1} \left[ \frac{1}{\frac{1}{d(s^t)} + 1} \right]. \quad (48)$$

Thus, we obtain two dynamic equations

$$\begin{aligned} \frac{M(s^t)}{M(s^{t-1})} &= \frac{1}{d(s^t)} \frac{\frac{1}{d(s^{t-1})} + 1}{\frac{1}{d(s^t)} + 1} E_t \left[ \left( \frac{A_{t+1}}{A_t} \right)^{1-\gamma} \right] \\ 1 - \frac{\alpha \left( 1 + \beta^{\frac{1}{\gamma}} \right)}{\beta^{\frac{1}{\gamma}}} &= \frac{1}{\frac{1}{d(s^{t-1})} + 1} + E_{t-1} \left[ \frac{1}{\frac{1}{d(s^t)} + 1} \right]. \end{aligned} \quad (49)$$

Since we are focusing on stationary equilibria, assume  $\frac{M(s^t)}{M(s^{t-1})} = \mu(s_t)$  and  $d(s^t) = d(s_t)$ . Then the first equation implies that  $d(s_{t-1})$  is independent of  $s_{t-1}$ . Thus  $d(s_t) = d$  for all  $s^t$  and for all  $t$ . Then the equation (49) can be solved for  $d$

$$d = \frac{\beta^{\frac{1}{\gamma}}(1-\alpha) - \alpha}{\beta^{\frac{1}{\gamma}}(1+\alpha) + \alpha}.$$

