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The Bank of Canada's Version of the Global Economy Model (BoC-GEM)

by

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The views expressed in this paper are those of the author.
No responsibility for them should be attributed to the Bank of Canada.

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Abstract

The Bank of Canada’s version of the Global Economy Model (BoC-GEM) is derived from the model created at the International Monetary Fund by Douglas Laxton (IMF) and Paolo Pesenti (FRBNY and NBER). The GEM is a dynamic stochastic general equilibrium model based on an optimizing representative agent framework with balanced growth, and some additional features to help mimic the overlapping generations’ class of models. Moreover, there is a concrete role for fiscal policy (albeit not fully optimized) and monetary policy. At the Bank, the model has been extended beyond the standard version with tradable and nontradable goods sectors to include both oil and non-oil commodities. Furthermore, the oil sector is decomposed into oil for production and oil for retail consumption. This report explains the structure of the model, and then outlines its calibration, which is based, in part, on previous estimation work and properties of the data. We also explore the properties of model, with both impulse responses to individual shocks, such as a simple demand shock, and more complex but practical applications, such as trade protectionism, global imbalances, and an increase in oil prices.

JEL classification: C68,E27,E37,F32,F47

Bank classification: Economic models; International topics; Business fluctuations and cycles

Résumé

La version de la Banque du Canada du modèle économique mondial (BoC-GEM) est dérivée du modèle créé initialement au Fond Monétaire International (FMI) par Douglas Laxton (FMI) et Paolo Pesenti (Réserve fédérale de New York et NBER). Le GEM est un modèle stochastique dynamique d’équilibre général basé sur des agents représentatifs optimisateurs. Le modèle prend également en compte la croissance de l’activité économique et inclut des mécanismes permettant de générer des propriétés similaires aux modèles de type inter-générationnel. De plus, les politiques fiscale et monétaire y jouent un rôle important. À la Banque du Canada, nous avons ajouté, aux secteurs des biens échangeables et non-échangeables, et les secteurs du pétrole et des produits de base non-énergétiques. De plus, le secteur pétrolier est divisé en pétrole destiné à la production et en pétrole destiné à la consommation. Ce rapport technique décrit la structure et la calibration du modèle qui est basée en partie sur des travaux d’estimation et sur les propriétés des données. Nous explorons également les propriétés du modèle en présentant des simulations de chocs individuels (par exemple, un choc de demande) et des simulations plus complexes visant à répondre à des enjeux concrets comme, par exemples, une resurgence possible du protectionisme, l’émergence des déséquilibres globaux et la hausse récente du prix du pétrole.

Classification JEL : C68,E27,E37,F32,F47

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1. Introduction and purpose of the model

The Bank of Canada has a rich history of modelling, focusing mainly on the economies of Canada and the United States. In the early 1990s, with advances in economic theory and increased computing power, the Bank developed a new Canadian projection model, the Quarterly Projection Model (QPM). This model reflected the state of the art in terms of theoretical structure and dynamic adjustment, and was the Bank of Canada's main model for Canadian economic projections and policy advice for the following decade. Recently, the Bank replaced QPM with a new projection and policy analysis model of the Canadian economy named ToTEM (Terms of Trade Economic Model). ToTEM was developed at the Bank of Canada and is considered to be at the cutting edge of the art and technology of macroeconomic modelling for projection and policy analysis (see Murchison and Rennison (2006)).

Although the different generations of models of Canada have undergone significant changes in terms of theoretical underpinnings or macroeconomic structure, they have always relied on other models or sources of information for forecasts of external economic activity. Until the introduction of the model presented in this Technical Report, the modelling efforts of the Bank of Canada regarding the external environment have mainly concentrated on the U.S. economy.¹ Over the past three years, staff have developed and used a new macroeconomic model, MUSE (Model of the United States Economy - see Gosselin and Lalonde (2005)), to analyze and forecast the U.S. economy. This model is an estimated forward-looking model with stock-flow dynamics based on the polynomial adjustment cost framework of Tinsley (1993). The model describes the interactions among the principal macroeconomic variables, such as gross domestic product and its components, inflation, interest rates, and the exchange rate.

However, with increasing openness to trade worldwide, the emergence of the economies of China and India, the rise of global imbalances and the recent increase in the price of oil, it is increasingly necessary to look at the external environment with a consistent global perspective. As a result, for many issues it has become unsatisfactory to take a stand on the current and future positions of the domestic economy through a lens focused mainly on the United States and Canada. As economic events of global importance occur outside of our traditional spheres of interest, such as the Asian financial crisis of the late 1990s, or the role of emerging Asia and the oil-exporting countries in financing the large (and continuing) current

¹Around the same time, the staff at the Bank of Canada also introduced a new small reduced-form model of the Euro Area and the U.K. economies called NEUQ (Piretti and St-Arnaud (2006))

account deficit of the United States, we have come to recognize the need to complement the existing tools with a global model at the Bank of Canada. This is the broad motivation for the project presented in this Technical Report.

More specifically, in 2004, the senior staff in the International Department of the Bank of Canada identified the key requirements for a new global model as follows:

- (i) An appropriate decomposition of the world into different regions beyond just Canada and the United States;
- (ii) Model properties that reflect both the analytical needs of the Bank of Canada and the current state of economic theory and empirics;
- (iii) A general equilibrium framework;
- (iv) Technical support from the creators of the model, to help maintain and extend the model over time, in conjunction with staff from the Bank of Canada;
- (v) But with an independent possibility for staff at the Bank of Canada to extend or modify the model; and
- (vi) Ease of use, in the sense that it fits with the current store of economic and technical knowledge at the Bank of Canada.

After considering several alternatives, the Global Economy Model (GEM) at the International Monetary Fund was seen as the best fit for these criteria, and a good starting point for further elaboration. So the version of the GEM presented in this Technical Report is far from exclusively a work of the staff at the Bank of Canada. Rather, this is a new extended version of a model that is the product of a network of central banks. The Bank of Canada is a member along with the originating institutions (the International Monetary Fund and the Federal Reserve Bank of New York) as well as other central banks who have learnt from and contributed to the experience of building a truly global, medium-scale DSGE model, such as the Norges Bank, the Bank of Japan and the Reserve Bank of New Zealand. It offers a theoretical coherence (yet flexibility) not found in many modelling tools, plus it is the product of the knowledge and experience not only of the Bank of Canada, but of other institutions with their own vast and informative modelling backgrounds.

The GEM belongs to the class of models known as dynamic stochastic general equilibrium models, or DSGE models for short. This puts it in the same class of model as the Bank of

Canada's main policy analysis and projection tool for the Canadian economy, ToTEM. The GEM is a representative agent model with a fully-optimizing framework based on microfoundations.² However, the GEM differs from ToTEM in that it fully utilizes the theory of New Open Economy Macroeconomics³. Practically, this implies a full articulation of the world within the GEM. The model is multi-region, encompassing the entire world economy, modelling explicitly all the bilateral trade flows and their relative prices for each region, including exchange rates. The GEM, as a tool, is conducive to large scale analysis of global issues, as well as country-specific issues. The Bank of Canada's version of the Global Economy Model (BoC-GEM) is comprised of five regional blocs - Canada (CA), the United States (US), emerging Asia (AS), the commodity exporter (CX), and the remaining countries (RC). Moreover, because of its composition, this version of the GEM can analyze either issues particular to Canada, or it can look at issues elsewhere in the world, and how they will affect Canada either directly or via effects on a third country, such as the United States.

The BoC-GEM is an extension of the model found in Faruquee, Laxton, Muir and Pesenti (2006a, 2006b) adapted to the needs of the Bank of Canada. It differs mainly along three important dimensions from the version developed in Faruquee et al. (2006a, 2006b). First of all, Canada is included as a separate region and the country composition of the regions is different. Second, the structure of the model is richer, because of the addition of sectors important for the Canadian economy, oil and non-oil commodities. Therefore, like the other prices modeled, the prices of oil and non-oil commodities are endogenous. Finally, the calibration is adjusted to reflect Bank of Canada staff views and the properties of ToTEM and MUSE. The last point is important for two reasons. First, ToTEM (see Murchison and Rennison (2006)) and MUSE (see Gosselin and Lalonde (2005)) are good representations of the Canadian and U.S. data. Second, for issues that need a global and/or a multisectoral perspective, the BoC-GEM is used to generate risk scenarios around the base-case staff projection generated using ToTEM and MUSE. Consequently, when applicable, the BoC-GEM's properties need to be consistent with those of ToTEM and MUSE.

