Equilibria in a Capacity-Constrained Differentiated Duopoly

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Preliminary and incomplete.

Abstract

In this paper I analyze the model of price-setting duopoly with capacity-constrained firms producing differentiated products. I show that a pure strategy equilibrium exists if the capacities of the firms are both small or both large. For the other capacity values, mixed strategy equilibria exist, with the support of the price distribution either being an interval or consisting of a finite number of points. I show that if before the price competition the firms simultaneously choose their capacity levels, their optimal choice of capacities would lead to a pure strategy equilibrium in prices. The optimal capacity levels decrease with an increase in consumer heterogeneity.

1 Introduction

Recent advances in estimations of demands for differentiated products necessitate research in the supply side of the market – how the firms act when facing such demands. While this question has been studied thoroughly for the firms that are not constrained in their production, the literature on the behavior of the capacity-constrained firms producing differentiated products is scarce. In this work, I characterize and compute the equilibria that occur in the situations, when the firms are capacity-constrained.

The literature on price competition with capacity constraints initially considered only homogeneous products (Bertrand-Edgeworth competition). Due to discontinuities in demand and profit functions, pure strategy equilibria do not always exist, thus, it was necessary to work with mixed strategy equilibria. First, the equilibria in symmetric Bertrand-Edgeworth models were computed for the proportional (Beckmann (1965)) and surplus-maximizing (Levitan and Shubik (1972)) rationing rules for unsatisfied demand. Kreps and Scheinkman (1983) characterized the mixed strategy equilibrium for asymmetric capacities using the surplus-maximizing rule. This allowed them to solve a two-stage game, in which the firms first choose capacities and then compete in prices. They found that the firms' choice of capacities in the first stage coincides with the Cournot equilibrium – the production levels the firms would choose if they were to compete in quantities.

Davidson and Deneckere (1986) argued that this result crucially depends on the assumption of the surplus-maximizing rationing rule, and it does not hold for almost all other rationing rules. They showed that for the proportional rationing rule, the firms necessarily will choose capacities that exceed the Cournot level, and the equilibria are asymmetric if the capacity cost is small. Furthermore, the mixed strategy equilibria for the general form of the demand function were characterized by Osborne and Pitchik (1986) for the surplus-maximizing rationing rule and Allen and Hellwig (1993) for the proportional rationing rule.

Only recently, models of price competition with capacity constraints were examined for differentiated products (Bertrand-Edgeworth-Chamberlin competition). Benassy (1989) proved that a pure strategy equilibrium in these models fails to exist if the degree of substitutability of the products is large enough.[1] In a related paper, Canoy (1996) provided a parametrized duopoly example in which a pure strategy equilibrium does not exist if

[1] Benassy (1989) also shows that for a given degree of product substitutability a pure strategy equilibrium exists if the number of competitors is large enough.

the products are sufficiently similar. Thus, it was established that when the products are sufficiently homogeneous only the mixed strategy equilibrium exists. However, no attempt has been made to characterize and study the equilibrium in mixed strategies for the Bertrand-Edgeworth-Chamberlin competition. Undertaking this study is the goal of this paper.

I take the demand to have a logit specification, which is a form of demand for differentiated products, widely used in empirical literature. The firms produce at a constant marginal cost, but are limited in production by their capacity constraints. I show that a pure strategy price equilibrium exists whenever the firms' capacities are both small or both large. For the other values of capacities, a pure strategy price equilibrium does not exist, and I compute the mixed strategy equilibrium. For the case of symmetric capacities in the intermediate range, this equilibrium involves a finite support for the optimal price distributions. For the case of asymmetric capacities, there are also mixed equilibria, for which the support of the price distributions is an interval.

The knowledge of the firms' profits in the pricing game allows me to find the equilibrium in a two-stage game, where the firms first choose the capacity levels. The optimal capacity levels lead to pure strategy pricing in the second stage. The capacities increase with a decline in consumer heterogeneity.

2 Basic Model

Consider a market where a particular good is produced by 2 firms at a zero marginal cost. Firms compete in prices p. The set of consumers has measure 1. Consumers have heterogeneous tastes for the goods produced by the firms. Each consumer receives utility $U_i = -p_i + \mu\varepsilon_i$ from purchasing a product from firm i. ε_i is iid standard double exponential. μ is the measure of consumer heterogeneity. An outside option gives the consumers a utility of $U_0 = -V_0 + \mu\varepsilon_0$. Thus, an outside option could be considered as another product which is sold at a fixed price of V_0. Consumers purchase the product that gives them the highest utility. Then, the demand faced by firm i is a standard logit demand: $G_i(p_i, p_j) = \frac{e^{-p_i/\mu}}{e^{-p_i/\mu}+e^{-p_j/\mu}+e^{-V_0/\mu}}$.