## F Figures

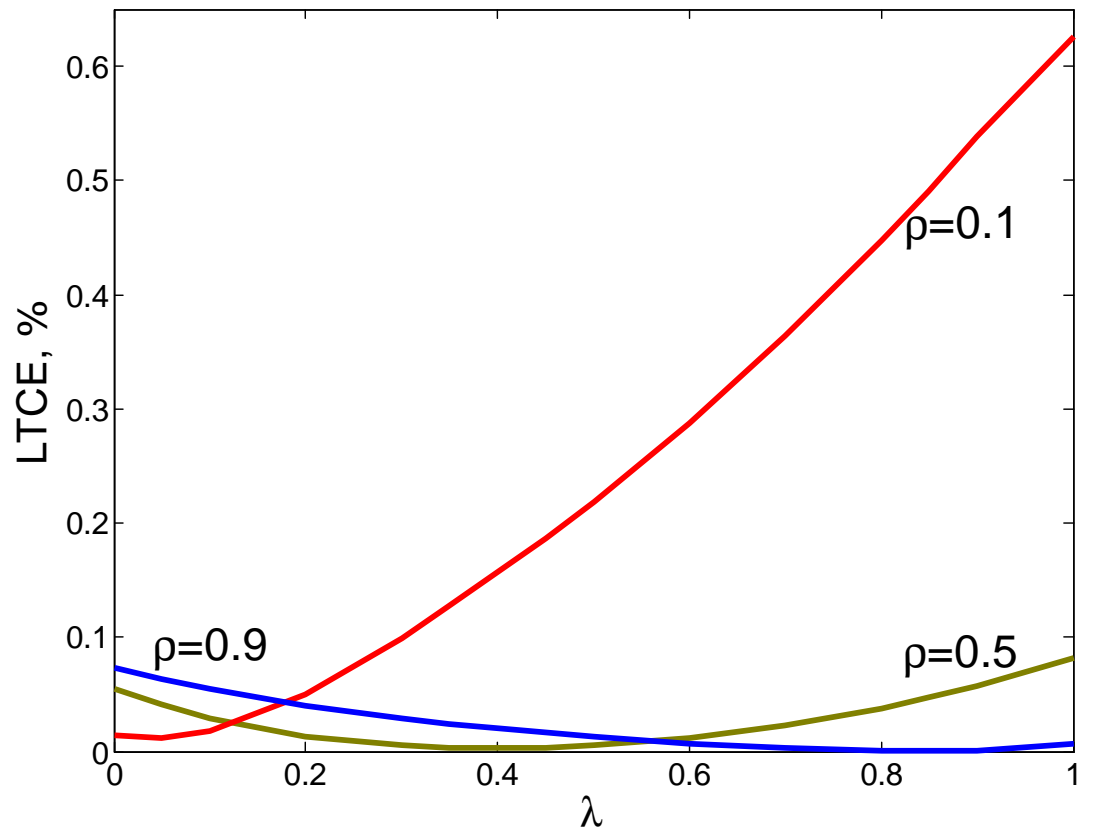


Figure 1: Parameters:  $\beta = 0.96^{30}$ ,  $\gamma = 1.5$ ,  $\sigma = 0.08$

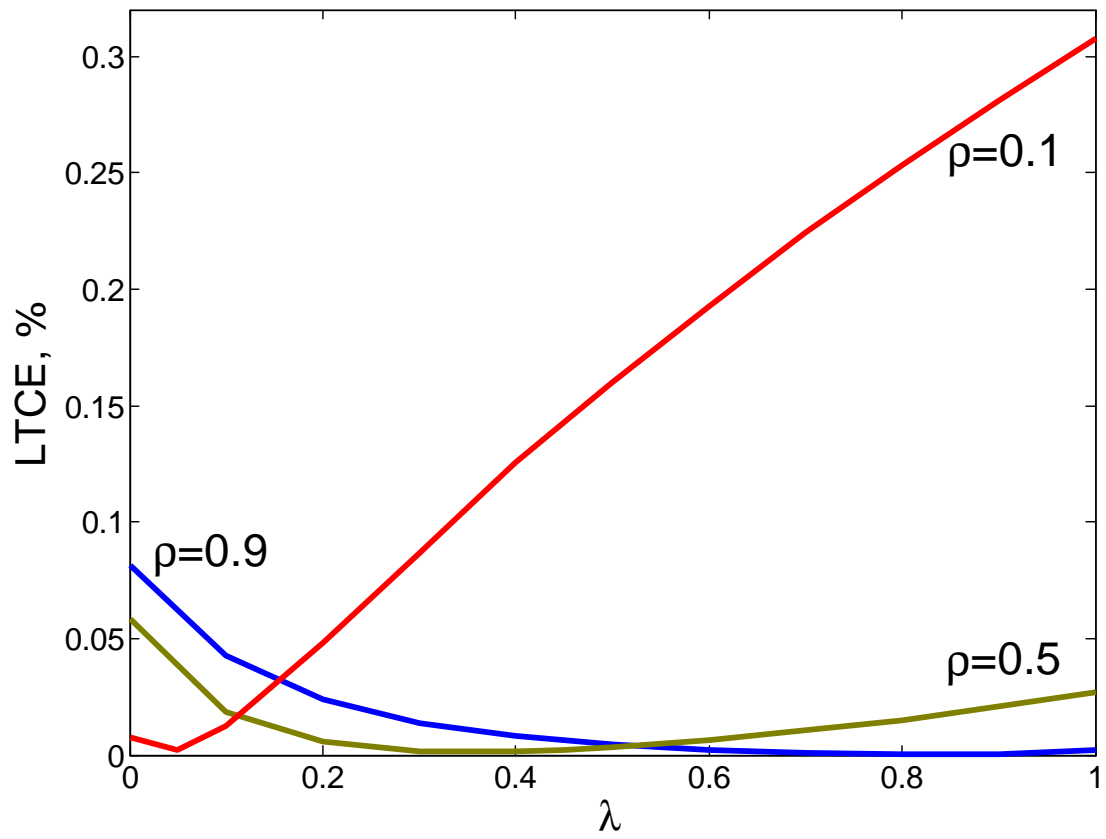


Figure 2: Parameters:  $\beta = 0.96^{30}$ ,  $\gamma = 1.5$ ,  $\sigma = 0.08$ ,  $\alpha = 0.12$

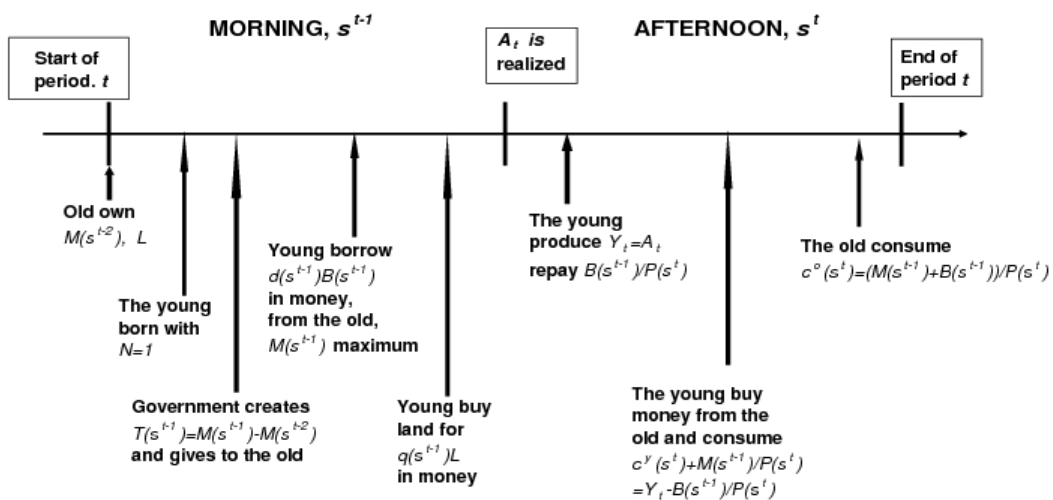


Figure 3: Timing of events in the model with land