One main feature that the BoC-GEM shares with its parent from the IMF is flexibility. This is in keeping with the philosophy of the original GEM - it is supposed to be a toolbox and a framework for exploring the global macroeconomy using the latest theory and techniques (Pesenti (2007)). Although the BoC-GEM is a large, complex DSGE model, it is not

²Goodfriend and King (1997) is a good starting point for a survey of the new neoclassical synthesis, the literature that best represents these models.

³For more on NOEM (New Open Economy Macroeconomics) as a whole, the reader is referred to Lane (2001). For a good explanation of many of the theoretical concepts in this technical report, the reader is referred to Corsetti and Pesenti (2005).

a monolith. It can be easily adapted to create other configurations, with either fewer regions or fewer sectors, with or without features such as a fully-functioning fiscal sector, or a distribution sector for imported goods.⁴ However, the main goal of this Technical Report is to present the BoC-GEM in its entirety and document its properties. Therefore, we present below the structure of the BoC-GEM and its properties, as well as demonstrate how the model can be used to address some challenges that are currently facing the world and the Canadian economies. These include:

- the impact of Emerging Asia on the price of traded goods and on the prices of oil and non-oil commodities,
- the risks associated with the emergence of global imbalances,
- the factors underlying the recent increase in the price of oil,
- the determinants of the Canadian real exchange rate over the medium and long run.

We start with a detailed presentation of the model's structure in Section 2, followed by its calibration in Section 3. Sections 4 and 5 explore a variety of the properties of the BoC-GEM, many of which are derived from previous versions of the GEM, or overlap with other models of interest to the Bank of Canada. Other properties and shocks are unique to the BoC-GEM, and this is the first time some of them have been presented in a published form. It is also the first time that the GEM has been applied to the Canadian economy. We conclude the Technical Report by discussing different alternatives for further development work on the BoC-GEM and the use of the model within the staff economic projection.

2. Structure of the model

This section outlines the entire structure of the BoC-GEM, beginning with a broad-brush exposition, followed by a more detailed (but far from exhaustive) technical presentation. The presentation of the model below (outside of the commodities, oil and gasoline sectors) relies heavily on Pesenti (2007); it should be referred to directly to gain a fuller understanding of the core structure of this model.

⁴Conceptually, there is also nothing to prevent an augmentation in the number of regions and / or sectors; currently the limitation in this direction is related to computational tractability.

2.1 Overview of the model

As mentioned in the introduction, there are five regional blocs - Canada (CA), the United States (US), emerging Asia (AS), the commodity exporter (CX), and the remaining countries (RC).⁵ Emerging Asia includes eight of the "Asian tigers" - China, India, Hong Kong SAR of China, Republic of Korea, Malaysia, the Philippines, Singapore and Thailand. The commodity exporter bloc includes the largest exporters of oil and non-oil commodities, such as the OPEC countries, Norway, Russia, South Africa, Australia, New Zealand, Argentina, Brazil, Chile and Mexico (among others). Finally, the remaining countries bloc includes all the other countries in the world, but effectively this means Japan and the members of the European Union (since Africa is very small economically).

Each of the five regions is modelled symmetrically. Each region consists of

- a continuum of firms (to allow for monopolistic competition, and by extension, price markups) that produce (and therefore both supply and demand) raw materials, intermediate goods and final goods.
- two types of households (to allow for a differentiation between liquidity-constrained and forward-looking consumers) that consume the (nontraded) final goods and supply differentiated labour inputs to firms.
- a government comprised of a fiscal authority that consumes nontradable goods and services, financed through taxation or borrowing and a monetary authority that manages short-term interest rates through monetary policy to provide a nominal anchor to the economy.

In general firms supply goods to domestic and foreign consumers, and demand labour from domestic consumers. In addition firms demand intermediate goods from other firms who supply them, both domestically and internationally. Consumers demand products from firms both at home and abroad and supply labour to domestic firms. So the entire model in its nonlinear form can be thought of as a system of demand, supply and pricing functions, usually using the mathematical form of the constant elasticity of substitution function (and its associated Dixit-Stiglitz representation).

⁵A list of the country membership for each of the aggregated regional blocs can be found in Appendix A.

In total, there are five sectors which produce goods from capital and labour and other factors. In general, the production of each of the five sectors is assumed to be monopolistically competitive. Monopolistic competition means that firms can still enter and exit the market, but because each firm's good is slightly differentiated from those produced by other firms, each firm is able to set a price above its marginal cost, allowing a markup. The five sectors are nontradable goods, tradable goods, oil and natural gas, non-oil-and-natural-gas commodities and heating fuel and automobile fuel. There is a special emphasis placed on oil and natural gas and commodities, since the Canadian economy has been historically subject to terms-of-trade shocks from these sectors, and we have no other tool currently at the Bank of Canada that can deal with these sorts of shocks in a fully-articulated global general equilibrium framework. Henceforth, in the parlance of the BoC-GEM, commodities excluding oil and natural gas are referred to as commodities; oil and natural gas are referred to as oil; heating and automobile fuels are referred to as gasoline.

The production structure for a single region is illustrated in three diagrams, Figure 1 to 3. The model will be described first in general, and then in detail, starting with the primary goods and moving upward in the production structure.

Figures 2 and 3 illustrate the production process for oil and commodities respectively. Each region has firms that produce oil from capital, labour and crude oil reserves⁶. Other firms produce commodities using capital, labour and land. Oil can be further processed (along with capital and labour) into gasoline.

Oil and commodities can be traded across regions. Figure 1 shows that oil and commodities are further combined with capital and labour to produce a tradable good (mostly financial services such as investment banking or manufactured items such as automobiles or semi-durables) and a nontradable good (mostly services outside of the financial sector, such as retailing, education or health care). The tradable good can also be traded across regions. In total then there are three intermediate goods - gasoline, the tradable good and the nontradable good. All three are combined to form a final consumption good, while the tradable and nontradable goods are combined to form a final investment good. The (nontraded) consumption and investment good can be consumed by private agents or by the fiscal agent (who also consumes some share of nontradable goods directly).

⁶Conceptually, there are three kinds of crude oil reserves - proven and exploited reserves; proven and unexploited reserves; unknown and/or unproven reserves. In the BoC-GEM, the first type of reserves are designated as crude oil reserves; although the second type can be easily approximated by shocks to the stock of crude oil reserves (as can the third kind, if desired).

All goods at all levels are assumed to be produced or aggregated using a CES technology. While a Cobb-Douglas form is in some sense more tractable, CES forms allow elasticities of substitution that differ from one between inputs in production, or between goods in formulating final demands.

Households consume the final goods, and provide labour to produce them. Moreover, only one class of consumer (the forward-looking consumers) ultimately own all firms and the capital stock used by firms for production; the other class of consumer (the liquidity-constrained consumers) have no access to capital markets and depend solely on their labour income.

Regarding international trade, all the bilateral flows (across regions) of the exports and imports of oil, commodities and tradable goods for consumption and investment are explicitly modeled as demands for imported goods from specific regions. In order to facilitate international trade, there is a single internationally-traded bond, denominated in U.S. dollars; thus financial markets are assumed to be incomplete. A region's bond holdings define its net foreign asset position, which is maintained at some exogenously specified NFA-to-GDP ratio using a modified risk-adjusted uncovered interest rate parity (UIRP) condition to define all the bilateral real exchange rates *vis-à-vis* the United States (from which all other cross-bilateral exchange rates can be deduced). There is also an explicit link between the level of government debt the fiscal agent holds and the level of net foreign assets, meaning that the representative agent in this model is non-Ricardian. There are also further non-Ricardian elements in the GEM - some consumers are liquidity-constrained as noted above (assumed to arise from a low labour skill set), and the government as fiscal agent raises revenues through distortionary taxation on labour income, capital income, and (possibly) through tariffs on imports.⁷ The monetary authority targets either core inflation (defined as the Consumer Price Index excluding gasoline prices and the effects of tariffs), headline CPI inflation (or a fixed nominal exchange rate) to achieve some objective related to price stability (or price certainty) with some standard reaction function.