On the supply side, assume that both firms have zero marginal cost, but can produce only up to capacity K_i. Given the nature of the consumers' utility functions the rationing rule for unsatisfied demand is proportional. If the first firm's demand $G_1(p_1, p_2)$ is greater than K_1, the remaining $(1-K_1)$ consumers have iid draws for ε_0 and ε_2, thus, the residual demand is $(1-K_1)\frac{e^{-p_2/\mu}}{e^{-p_2/\mu}+e^{-V_0/\mu}}$. In summary, the contingent demand of firm i is

$$\widetilde{G}_i(p_i, p_j) = \begin{cases} G_i(p_i, p_j) & \text{if } G_j(p_j, p_i) \leqslant K_j \\ (1-K_j)\frac{e^{-p_i/\mu}}{e^{-p_i/\mu}+e^{-V_0/\mu}} & \text{if } G_j(p_j, p_i) > K_j \end{cases}$$

This section deals with the symmetric case when both firms have the same capacity constraint $K = K_1 = K_2$, and the next section will address the case $K_1 \neq K_2$. First, I will examine for what values of K and μ a pure strategy equilibrium exists. There are two different cases: (a) both firms produce at the capacity constraint; and (b) both firms produce at the level below the capacity constraint.

Production at the capacity constraint.

A single-price equilibrium with both firms operating at capacity constraints exists when the parameters satisfy the following conditions[2]:

$$\begin{aligned} \mu &< \frac{V_0}{\frac{1-K}{1-2K} + \ln\left(\frac{K}{1-2K}\right)}, \text{ if } \frac{1-K}{1-2K} + \ln\left(\frac{K}{1-2K}\right) > 0 \\ \mu &> 0, \qquad\qquad\qquad\qquad \text{if } \frac{1-K}{1-2K} + \ln\left(\frac{K}{1-2K}\right) \leqslant 0 \end{aligned} \tag{1}$$

The prices charged by the firms are

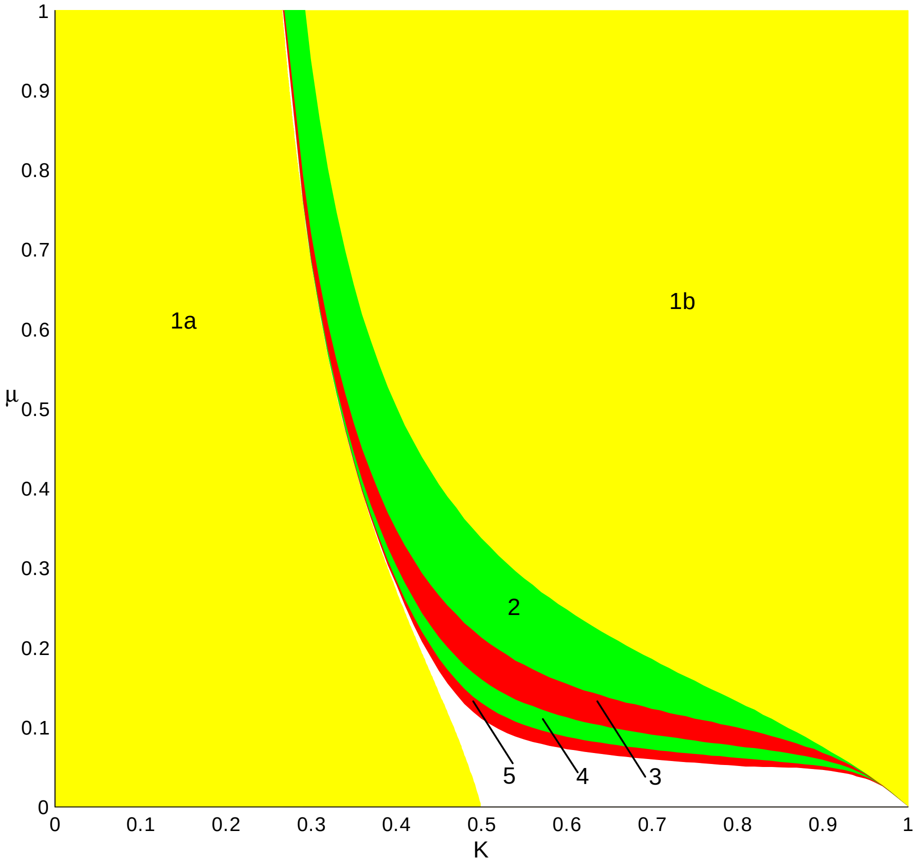
$$p^* = V_0 - \mu \ln\left(\frac{K}{1-2K}\right) \tag{2}$$

[2]All derivations are in the Appendix.

$\frac{1-K}{1-2K} + \ln\left(\frac{K}{1-2K}\right) = 0$ for $K = K_a \approx 0.178855$. For the smaller values of K, $\frac{1-K}{1-2K} + \ln\left(\frac{K}{1-2K}\right)$ is less than zero. Therefore, a single price equilibrium exists for all levels of consumer heterogeneity when the capacity constraints are small (K is less than K_a). For the larger values of K, when the consumer heterogeneity is large enough, a single price equilibrium with both firms reaching their capacity levels fails to exist. When V_0 is equal to zero or is negative and $K > K_a$, this type of a single price equilibrium never exists.

Region 1a in figure 1 shows the parameter values for which (1) holds.

Figure 1: Types of equilibrium in the symmetric case for $V_0 = 1$.



Production at the level below the capacity constraint.

If both firms are producing at the level below their capacity constraint, they charge the prices p^{**} that satisfy the following equation:

$$\mu = p^{**} \frac{e^{-\frac{p^{**}}{\mu}} + e^{-\frac{V_0}{\mu}}}{2e^{-\frac{p^{**}}{\mu}} + e^{-\frac{V_0}{\mu}}} \tag{3}$$

The firm earns a profit of $\pi^{**} = p^{**} G_i(p^{**}, p^{**})$.

For these prices to be a global maximum, π^{**} must be higher than the monopoly profit on residual demand. Another potential local maximum could occur at a higher price $\widehat{p}$, when the rival is capacity-constrained. This price solves the equation

$$\mu = \widehat{p} \frac{e^{-\frac{V_0}{\mu}}}{e^{-\frac{\widehat{p}}{\mu}} + e^{-\frac{V_0}{\mu}}}, \tag{4}$$

and gives a profit $\widehat{\pi} = \widehat{p}(1-K)\frac{e^{-\frac{\widehat{p}}{\mu}}}{e^{-\frac{\widehat{p}}{\mu}}+e^{-\frac{V_0}{\mu}}}$. p^{**} is a global maximum only if $\pi^{**} > \widehat{\pi}$ or if the rival is not capacity-constrained when the price is $\widehat{p}$. That is, the following inequalities must hold:

$$\pi^{**} > \widehat{\pi} \text{ or } G_i(p^{**}, \widehat{p}) \leqslant K, \tag{5}$$

where p^{**} is the solution to (3), and $\widehat{p}$ is the solution to (4). The region where both firms are producing at the level below their capacity constraint is depicted in Figure 1 as 1b.

As μ goes to infinity, the boundary of the region 1b converges to the value K_b. It is possible to find the value of K_b. First, (3) could be rewritten as $1 = \frac{p^{**}}{\mu}\frac{e^{-\frac{p^{**}}{\mu}}+e^{-\frac{V_0}{\mu}}}{2e^{-\frac{p^{**}}{\mu}}+e^{-\frac{V_0}{\mu}}}$. When $\mu \to \infty$, $e^{-\frac{V_0}{\mu}} \to 1$. So, $\frac{p^{**}}{\mu} \to t \approx 1.2268$. Similarly, (4) could be rewritten as $1 = \frac{\widehat{p}}{\mu}\frac{e^{-\frac{V_0}{\mu}}}{e^{-\frac{\widehat{p}}{\mu}}+e^{-\frac{V_0}{\mu}}}$, from where $\frac{\widehat{p}}{\mu} \to q \approx 1.2785$. The boundary of the region 1b is described by the equation $\pi^{**} = \widehat{\pi}$ or $\widehat{p}(1-K)\frac{e^{-\frac{\widehat{p}}{\mu}}}{e^{-\frac{\widehat{p}}{\mu}}+e^{-\frac{V_0}{\mu}}} = p^{**}\frac{e^{-\frac{p^{**}}{\mu}}}{2e^{-\frac{p^{**}}{\mu}}+e^{-\frac{V_0}{\mu}}}$, from where $K = 1 - \left(\frac{p^{**}}{\mu}\frac{e^{-\frac{p^{**}}{\mu}}}{2e^{-\frac{p^{**}}{\mu}}+e^{-\frac{V_0}{\mu}}}\right) / \left(\frac{\widehat{p}}{\mu}\frac{e^{-\frac{\widehat{p}}{\mu}}}{e^{-\frac{\widehat{p}}{\mu}}+e^{-\frac{V_0}{\mu}}}\right)$. Using the values for t and q found previously, $K \to K_b \approx 0.1858$ as $\mu \to \infty$.

The following statements summarize the findings about the existence of a pure strategy equilibrium:

1) For the low levels of the capacity constraint ($K \leqslant K_a \approx 0.178855$) a pure strategy equilibrium always exist with both firms producing at the capacity constraint level and charging price p^* from (2).