Finally, as is generally the case in the DSGE modeling literature, in order to match the persistence observed in the data, the model includes real adjustment costs and nominal rigidities that are allowed to be different across regions.⁸ We assume real adjustment costs in capital, investment, labour and imports. We also suppose that the real adjustment costs are

⁷The fiscal agent can also subsidize trade if it so chooses.

⁸Two practical examples are the Bank of Canada's ToTEM (Terms of Trade Model - Murchison and Rennison (2006)) and the Board of Governors of the Federal Reserve's SIGMA model (Erceg et al. (2005b)).

very important in the production and demand of oil and commodities. Nominal rigidities are found in wages and tradable and nontradable good prices (but not for the oil and commodities sectors).

2.2 General points

Before embarking on a comprehensive overview of the model, several technical matters need to be addressed.

First, in most sections that focus on region-specific equations that are independent of foreign variables and are thus qualitatively similar across countries, region indices are dropped for notational simplicity. For the sections involving international transactions, region indices are explicitly incorporated into the notation, where H is the domestic region, and R is a representative region from the rest of the world.

Second, there is a common trend in productivity for the world economy ($TREND$) whose gross rate of growth between time t and time τ is $g_{t,\tau}$. All quantity variables in the model are expressed in detrended terms, as ratios relative to $TREND$. Furthermore, all prices in the model are stated as relative prices, where the *numéraire* price is the headline consumer price index (CPI). CPI is normalized to unity, and all other prices are stated in relation to it. The GEM is detrended in this way to allow for ease of computation.⁹

Third, variables which are not explicitly indexed to firms or households or the government are understood to be expressed in per-capita (average) terms. For instance with the final consumption good A_t is the sum of the output of all firms over the continuum ss , $(1/ss) \int_0^{ss} A_t(x) dx$ where ss is the size of a region in terms of its labour force and $\sum_{ss} ss = 1$.

Fourth, as a general convention throughout the model, when we state that variable X follows is a stochastic process, we mean that it uses an autoregressive formulation, in either levels or logarithmic levels:

$$X_t = (1 - \lambda_X) X + \lambda_X X_{t-1} + e_{X,t} \tag{1}$$

where $0 < \lambda_X < 1$, X is the steady-state value of X_t , and $e_{X,t}$ is a noise term.

⁹Most DSGE models assume zero growth in the steady state or either (or both) the real and nominal sides of the economy. We hope that by allowing such growth, it will permit us in the future to better match the model to real data without having to rely on arbitrary data-detrending methods such as the band-pass or Hodrick-Prescott (H-P) filter.

Finally, the following terminology is employed regularly below:

- a period is assumed to be one quarter;
- all variables are stated in annualized terms unless otherwise stated;
- when there are superscripts for the regions in bilateral equations, a generic region is defined as R while the importing region is defined as H ;
- recall that "oil" means the commodities oil and natural gas, "commodities" means all other commodities, "raw materials" means the oil and commodities sectors together, and "gasoline" means automobile and heating fuels.

2.3 The firms' problem

There are three levels of production by firms in the model. We will consider each in turn, starting with the lowest level of raw materials, which in the BoC-GEM are oil and commodities. Then we will discuss the intermediate goods sectors (tradables, nontradables and gasoline) and lastly, the top level of final goods (the consumption good and the investment good).

2.3.1 Raw materials

For the two raw materials sectors, we will discuss the commodities sector only, with the understanding that this exposition is equally applicable to the oil sector as well.

In each region there is a continuum of firms that can produce commodities in a monopolistically competitive framework. Each firm is indexed by $s \in [0, ss]$ where ss is the size of the region in the world ($0 < ss < 1$). Firm s produces $S_t(s)$ at period t of its variety of commodities by combining capital $K(s)$, labour $L(s)$ and a fixed factor which is a non-reproducible resource $LAND(s)$ using a constant elasticity of substitution (CES) technology:

$$\begin{aligned}
 S_t(s)^{1-\frac{1}{\xi_S}} &= Z_{S,t}[\alpha_{LANDS,t}^{\frac{1}{\xi_S}}(Z_{LANDS,t}LAND_t(s))^{1-\frac{1}{\xi_S}} \\
 &\quad + (1 - \alpha_{KS,t} - \alpha_{LANDS,t})^{\frac{1}{\xi_S}}(Z_{LS,t}l_t(s)(1 - \Gamma_{LS,t}))^{1-\frac{1}{\xi_S}} \\
 &\quad + \alpha_{KS,t}^{\frac{1}{\xi_S}}(Z_{KS,t}K_t(s)(1 - \Gamma_{KS,t}))^{1-\frac{1}{\xi_S}}] \tag{2}
 \end{aligned}$$

where Z_S is a shock that follows a stochastic process common to all firms producing commodities to the level of productivity in the entire sector. Z_{LS} , Z_{KS} and Z_{LANDS} are productivity

shocks following stochastic processes that are common to all firms producing commodities specific to the factors in that sector (labour, capital and land respectively). Γ_{KS} and Γ_{LS} are real adjustment costs incurred when changing the levels of capital and labour.

Because the production of commodities does not respond immediately to movements in demand, we need to differentiate between a short-run and a long-run supply curve, so that we can match price elasticities that are smaller in the short term than in the long run. Therefore, we model the use of labour and capital in the production process as being subject to real adjustment costs of a quadratic form. In the case of capital, we assume that real adjustment costs can be represented as :

$$\Gamma_{KS,t} \left[\frac{K_{S,t}(s)}{S_t(s)} / \frac{K_{S,t-1}}{S_{t-1}} \right] = \frac{\phi_{KS}}{2} \left[(K_{S,t}(s)/S_t(s)) / (K_{S,t-1}/S_{t-1}) - 1 \right]^2 \quad (3)$$

where $\Gamma_{KS}[1] = 0$. This functional form is also assumed for labour. These adjustment costs reduce the ability of firms to change the input composition of their production. Therefore, in the short run, the elasticity of substitution among inputs is lower than in the long run. The size of ϕ_{KS} determines by how much and for how long the elasticity of substitution will be lower than its permanent value ε_s .

Defining w_t and r_t as the real prices of labour and capital (relative to the CPI) common across all sectors of production, the real marginal cost of commodities production is:

$$mc_t(s) = \frac{1}{Z_{S,t}} \left[(1 - \alpha_{KS,t} - \alpha_{LANDS,t}) \left(\frac{w_t}{Z_{LS,t}} \right)^{1-\xi_s} + \alpha_{KS,t} \left(\frac{r_t}{Z_{KS,t}} \right)^{1-\xi_s} + \alpha_{LANDS,t} \left(\frac{PLAND,t}{Z_{LANDS,t}} \right)^{1-\xi_s} \right]^{\frac{1}{1-\xi_s}} \quad (4)$$

and the capital-labour ratio (subject to real adjustment costs) is:

$$\frac{K_t(s)}{\ell_t(s)} = \frac{Z_{LS,t} (1 - \Gamma_{LS,t})}{Z_{KS,t} (1 - \Gamma_{KS,t})} \frac{\alpha_{KS,t}}{1 - \alpha_{KS,t} - \alpha_{LANDS,t}} \times \left(\frac{Z_{LS,t} r_t (1 - \Gamma_{KS,t} - K_t(s) \Gamma'_{KS,t})}{Z_{KS,t} w_t (1 - \Gamma_{LS,t} - \ell_t(s) \Gamma'_{LS,t})} \right)^{-\xi_s} \quad (5)$$