2) For the very small region of low capacities ($K_a < K \leqslant K_b \approx 0.1858$) a pure strategy equilibrium exists for the low enough levels of consumer heterogeneity (region 1a in Figure 1). For the high levels of μ a pure strategy equilibrium does not exist.

2) For the intermediate levels of the capacity constraint ($K_b < K < 0.5$) a pure strategy equilibrium exists for the high levels of consumer heterogeneity μ (region 1b in Figure 1) and for the low levels of consumer heterogeneity (region 1a in Figure 1). There always exist intermediate values of μ, for which a pure strategy equilibrium does not exist.

3) For the high levels of the capacity constraint ($K \geqslant 0.5$) a pure strategy equilibrium exists only when the consumer heterogeneity is high enough (region 1b in Figure 1).

4) For any level of the consumer heterogeneity a pure strategy equilibrium exists for the low levels of capacity constraint (region 1a in Figure 1) and for the high levels of capacity constraint (region 1b in Figure 1), but not for the intermediate values. As μ decreases, the region, where a pure strategy equilibrium does not exist, increases.

These conclusions are robust to the changes in V_0. If V_0 is increasing, region 1a increases in size with its boundary approaching the vertical line at 0.5 while region 1b decreases. If V_0 is approaching zero, region 1a decreases in size with its boundary approaching zero for the values of $K > K_a$, while region 1b increases.

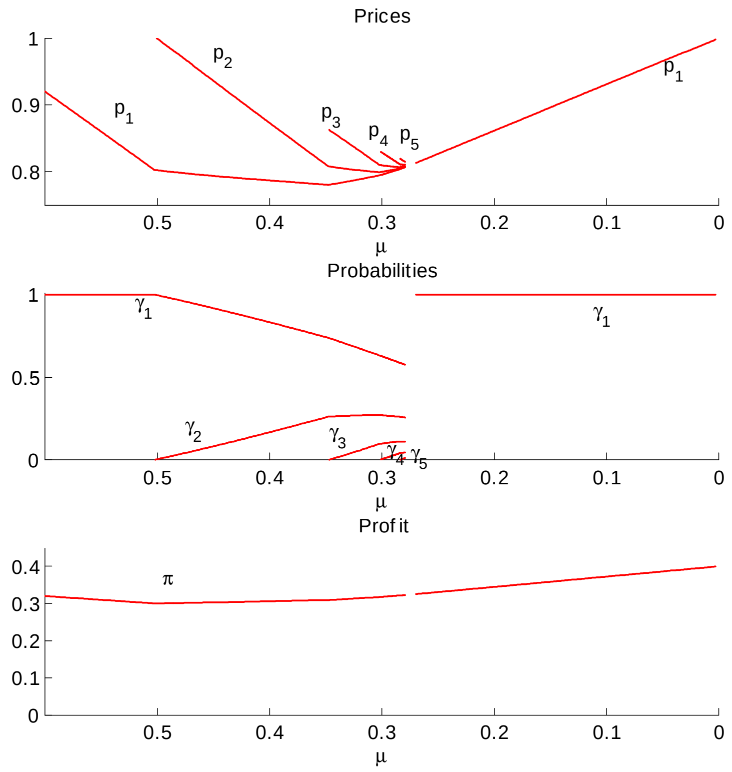
In the area, where a pure strategy equilibrium does not exist, a mixed strategy equilibrium exists.[3] Theorem 1 in Sinitsyn (2006) shows that if the demands are analytic, a mixed strategy price equilibrium has to have a finite support. In our case, the demands are not analytic – they have a kink at the price which makes the rival capacity-constrained.

[3] The existence result is due to Glicksberg (1952).

Nevertheless, I found that for the symmetric case a mixed strategy equilibrium also has a finite support.

Figure 2 illustrates the pricing strategies of the firms for the small capacity levels.

Figure 2: Equilibrium pricing strategies and profits for $K = 0.4$.



I fix K to be 0.4 and change the level of consumer heterogeneity μ. I start with a relatively high level of μ ($\mu = 0.6$) and then decrease it. For the high values of μ a pure strategy price equilibrium exists (the parameters fall in region 1b from Figure 1) with both firms operating under their capacity constraints. As μ declines, the demands become more elastic, the equilibrium more competitive, and the optimal prices decrease. Finally, the prices reach the level $\underline{p}$, where it becomes equally profitable for the firms to deviate to the high price $\overline{p}$ as $\pi(\overline{p}, \underline{p}) = \pi(\underline{p}, \underline{p})$ (μ is approximately equal to 0.5). This point is the boundary of the region 1b from Figure 1. If μ keeps decreasing, a pure strategy equilibrium does not exist anymore as the firms prefer deviation to the high price. However, an equilibrium with two prices appears.