Labour inputs are differentiated and come in different varieties (skill levels). They are defined over a continuum of mass ss and indexed by $j \in [0, ss]$. Each firm s uses a CES

combination of labour inputs:

$$\ell_t(s) = \left[\left(\frac{1}{ss} \right)^{\frac{1}{\psi_{L,t}}} \int_0^{ss} \ell(s, j)^{1 - \frac{1}{\psi_{L,t}}} dj \right]^{\frac{\psi_{L,t}}{\psi_{L,t} - 1}} \quad (6)$$

where $\ell(s, j)$ is the demand for labour input of type j by the producer of good s and $\psi_L > 1$ is the elasticity of substitution among labour inputs (differentiated by skill level). Cost minimization implies that $\ell(s, j)$ is a function of the relative wage:

$$\ell_t(s, j) = \left(\frac{1}{ss} \right) \left(\frac{w_t(j)}{w_t} \right)^{-\psi_{L,t}} \ell_t(s) \quad (7)$$

where $w(j)$ is the wage paid to the domestic labour input j and the average real wage w is defined as:

$$w_t = \left[\left(\frac{1}{ss} \right) \int_0^s w_t(j)^{1 - \psi_{L,t}} dj \right]^{\frac{1}{1 - \psi_{L,t}}} \quad (8)$$

Finally, cost minimization implies that firm s 's demand for the fixed factor of production, $LAND$, is:

$$LAND_t(s) = \frac{\alpha_{LANDS,t}}{Z_{LANDS,t}} \left(\frac{p_{LAND,t}}{Z_{LANDS,t}} / \frac{mc_t(s)}{Z_{S,t}} \right)^{-\xi_S} \frac{S_t(s)}{Z_{S,t}} \quad (9)$$

which implies that as the price of fixed factor (p_{LAND}) diverges further above (below) from the real marginal cost of producing commodities, the demand for $LAND$ will fall (rise). The elasticity of substitution between input factors (ξ_S) determines how much demand will react. Since ξ_S is less than unity, the demand for $LAND$ is relatively inelastic, as the expression $\left(\frac{p_{LAND,t}}{Z_{LANDS,t}} / \frac{mc_t(s)}{Z_{S,t}} \right)^{-\xi_S}$ implies the negative effect on demand will be diminished

Now that we have defined the supply of commodities, we have to consider the demands for those goods by the types of two intermediate goods firms - the representative firm h which produces tradables (T) the representative firm n which produces nontradables (N). Therefore we will consider, in turn, the demand for domestically-produced and imported commodities.

Demand for domestically-produced commodities The aggregate demand for domestically-produced commodities by intermediate-goods-producing firms is summarized by:

$$\left(\frac{p_t(s)}{p_{SN,t}}\right)^{-\theta_{S,t}} S_{N,t} + \left(\frac{p_t(s)}{p_{ST,t}}\right)^{-\theta_{S,t}} S_{T,t} = \int_0^{ss} S_t(s, n) dn + \int_0^{ss} S_t(s, h) dh \quad (10)$$

where $S(s, n)$ is the demand by firm n producing nontradables for commodities produced by firm s and $S(s, h)$ is the demand by firm h producing tradables for commodities produced by firm s . S_N is the amount of commodities produced for use by the nontradables-producing firms, given the commodities' firms' price $p_t(s)$ relative to the aggregate price p_{SN} that nontradables-producing firms are willing to pay. S_T is defined similarly for firms producing tradables.

We will focus first on the basket $S(s, n)$, which is a CES index of all domestic varieties of commodities used in the production of nontradable goods by firm n . So for $S_t(s, n)$:

$$Q_{S,t}(n) = \left[\left(\frac{1}{ss}\right)^{\frac{1}{\theta_{S,t}}} \int_0^{ss} S_t(s, n)^{1-\frac{1}{\theta_{S,t}}} ds \right]^{\frac{\theta_{S,t}}{\theta_{S,t}-1}} \quad (11)$$

where $\theta_{S,t} > 1$ denotes the elasticity of substitution among commodities produced by different firms and $Q_S(n)$ is the demand by all nontradable-goods-producing firms for domestically-produced commodities.

Any firm n takes as given the price of commodities $p(s)$ produced domestically (Q), giving the price of domestically-produced commodities $p_Q(s)$. Cost minimization implies:

$$S_t(s, n) = \frac{1}{ss} \left(\frac{p_{Q,t}(s)}{p_{SN,t}}\right)^{-\theta_{S,t}} S_t(s) \quad (12)$$

where p_{SN} is the price of one unit of the commodities basket designated for the production of nontradable goods. As firm s 's price $p_Q(s)$ becomes larger (smaller) than the aggregate price of commodities used in nontradable-goods production (p_{SN}), demand for commodities from firm s by firm n ($S(s, n)$) will fall (increase), usually by an amount larger than the price difference, since the elasticity of substitution between the varieties of commodities produced by firms is greater than unity. The demand by all firms producing tradable goods for domestically-produced commodities ($Q_S(t)$) is similarly characterized.

Demand for imported commodities Consider now the demand for imported commodities. It occurs at two different levels. First, the importing region H demands commodities

from the other regions R . Second, different sectors within the importing region have demands that must be met. The representative firm in the commodities sector is $s^H \in [0, ss^H]$. Its imports $M_S^H(s^H)$ are a CES function of baskets of goods imported from the other regions, or:

$$M_{S,t}^H(s^H)^{1-\frac{1}{\rho_S^H}} = \sum_{R \neq H} \left(b_{S,t}^{H,R} \right)^{\frac{1}{\rho_S^H}} \left(M_{S,t}^{H,R}(s^H) (1 - \Gamma_{MS,t}^{H,R}(s^H)) \right)^{1-\frac{1}{\rho_S^H}} \quad (13)$$

where:

$$0 \leq b_{S,t}^{H,R} \leq 1, \quad \sum_{R \neq H} b_{S,t}^{H,R} = 1 \quad (14)$$

In (13) above ρ_S^H is the elasticity of import substitution across regions: the higher is ρ_S^H , the easier it is for firm s^H to substitute imports of commodities from one region with imports from another. The parameter $b_S^{H,R}$ is the percent amount of the commodities imported by region H from region R , subject to the elasticity of import substitution ρ_S^H . $M_S^{H,R}(s^H)$ denotes imports of commodities by region H 's firm s from region R .¹⁰

Denoting $p_{MS}^{H,R}$ the price in region H of a basket of commodities imported from region R , cost minimization implies:

$$M_{S,t}^{H,R}(s^H) = b_{S,t}^{H,R} \left(\frac{p_{MS,t}^{H,R}}{p_{MS,t}^H(s^H)} \right)^{-\rho_S^H} M_{S,t}^H(s^H) \quad (15)$$

so that as $p_{MS}^{H,R}$ rises above (falls below) the aggregate price of imported commodities $p_{MS}^H(s^H)$, region H will import less (more) from region R , and this amount will shift readily since the elasticity of import substitution ρ_S^H is set quite high (well above unity) for commodities.

The import price for commodities in region H , p_{MS}^H , is defined as:

$$p_{MS,t}^H(s^H) = \left[\sum_{R \neq H} b_{S,t}^{H,R} \left(p_{MS,t}^{H,R} \right)^{1-\rho_S^H} \right]^{\frac{1}{1-\rho_S^H}} \quad (16)$$

In principle, the cost-minimizing import price $p_{MS}^H(s^H)$ is firm-specific, as it depends on firm s^H 's import share. To the extent that all firms s^H are symmetric within the commodities sector, however, there will be a unique import price p_{MS}^H .