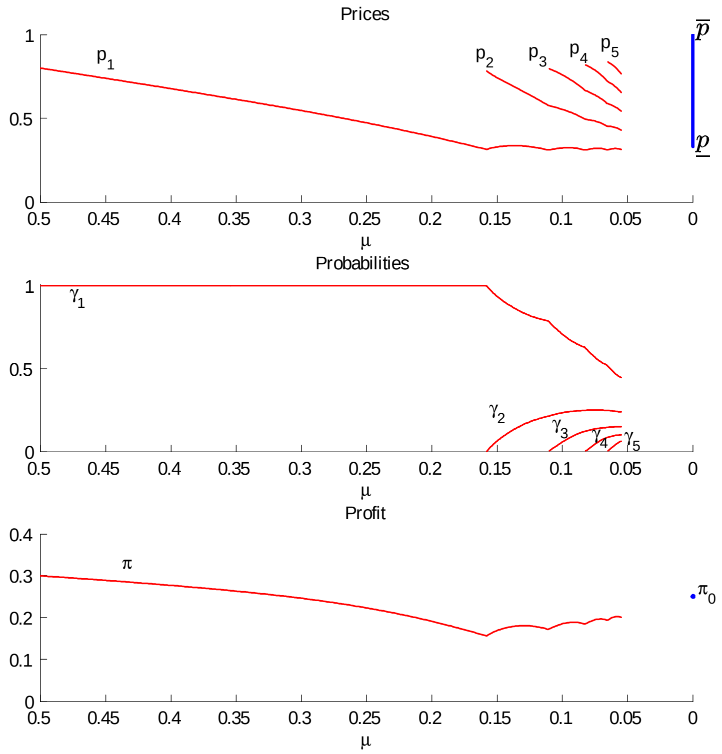
Each firm charges two prices – p_1 and p_2 with corresponding probabilities γ_1 and γ_2. From figure 2 it is evident that when the two prices are charged the lower of two prices – p_1 – does not decrease as steeply as in the region with one price only. This means that if p_1 were the only price charged, not only there would be a profitable deviation to the higher price p_2, but also there would be an incentive to undercut p_1. This does not happen because the presence of a higher price p_2 charged with some probability γ_2 reduces the undercutting incentives. Similarly, a price p_2 charged with probability 1 will also be undercut, but the fact that there is a lower price p_1 charged with some probability γ_1 keeps the high price at p_2. This happens because in the neighborhood of p_2 the rival charging a lower price is always capacity-constrained, so there is an incentive to charge a price higher than p_2. In equilibrium, this incentive is exactly offset by the undercutting incentive and the optimal price is p_2.

As μ keeps decreasing, the incentive to undercut also increase. This puts a greater weight on p_2 (γ_2 increases) and decreases both prices. Surprisingly, the profit increases slightly as the effect of putting a greater weight on a higher price p_2 outweighs the effect of declining prices. Finally, μ declines to such a level (approximately 0.345 in Figure 2) that again a profitable deviation appears. The firms start charging three prices and the cycle keeps repeating.

In Figure 2 I calculated the mixed strategy equilibria with up to five prices charged by the firms. It could be observed that the range of the prices charged decreases and finally collapses to a point when μ reaches approximately 0.27. This is a boundary point of the region 1a from Figure 1 – so both firms operate at the capacity constraint and the single price equilibrium reappears. As could be seen from (2), the optimal price p^* will increase with decline of μ only if $\ln\left(\frac{K}{1-2K}\right)$ is positive, which holds for $K > 1/3$. Since in Figure 2 $K = 0.4$, the optimal price and the profit increase as μ declines.

Figure 3 illustrates the optimal pricing strategies for the large capacity levels (K is taken to be 0.75 for this figure).

Figure 3: Equilibrium pricing strategies and profits for $K = 0.75$.



The movement of prices and the emergence of new equilibria with multiple prices is similar to the process described for the small capacity levels and illustrated in Figure 2. The only major difference between the cases with small ($K < 0.5$) and large capacities is that for the large capacities, the equilibria with multiple prices do not converge to a single price equilibrium as μ decreases. Instead they approach a mixed strategy equilibrium with an interval for a price support. In Figure 3 the price range of the mixed strategy equilibrium for $\mu = 0$ is illustrated by the segment $[\underline{p}; \overline{p}]$ and the corresponding profit is labeled π_0.

Figure 1 shows the regions, where mixed strategy equilibria with the firms charging multiple prices exist. Each region is numbered in accordance with the number of prices the firms use. In the unshaded region in Figure 1 both firms use more than five prices in the equilibrium.