We know that in aggregate each region's representative firm demands $M_{S,t}^H(s^H)$; it is

¹⁰The parameter $b_S^{H,R}$ can be time-varying; it is represented in the model by the stochastic process found in equation (1).

distributed across the nontradable and tradable sectors according to the following two demand equations:

$$M_{S,t}(n) = (1 - \nu_{SN,t}) (p_{MS,t}/p_{SN,t})^{-\mu_{SN}} S_t(n) \times \\ (1 - \Gamma_{MS,t}(n) - M_{S,t}(n)\Gamma'_{MS,t}(n))/(1 - \Gamma_{MS,t}(n)) \quad (17)$$

$$M_{S,t}(h) = (1 - \nu_{ST,t}) (p_{MS,t}/p_{ST,t})^{-\mu_{ST}} S_t(h) \times \\ (1 - \Gamma_{MS,t}(h) - M_{S,t}(h)\Gamma'_{MS,t}(h))/(1 - \Gamma_{MS,t}(h)) \quad (18)$$

where ν_{SN} and ν_{ST} represent the bias of the home region towards commodities produced domestically. We also assume that in the short-run, there is an additional lag required for firms to shift from domestically produced commodities to imported commodities (because of fixed supply contracts, for example). Hence the presence of the real adjustment costs $\Gamma_{MS}(n)$ and $\Gamma_{MS}(h)$:

$$\Gamma_{MS,t}(n) \left[\frac{M_{S,t}(n)}{N_t(n)} / \frac{M_{SN,t-1}}{N_{t-1}} \right] = \\ \frac{\phi_{MSN}}{2} \frac{[(M_{S,t}(n)/N_t(n)) / (M_{SN,t-1}/N_{t-1}) - 1]^2}{(1 + [(M_{S,t}(n)/N_t(n)) / (M_{SN,t-1}/N_{t-1}) - 1]^2)} \quad (19)$$

$$\Gamma_{MS,t}(h) \left[\frac{M_{S,t}(h)}{T_t(h)} / \frac{M_{ST,t-1}}{T_{t-1}} \right] = \\ \frac{\phi_{MST}}{2} \frac{[(M_{S,t}(h)/T_t(h)) / (M_{ST,t-1}/T_{t-1}) - 1]^2}{(1 + [(M_{S,t}(h)/T_t(h)) / (M_{ST,t-1}/T_{t-1}) - 1]^2)} \quad (20)$$

such that $\Gamma_{MS}^{H,R}[1] = 0$, $\Gamma_{MS}^{H,R}[\infty] = \phi_{MS}^{H,R}/2$, and $\Gamma_{MS}^{H,R}[0] = \Gamma_{MS}^{H,R}[2] = \phi_{MS}^{H,R}/4$ for both firms h and n .

The price of imported commodities used in the production of nontradables $p_{SN,t}$ and the price of imported commodities used in the production of tradables $p_{ST,t}$ are both CES aggregators of the price of domestically-produced commodities p_{QS} and imported commodities p_{MS} , differentiated by the real adjustment costs in the import sector, and the elasticity of substitution between domestically-produced and imported commodities μ_{SN} and μ_{ST} :

$$p_{SN,t} = [\nu_{SN,t} p_{QS,t}^{1-\mu_{SN}} + (1 - \nu_{SN,t}) p_{MS,t}^{-\mu_{SN}} \times \\ (1 - \Gamma_{MS,t}(n) - M_{S,t}(n)\Gamma'_{MS,t}(n))^{\mu_{SN}-1}]^{\frac{1}{1-\mu_{SN}}} \quad (21)$$

$$p_{ST,t} = [\nu_{ST,t} p_{QS,t}^{1-\mu_{ST}} + (1 - \nu_{ST,t}) p_{MS,t}^{-\mu_{ST}} \times \\ (1 - \Gamma_{MS,t}(h) - M_{S,t}(h)\Gamma'_{MS,t}(h))^{\mu_{ST}-1}]^{\frac{1}{1-\mu_{ST}}} \quad (22)$$

Imported commodities can be expressed as a sum of commodities imported for use in the production of tradable and nontradable good, or as a sum of the imported commodities from the different regions, subject to real adjustment costs:

$$\begin{aligned}
p_{MS,t}^H(s)M_{S,t}^H(s) &= p_{MSN,t}^H M_{SN,t}^H + p_{MST,t}^H M_{ST,t}^H \\
&= \sum_{R \neq H} p_{MS,t}^{H,R}(s)M_{S,t}^{H,R}(s) * \Gamma_{MSagg,t}^H
\end{aligned} \tag{23}$$

where Γ_{MSagg} is the aggregated effects of $\Gamma_{MS}(n)$ and $\Gamma_{MS}(h)$ on both prices and volumes.

We can now equate the demand for commodities by firms producing nontradable and tradable goods by aggregating across firms in the nontradable and tradable sectors that use commodities produced domestically or imported from abroad:

$$\begin{aligned}
&\left(\frac{p_t(s)}{p_{SN,t}}\right)^{-\theta_{S,t}} S_{N,t} + \left(\frac{p_t(s)}{p_{ST,t}}\right)^{-\theta_{S,t}} S_{T,t} = \\
&\int_0^{ss} Q_{S,t}(s, n)dn + \int_0^{ss} Q_{S,t}(s, h)dh \\
&+ \int_0^{ss} M_{S,t}(s, n)dn + \int_0^{ss} M_{S,t}(s, h)dh
\end{aligned} \tag{24}$$

The oil sector For the oil sector, the technology of production can be quantitatively different from the commodities sector, but its formal characterization is very similar, with self-explanatory changes in notation. For instance, a firm $o \in [0, ss]$ that produces oil uses the production technology:

$$\begin{aligned}
O_t(o)^{1-\frac{1}{\xi_o}} &= Z_{O,t}[\alpha_{OILO,t}^{\frac{1}{\xi_o}}(Z_{OILO,t}OIL_t(o))^{1-\frac{1}{\xi_o}} \\
&+ (1 - \alpha_{KO,t} - \alpha_{OILO,t})^{\frac{1}{\xi_o}} (Z_{LO,t}L_t(o)(1 - \Gamma_{LO,t}))^{1-\frac{1}{\xi_o}} \\
&+ \alpha_{KO,t}^{\frac{1}{\xi_o}}(Z_{KO,t}K_t(o)(1 - \Gamma_{KO,t}))^{1-\frac{1}{\xi_o}}]
\end{aligned} \tag{25}$$

where in this case crude oil reserves $OIL(o)$ is the fixed factor.¹¹ The demand for oil in the intermediate goods firms, both domestically produced and imported is also analogous to the commodities sector. The only difference from the demand of commodities is that there is a third type of firm in the intermediate goods sector, firm g , that produces gasoline GAS which has a demand for oil as well, meaning the aggregate constraint in the oil sector (analogous

¹¹Real marginal cost for the oil sector is the cost dual of this equation, with one addition - τ_{ROYAL} is the royalties paid on the holdings of crude oil reserves by the oil-producing firms to the government.

to equation (24)) becomes:

$$\begin{aligned}
& \left(\frac{p_t(o)}{p_{ON,t}} \right)^{-\theta_{O,t}} O_{N,t} + \left(\frac{p_t(o)}{p_{OT,t}} \right)^{-\theta_{O,t}} O_{T,t} + \left(\frac{p_t(o)}{p_{OGAS,t}} \right)^{-\theta_{O,t}} O_{GAS,t} = \\
& \int_0^{ss} Q_{O,t}(o, n) dn + \int_0^{ss} Q_{O,t}(o, h) dh + \int_0^{ss} Q_{O,t}(o, g) dg \\
& + \int_0^{ss} M_{O,t}(o, n) dn + \int_0^{ss} M_{O,t}(o, h) dh + \int_0^{ss} M_{O,t}(o, g) dg
\end{aligned} \tag{26}$$

2.3.2 Intermediate goods

For intermediate goods we will consider the nontradable good in detail, and treat the tradable and gasoline sectors as analogues. Intermediate inputs come in different varieties (brands) and are produced under conditions of monopolistic competition. In each region there are three kinds of intermediate goods - nontradables, tradables and gasoline. Each kind is defined over a continuum of mass ss (the size of the region in the world according to its labour force) Each nontradable good is produced by a domestic firm indexed by $n \in [0, ss]$, each tradable good is produced by a firm $h \in [0, ss]$, and each gasoline good is produced by a firm $g \in [0, ss]$.

Nontradables sector The nontradable N is produced by firm n with the following CES technology:

$$\begin{aligned}
N_t(n) = & Z_{N,t} [(1 - \alpha_{KN,t} - \alpha_{SN,t} - \alpha_{ON,t})^{\frac{1}{\xi_N}} (Z_{LN,t} \ell_t(n))^{1 - \frac{1}{\xi_N}} \\
& + \alpha_{KN,t}^{\frac{1}{\xi_N}} (Z_{KN,t} K_t(n))^{1 - \frac{1}{\xi_N}} + \alpha_{SN,t}^{\frac{1}{\xi_N}} (S_t(n) (1 - \Gamma_{SN,t}))^{1 - \frac{1}{\xi_N}} \\
& + \alpha_{ON,t}^{\frac{1}{\xi_N}} (O_t(n) (1 - \Gamma_{ON,t}))^{1 - \frac{1}{\xi_N}}]^{\frac{\xi_N}{\xi_N - 1}}
\end{aligned} \tag{27}$$

Firm n uses four variable factors in the production of its good, and there is no fixed factor present. It uses labour $\ell(n)$, capital $K(n)$, commodities $S(n)$ and oil $O(n)$ to produce $N(n)$ units of its variety. $0 < \xi_N < 1$ is the elasticity of substitution among factor inputs. As in the commodities and oil sectors, Z_N is a sector-wide productivity shock common to all firms n producing a nontradable good, while Z_{LN} and Z_{KN} are productivity shocks that are specific to labour and capital respectively in the nontradables sector. In the short run, the capacity of the nontradable firms to adjust their demand for commodities and oil is very small, therefore we assume that they are facing real adjustment costs Γ . Using commodities as an example, the real adjustment costs take the form:

$$\Gamma_{SN,t} \left[\frac{S_t(n)}{N_t(n)} / \frac{S_{N,t-1}}{N_{t-1}} \right] = \frac{\phi_{SN}}{2} \left[(S_t(n)/N_t(n)) / (S_{N,t-1}/N_{t-1}) - 1 \right]^2 \quad (28)$$

Therefore, in the short run, the elasticity of substitution among factor inputs is lower than in the long run. The size of the parameter ϕ_{SN} determines by how much and for how long the effective elasticity of substitution of the use of commodities as a factor input with other factors will be lower than its long-run value of ε_N . The real marginal cost in nontradables production is:

$$\begin{aligned} mc_t(n) = & \frac{1}{Z_{N,t}} \left[(1 - \alpha_{KN,t} - \alpha_{SN,t} - \alpha_{ON,t}) \left(\frac{w_t}{Z_{LN,t}} \right)^{1-\xi_N} + \alpha_{KN,t} \left(\frac{r_t}{Z_{KN,t}} \right)^{1-\xi_N} \right. \\ & + \alpha_{SN,t} (p_{SN,t}^{1-\xi_N} (1 - \Gamma_{S,t}(n) - S_t(n) \Gamma'_{S,t}(n))^{\xi_N - 1})^{1-\xi_N} \\ & \left. + \alpha_{ON,t} (((1 + \tau_{OIL,t}) p_{ON,t})^{1-\xi_N} (1 - \Gamma_{O,t}(n) - O_t(n) \Gamma'_{O,t}(n))^{\xi_N - 1}) \right]^{\frac{1}{1-\xi_N}} \quad (29) \end{aligned}$$

and the capital-labour ratio is:

$$\frac{K_t(n)}{\ell_t(n)} = \frac{Z_{KN,t}}{Z_{LN,t}} \frac{\alpha_{KN,t}}{1 - \alpha_{KN,t} - \alpha_{SN,t} - \alpha_{ON,t}} \left(\frac{Z_{LN,t} r_t}{Z_{KN,t} w_t} \right)^{-\xi_N} \quad (30)$$

where the real marginal cost equation is simply the cost dual of equation (27).

As in the commodities sector, each firm n uses a CES combination of labour inputs:

$$\ell_t(n) = \left[\left(\frac{1}{ss} \right)^{\frac{1}{\psi_{L,t}}} \int_0^s \ell(n, j)^{1 - \frac{1}{\psi_{L,t}}} dj \right]^{\frac{\psi_{L,t}}{\psi_{L,t} - 1}} \quad (31)$$

where $\ell(n, j)$ is the demand of labour input of type j by the producer of good n . Cost minimization implies that $\ell(n, j)$ is a function of the relative wage:

$$\ell_t(n, j) = \left(\frac{1}{ss} \right) \left(\frac{w_t(j)}{w_t} \right)^{-\psi_{L,t}} \ell_t(n) \quad (32)$$

where $w(j)$ is the wage as defined before in equation (8).

Tradables sector Production of the tradable good follows the same line of reasoning as that of nontradables. $T(h)$ is the supply of each intermediate tradable h produced by firm x

in the following manner from labour $\ell(h)$, capital $K(h)$, commodities $S(h)$ and oil $O(h)$:

$$\begin{aligned}
T_t(h) = & Z_{T,t}[(1 - \alpha_{KT,t} - \alpha_{ST,t} - \alpha_{OT,t})^{\frac{1}{\xi_T}} (Z_{LT,t}\ell_t(h))^{1-\frac{1}{\xi_T}} \\
& + \alpha_{KT,t}^{\frac{1}{\xi_T}} (Z_{KT,t}K_t(h))^{1-\frac{1}{\xi_T}} + \alpha_{ST,t}^{\frac{1}{\xi_T}} (S_t(h)(1 - \Gamma_{ST,t}))^{1-\frac{1}{\xi_T}} \\
& + \alpha_{OT,t}^{\frac{1}{\xi_T}} (O_t(h)(1 - \Gamma_{OT,t}))^{1-\frac{1}{\xi_T}}]^{\frac{\xi_T}{\xi_T-1}}
\end{aligned} \tag{33}$$

where Z_T , Z_{LT} and Z_{KT} are productivity shocks that follow stochastic processes. The rest follows as above for the nontradables sector.

Gasoline sector The supply (i.e. production) of gasoline is similar to that of nontradables and tradables. However there are two differences - there is no role for commodities, plus there are real adjustment costs of the form found in the oil and commodities sectors. Firm g supplies gasoline in the amount $GAS(g)$:

$$\begin{aligned}
GAS_t(g) = & Z_{GAS,t}[(1 - \alpha_{KGAS,t} - \alpha_{OGAS,t})^{\frac{1}{\xi_{GAS}}} (Z_{LGAS,t}\ell_t(g)(1 - \Gamma_{LGAS,t}))^{1-\frac{1}{\xi_{GAS}}} \\
& + \alpha_{KGAS,t}^{\frac{1}{\xi_{GAS}}} (Z_{KGAS,t}K_t(g)(1 - \Gamma_{KGAS,t}))^{1-\frac{1}{\xi_{GAS}}} \\
& + \alpha_{OGAS,t}^{\frac{1}{\xi_{GAS}}} (O_t(g)(1 - \Gamma_{OGAS,t}))^{1-\frac{1}{\xi_{GAS}}}]^{\frac{\xi_{GAS}}{\xi_{GAS}-1}}
\end{aligned} \tag{34}$$

where Z_{GAS} , Z_{LGAS} and Z_{KGAS} are productivity shocks following stochastic processes. Notice that there are real adjustment costs present for all factor inputs (Γ_{LGAS} , Γ_{KGAS} and Γ_{OGAS}), meaning that the short-run supply curve is much more inelastic than its long-run equivalent.

Now that we know the supply of intermediate goods, we have to consider the demands for those goods by the types of two final goods firms x and e for consumption goods (A) and investment goods (E) respectively, and if need be the government (in the case of nontradables). Therefore we will consider, in turn, the demand for nontradable goods, then domestically-produced and imported tradable goods. The demand for gasoline (which is a nontraded good in the BoC-GEM) will be covered in section 2.3.3 on final goods.