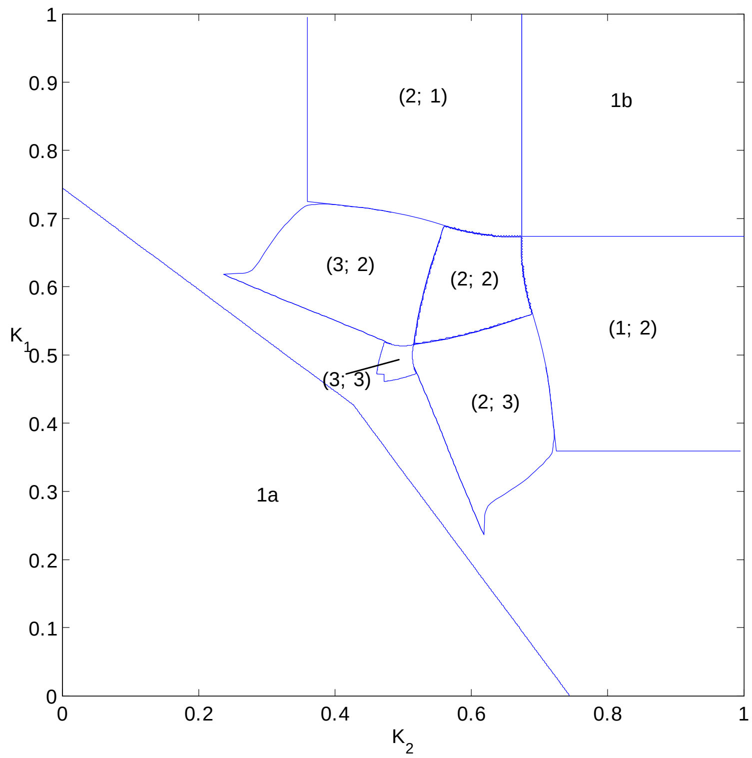
3 Asymmetric Capacities

Now, I will consider the case $K_1 \neq K_2$. This will serve as a building block for solving the two-stage game, in which the firms first choose capacities and then compete in prices.

Pure strategy equilibria

The case where both firms produce at their capacity level or both firms produce at the level below their capacity is handled exactly the same way as in the symmetric case. Figure 4 shows the regions where such equilibria exist as 1a and 1b, correspondingly.

Figure 4: Types of equilibria in the case of asymmetric capacities for $V_0 = 1$ and $\mu = 0.2$.



It seems intuitively plausible that there could exist pure strategy equilibria with one (larger) firm producing below its capacity level and another (smaller) firm producing at its capacity level. However, the following proposition proves that such equilibria do not exist.

Proposition 1 *A pure strategy equilibrium with one firm producing at the level below its capacity constraint, and another firm producing at its capacity constraint does not exist.*

Proof. Without loss of generality assume that the first firm produces at the level below its capacity constraint ($\widetilde{G}_1(p_1, p_2) < K_1$) and the second firm produces at its capacity constraint ($\widetilde{G}_2(p_2, p_1) = K_2$). I will examine the behavior of the profit function of the first firm at p_1. If the first firm charges a price lower than p_1 the second firm operates at the level below its capacity constraint, otherwise the second firm could profitably increase its price. Therefore, to the left of p_1 the profit function of the first firm is $\pi_-(p_1, p_2) = p_1 G_1(p_1, p_2)$. When the first firm charges a price above p_1 the second firm is capacity-constrained, thus the profit to the right of p_1 is $\pi_+(p_1, p_2) = (1 - K_2)p_1 \frac{e^{-p_1/\mu}}{e^{-p_1/\mu} + e^{-V_0/\mu}} = (1 - K_2)p_1 \widehat{G}_1(p_1)$, where $\widehat{G}_1(p_1) = \frac{e^{-p_1/\mu}}{e^{-p_1/\mu} + e^{-V_0/\mu}}$. The left hand side derivative of the profit function at p_1 is equal to $\frac{\partial \pi_-(p_1,p_2)}{\partial p_1} = G_1(p_1, p_2) - p_1 \frac{G_1(p_1,p_2)(1-G_1(p_1,p_2))}{\mu}$. The right hand side derivative of the profit function at p_2 is equal to $\frac{\partial \pi_+(p_1,p_2)}{\partial p_1} = (1 - K_2)\widehat{G}_1(p_1) - p_1(1 - K_2)\frac{\widehat{G}_1(p_1)(1-\widehat{G}_1(p_1))}{\mu}$. Using the fact that $(1 - K_2)\widehat{G}_1(p_1) = G_1(p_1, p_2)$, we obtain $\frac{\partial \pi_+(p_1,p_2)}{\partial p_1} - \frac{\partial \pi_-(p_1,p_2)}{\partial p_1} = -p_1 \frac{G_1(p_1,p_2)(G_1(p_1,p_2)-\widehat{G}_1(p_1))}{\mu} > 0$. Thus, the right hand side derivative of the first firm's profit function at p_1 is greater than the left hand side derivative. The left hand side derivative has to be greater or equal to zero at p_1 (otherwise, there exists a maximum to the left of p_1), thus, the right hand side derivative has to be strictly greater than zero. This means that the profit function of the first firm increases to the right of p_1, so p_1 can not be the maximum. ■