Demand for nontradable intermediate goods The aggregate demand for nontradable intermediate goods can be summarized by:

$$p_{N,t}N_t = \int_0^{ss} N_{A,t}(n, x)dx + \int_0^{ss} N_{E,t}(n, e)de + G_{N,t}(n) = \left(\frac{p_t(n)}{p_{N,t}} \right)^{-\theta_{N,t}} (N_{A,t} + N_{E,t} + G_{N,t}) \quad (35)$$

where $N_{A,t}(n, x)$ is the demand from firms x for nontradables for the consumption good, $N_{E,t}(n, e)$ is the demand for nontradables from firms e for the investment good and $G_{N,t}(n)$ is the demand for nontradables by the government sector.

Focusing first on the basket N_A , this is a CES index of all domestic varieties of nontradables. Denoting as $N_A(n, x)$ the demand by firm x of an intermediate good produced by firm n , the basket $N_A(x)$ is:

$$N_{A,t}(x) = \left[\left(\frac{1}{ss} \right)^{\frac{1}{\theta_{N,t}}} \int_0^{ss} N_{A,t}(n, x)^{1-\frac{1}{\theta_{N,t}}} dn \right]^{\frac{\theta_{N,t}}{\theta_{N,t}-1}} \quad (36)$$

where $\theta_{N,t} > 1$ denotes the elasticity of substitution among intermediate nontradables.

Firm x takes as given the prices of the nontradable goods $p(n)$. Cost minimization implies:

$$N_{A,t}(n, x) = \frac{1}{ss} \left(\frac{p_t(n)}{p_{N,t}} \right)^{-\theta_{N,t}} N_{A,t}(x) \quad (37)$$

where $p_{N,t}$ is the price of one unit of the nontradable basket, or:

$$p_{N,t} = \left[\left(\frac{1}{ss} \right) \int_0^{ss} p_t(n)^{1-\theta_{N,t}} dn \right]^{\frac{1}{1-\theta_{N,t}}} \quad (38)$$

The basket N_E is similarly characterized.

Demand for domestically-produced tradable goods Following the same steps we can derive the domestic demand schedules for the intermediate goods h :

$$\int_0^{ss} Q_{A,t}(h, x)dx + \int_0^{ss} Q_{E,t}(h, e)de = \left(\frac{p_t(h)}{p_{Q,t}} \right)^{-\theta_{T,t}} (Q_{A,t} + Q_{E,t}). \quad (39)$$

where $Q_{A,t}(h, x)$ is the demand from firms x for tradables for the consumption good and $Q_{E,t}(h, e)$ is the demand for nontradables from firms e for the investment good.

Demand for imported tradable goods The derivation of the foreign demand schedule for good h from the home country is analytically more complex but, as we show in equation (47) at the end of this section, it shares the same functional form as equations (35) and (39) above and can be written as a function of the relative price of good h (with elasticity $\theta_{T,t}$) and total foreign demand for imports of goods from the home country.

We will focus first on import demand in the consumption good sector. Since we deal with goods produced in different regions, there are region indices in the notation. The imports M_A^H by firm x^H for the home region H are a CES function of baskets of goods imported from the other regions R , or:

$$M_{A,t}^H(x^H)^{1-\frac{1}{\rho_A^H}} = \sum_{R \neq H} \left(b_{A,t}^{H,R} \right)^{\frac{1}{\rho_A^H}} \left(M_{A,t}^{H,R}(x^H)(1 - \Gamma_{MA,t}^{H,R}(x^H)) \right)^{1-\frac{1}{\rho_A^H}} \quad (40)$$

where:

$$0 \leq b_{A,t}^{H,R} \leq 1, \quad \sum_{R \neq H} b_{A,t}^{H,R} = 1 \quad (41)$$

In equation (40) above ρ_A^H is the elasticity of import substitution across countries: the higher is ρ_A^H , the easier it is for firm x^H to substitute away from importing goods from one region to importing goods from another region. The parameter $b_{A,t}^{H,R}$ helps determine the percentage share of imports from a particular region, subject to the elasticity of substitution ρ_A^H .¹²

The response of imports to changes in fundamentals and their price elasticities are typically observed to be smaller in the short run than in the long run. To model realistic dynamics of import volumes (such as delayed and sluggish adjustment to changes in relative prices) we assume that imports are subject to real adjustment costs $\Gamma_{MA}^{H,R}$. These costs are specified as a function of the one-period change in import shares relative to firm x^H 's output and can be different across exporters. They are zero in the steady state. Specifically, we

¹²The parameter $b_A^{H,R}$ can be time-varying; it is represented in the model by the stochastic process found in equation (1).

adopt the parameterization:

$$\Gamma_{MA,t}^{H,R} \left[\frac{M_{A,t}^{H,R}(x^H)}{A_t^H(x)} / \frac{M_{A,t-1}^{H,R}}{A_{t-1}^H} \right] = \frac{\phi_{MA}^{H,R}}{2} \frac{\left[\left(M_{A,t}^{H,R}(x^H) / A_t^H(x) \right) / \left(M_{A,t-1}^{H,R} / A_{t-1}^H \right) - 1 \right]^2}{\left(1 + \left[\left(M_{A,t}^{H,R}(x^H) / A_t^H(x) \right) / \left(M_{A,t-1}^{H,R} / A_{t-1}^H \right) - 1 \right]^2 \right)} \quad (42)$$

such that $\Gamma_{MA}^{H,R}[1] = 0$, $\Gamma_{MA}^{H,R}[\infty] = \phi_{MA}^{H,R}/2$, and $\Gamma_{MA}^{H,R}[0] = \Gamma_{MA}^{H,R}[2] = \phi_{MA}^{H,R}/4$.¹³

Denoting $p_M^{H,R}$ the price in region H of a basket of intermediate inputs imported from region R , cost minimization implies:

$$M_{A,t}^{H,R}(x^H) = b_{A,t}^{H,R} \left(\frac{p_{M,t}^{H,R}}{p_{MA,t}^H(x^H)} \right)^{-\rho_A^H} M_{A,t}^H(x^H) \frac{\left(1 - \Gamma_{MA,t}^{H,R}(x^H) - M_{A,t}^{H,R}(x^H) \Gamma_{MA,t}'^{H,R}(x^H) \right)^{\rho_A^H}}{\left(1 - \Gamma_{MA,t}^{H,R}(x^H) \right)} \quad (43)$$

where $\Gamma_{MA}^{\prime H,R}(x^H)$ is the first derivative of $\Gamma_{MA}^{H,R}(x^H)$ with respect to $M_A^{H,R}(x^H)$. The import price in the consumption sector, p_{MA}^H , is defined as:

$$p_{MA,t}^H(x^H) = \left[\sum_{r \neq H} b_{A,t}^{H,R} \left(\frac{p_{M,t}^{H,R}}{1 - \Gamma_{MA,t}^{H,R}(x^H) - M_{A,t}^{H,R}(x^H) \Gamma_{MA,t}'^{H,R}(x^H)} \right)^{1-\rho_A^H} \right]^{\frac{1}{1-\rho_A^H}} \quad (44)$$

In principle, the cost-minimizing import price $p_{MA}^H(x^H)$ is firm-specific, as it depends on firm x^H 's import share. To the extent that all firms x^H are symmetric within the consumption sector, however, there will be a unique import price p_{MA}^H . This means that the aggregate level of nominal imports for the consumption good is the following function of imports from the differing regions, subject the real adjustment costs of $\Gamma_{MA}^{H,R}$:

$$p_{MA,t}^H(x^H) M_{A,t}^H(x^H) = \sum_{R \neq H} p_{M,t}^{H,R} \frac{M_{A,t}^{H,R} (1 - \Gamma_{MA,t}^{H,R}(x^H))}{\left(1 - \Gamma_{MA,t}^{H,R}(x^H) - M_{A,t}^{H,R}(x^H) \Gamma_{MA,t}'^{H,R}(x^H) \right)} \quad (45)$$

$M_A^{H,R}(x^H)$ is the basket of imported consumption goods in region H from imported from region R . It is a CES index of all varieties of tradable intermediate goods destined for consumption produced by firms h^R operating in region R and exported to region H (similar

¹³This parameterization of import adjustment costs allows the non-linear model to deal with potentially large shocks relative to the quadratic specification adopted originally in Laxton and Pesenti (2003).

to equation (36) above). The imported good $M_A^{H,R}(x^H)$ can also be defined as the sum of the demands by domestic consumption-good-producing firms x^H of an intermediate good produced by firms in region R producing the tradable good h^R (denoted as $M_A^{H,R}(h^R, x^H)$):

$$M_{A,t}^{H,R}(x^H) = \left[\left(\frac{1}{ss^R} \right)^{\frac{1}{\theta_{T,t}^R}} \int_0^{ss^R} M_{A,t}^{H,R}(h^R, x^H)^{1-\frac{1}{\theta_{T,t}^R}} dh^R \right]^{\frac{\theta_{T,t}^R}{\theta_{T,t}^R-1}} \quad (46)$$

where $\theta_T^R > 1$ is the elasticity of substitution among intermediate tradables, the same elasticity entering equation (39) in region R .