Therefore, pure strategy equilibria exist only in regions 1a and 1b in Figure 4. For all other values of K_1 and K_2 it is necessary to search for the mixed strategy equilibria.

Mixed Strategy Equilibria

Figure 4 shows several regions, in which mixed equilibria exist. The numbers in brackets show the number of prices charged by each firm, i.e. $(3; 2)$ indicates that the first firm charges 3 prices, and the second firm charges 2 prices. The large spaces in the upper-left and bottom-right correspond to the mixed strategy equilibrium, in which the support of the price distributions is an interval. A complete characterization of these equilibria and precise boundaries of the region, where they exist, are to be added.

4 Choice of Capacities (preliminary)

Now, consider a two-stage game, in which the firms first costlessly accumulate capacities, and then compete in prices. Figure 5 illustrates how the choice of the capacity of the second firm affects its profits given that the first firm's capacity is fixed.

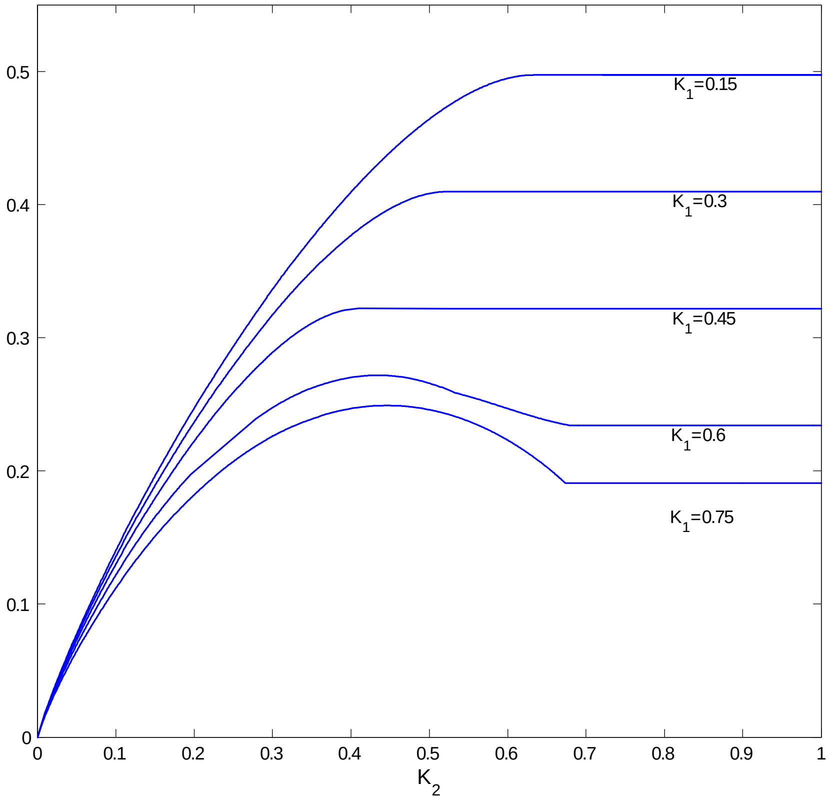
It could be seen from Figure 5 that the optimal choice of capacity for the second firm first declines as K_1 decreases, but always stays smaller than K_1. It keeps declining until $K_1 = K_2 = K$ that solves $\frac{1-K}{1-2K} + \ln\left(\frac{K}{1-2K}\right) = \frac{V_0}{\mu}$. Afterwards, the optimal value of K_2 increases. Thus, there is a unique symmetric equilibrium in a two-stage game. First, both firms choose the capacity level K that solves $\frac{1-K}{1-2K} + \ln\left(\frac{K}{1-2K}\right) = \frac{V_0}{\mu}$. Then, they both charge prices p^* from (2).

To be added.

5 Conclusion

To be added.

Figure 5: Profits of the second firm for the different capacity levels of the first firm.



Appendix

Production at the capacity constraint.

First, I will find price p^*, at which the capacity constraint is reached. $G_i(p^*, p^*) = K$, so $\frac{e^{-p^*/\mu}}{2e^{-p^*/\mu}+e^{-V_0/\mu}} = K$. Then, $e^{-p^*/\mu}(1-2K) = Ke^{-V_0/\mu}$, from where $p^* = -\mu \ln\left(\frac{Ke^{-\frac{V_0}{\mu}}}{1-2K}\right)$, and (2) follows.

Now I need to establish conditions under which p^* is the global maximum. If the firm charges any price p below p^* it will be capacity-constrained, so its profit at p will be below the one at p^*. In order to check that no prices above p^* result in a higher profit it is enough to check that the right-hand side derivative of the profit function $\pi_i(p_i, p^*) = p_i\widetilde{G}_i(p_i, p^*)$

is negative at p^*.[4]

$$\frac{\partial \pi_i(p^*, p^*)}{\partial p_i} = \widetilde{G}_i(p^*, p^*) + p^* \frac{\partial \widetilde{G}_i(p^*, p^*)}{\partial p_i} \tag{6}$$

When p_i is greater than p^*, $\widetilde{G}_i(p_i, p^*) = (1-K)\frac{e^{-p_i/\mu}}{e^{-p_i/\mu}+e^{-V_0/\mu}}$. It is known (see, for example, Anderson and de Palma (1992), Lemma 1) that if $\Phi_i = \frac{e^{-p_i/\mu}}{e^{-p_i/\mu}+e^{-V_0/\mu}}$ is the probability given by the multinomial logit, then $\frac{\partial \Phi_i}{\partial p_i} = \frac{-\Phi_i(1-\Phi_i)}{\mu}$. In our case, $K = \widetilde{G}_i(p^*, p^*) = (1-K)\frac{e^{-p_i/\mu}}{e^{-p_i/\mu}+e^{-V_0/\mu}}$, so $\frac{e^{-p_i/\mu}}{e^{-p_i/\mu}+e^{-V_0/\mu}} = \frac{K}{1-K}$. Thus,

$$\frac{\partial \widetilde{G}_i(p^*, p^*)}{\partial p_i} = (1-K)\frac{-\frac{K}{1-K}\left(1-\frac{K}{1-K}\right)}{\mu} = -\frac{K(1-2K)}{\mu(1-K)} \tag{7}$$

Substituting $\frac{\partial \widetilde{G}_i(p^*,p^*)}{\partial p_i}$ from (7) and p^* from (2) into (6), we get

$$\frac{\partial \pi_i(p^*, p^*)}{\partial p_i} = K + \ln\left(\frac{Ke^{-\frac{V_0}{\mu}}}{1-2K}\right)\frac{K(1-2K)}{(1-K)} \tag{8}$$

$\frac{\partial \pi_i(p^*,p^*)}{\partial p_i}$ is less than 0 when $1 + \frac{1-2K}{1-K}\ln\left(\frac{K}{1-2K}\right) - \frac{V_0}{\mu}\frac{(1-2K)}{(1-K)} < 0$ or $\frac{V_0}{\mu} > \frac{1-2K}{1-K} + \ln\left(\frac{K}{1-2K}\right)$. This is equivalent to (1).

Production at the level below the capacity constraint.

If both firms operate below their capacity constraints, then $\widetilde{G}_i(p_i, p_j) = G_i(p_i, p_j)$. $\frac{\partial \pi_i(p_i,p_j)}{\partial p_i} = G_i(p_i, p_j) - p_i\frac{\partial G_i(p_i,p_j)}{\partial p_i} = G_i(p_i, p_j) - \frac{p_i G_i(p_i,p_j)(1-G_i(p_i,p_j))}{\mu}$. $\frac{\partial \pi_i(p^{**},p^{**})}{\partial p_i} = 0$, so $\mu = p^{**}\left(1 - G_i(p^{**}, p^{**})\right)$, which is equivalent to (3).

Firm i can choose to charge a higher price p_i for which its rival is capacity-constrained ($G_j(p^{**}, p_i) > K$). For such price the profit is $\pi_i(p_i) = (1-K)\frac{e^{-\frac{p_i}{\mu}}}{e^{-\frac{p_i}{\mu}}+e^{-\frac{V_0}{\mu}}}$. Following the same argument as in the paragraph above, the optimal price $\widehat{p}$ should solve $\mu = \widehat{p}\frac{e^{-\frac{V_0}{\mu}}}{e^{-\frac{p_i}{\mu}}+e^{-\frac{V_0}{\mu}}}$.

[4] $\pi_i(p_i, p_j) = p_i\widetilde{G}_i(p_i, p_j)$, where $\widetilde{G}_i(p_i, p_j)$ is logit has a unique maximum, thus once it starts decreasing, it decreases forever.

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