The cost-minimizing firm x^H takes as given the prices of the imported goods $p^H(h^R)$ and determines its demand for good h^R according to:

$$M_{A,t}^{H,R}(h^R, x^H) = \frac{1}{ss^R} \left(\frac{p_t^H(h^R)}{p_{M,t}^{H,R}} \right)^{-\theta_{T,t}^R} M_{A,t}^{H,R}(x^H) \quad (47)$$

where $M_{A,t}^{H,R}(x^H)$ has been defined in (43) and $p_M^{H,R}$ is:

$$p_{M,t}^{H,R} = \left[\left(\frac{1}{ss^R} \right) \int_0^{ss^R} (1 + \tau_{TRF}^{H,R}) p_t^H(h^R)^{1-\theta_{T,t}^R} dh^R \right]^{\frac{1}{1-\theta_{T,t}^R}} \quad (48)$$

where τ_{TRF} is the tariff imposed by the fiscal agent in region H on the nominal value of imports (and hence their price $p_t^H(h^R)$) from region R .

Import demand in the investment good sector is derived in the same manner as above. As a last step, we can derive region R 's demand for region H 's intermediate good h^H , that is, the analogue of (39). Aggregating across firms gives us the result that imports supplied to region R are equal to the demand for imports in region R :

$$\begin{aligned} & \int_0^{ss^R} M_{A,t}^{R,H}(h^H, x^R) dx^R + \int_0^{ss^R} M_{E,t}^{R,H}(h^H, e^R) de^R \\ &= \frac{ss^R}{ss^H} \left(\frac{p_t^R(h^H)}{p_{M,t}^{R,H}} \right)^{-\theta_{T,t}^H} (M_{A,t}^{R,H} + M_{E,t}^{R,H}) \end{aligned} \quad (49)$$

2.3.3 Final goods - consumption and investment

In each region there is a continuum of symmetric firms producing the two nontraded final goods, A (the consumption good) and E (the investment good) under perfect competition. Consider first the consumption sector. Each firm producing the final consumption good is indexed by $x \in [0, ss]$. Firm x 's output at time t is denoted $A_t(x)$. The consumption good is produced with the following double-nested constant elasticity of substitution (CES) technology:

$$\begin{aligned}
A_t(x)^{1-\frac{1}{\epsilon_{GAS}}} &= \gamma_{GAS,t}^{\frac{1}{\epsilon_{GAS}}} GAS_t(x)^{1-\frac{1}{\epsilon_{GAS}}} \\
&+ (1 - \gamma_{GAS,t})^{\frac{1}{\epsilon_{GAS}}} [(1 - \gamma_{A,t})^{\frac{1}{\epsilon_A}} N_{A,t}(x)^{1-\frac{1}{\epsilon_A}} \\
&+ \gamma_{A,t}^{\frac{1}{\epsilon_A}} [\nu_{A,t}^{\frac{1}{\mu_A}} Q_{A,t}(x)^{1-\frac{1}{\mu_A}} + (1 - \nu_{A,t})^{\frac{1}{\mu_A}} M_{A,t}(x)^{1-\frac{1}{\mu_A}}]^{\frac{\mu_A}{\mu_A-1}} (1-\frac{1}{\epsilon_A})^{\frac{\epsilon_A}{\epsilon_A-1}}]^{1-\frac{1}{\epsilon_{GAS}}} \quad (50)
\end{aligned}$$

Four intermediate inputs are used in the production of the consumption good A . First there is a combination of a basket Q_A of domestic tradable goods, and a basket M_A of imported goods to obtain a basket of tradable goods (notionally called T_A), which is then combined with a basket N_A of nontradable goods to obtain a basket of non-gasoline goods (notionally called F_A), which is finally combined basket GAS_A of gasoline goods to produce the consumption good A . The double-nested CES technology generates more flexibility in the calibration by allowing different elasticities of substitution between the components of the consumption basket. The elasticity of substitution between tradables and nontradables is $\epsilon_A > 0$, the elasticity of substitution between domestic and imported tradables is $\mu_A > 0$ and the elasticity of substitution between gasoline and the composite good is $\mu_{GAS} > 0$. The biases towards the use of the four inputs in the production of the consumption good are, γ_{GAS} for gasoline $(1 - \gamma_{GAS})(1 - \gamma_A)$ for the nontradable good, $(1 - \gamma_{GAS})\gamma_A\nu_A$ for the domestically-produced tradable good and $(1 - \gamma_{GAS})\gamma_A(1 - \nu_A)$ for the imported tradable good, with $0 < \gamma_A, \gamma_{GAS}, \nu_A < 1$.

Firm x takes as given the prices of the four inputs and minimizes its costs subject to the supply function (50). Cost minimization implies that firm x 's demands for intermediate inputs are:

$$GAS_t(x) = \gamma_{GAS,t}((1 + \tau_{GAS,t})p_{GAS,t})^{-\epsilon_{GAS}} A_t(x) \quad (51)$$

$$N_{A,t}(x) = (1 - \gamma_{GAS,t})(1 - \gamma_{A,t}) p_{N,t}^{-\epsilon_A} p_{FA,t}^{\epsilon_A - \epsilon_{GAS}} A_t(x) \quad (52)$$

$$Q_{A,t}(x) = (1 - \gamma_{GAS,t}) \gamma_{A,t} \nu_{A,t} p_{Q,t}^{-\mu_A} p_{TA,t}^{\mu_A - \epsilon_A} p_{FA,t}^{\epsilon_A - \epsilon_{GAS}} A_t(x) \quad (53)$$

$$M_{A,t}(x) = (1 - \gamma_{GAS,t}) \gamma_{A,t} (1 - \nu_{A,t}) p_{MA,t}^{-\mu_A} p_{TA,t}^{\mu_A - \epsilon_A} p_{FA,t}^{\epsilon_A - \epsilon_{GAS}} A_t(x) \quad (54)$$

where τ_{GAS} is the tax rate on gasoline, p_{GAS} , p_N , p_Q and p_{MA} are the relative prices of the inputs, p_{TA} is the shadow relative price of the composite basket of domestic and foreign tradables:

$$p_{TA,t} \equiv \left[\nu_{A,t} p_{Q,t}^{1-\mu_A} + (1 - \nu_{A,t}) p_{MA,t}^{1-\mu_A} \right]^{\frac{1}{1-\mu_A}} \quad (55)$$

and p_{FA} is the shadow relative price of the composite basket of the non-gasoline final consumption good:

$$p_{FA,t} \equiv \left[\gamma_{A,t} p_{TA,t}^{1-\epsilon_A} + (1 - \gamma_{A,t}) p_{N,t}^{1-\epsilon_A} \right]^{\frac{1}{1-\epsilon_A}} \quad (56)$$

while the price of the consumption good is normalized to one, as the Consumer Price Index (*CPI*) is the *numéraire* of the economy:

$$1 \equiv \left[\gamma_{GAS,t} (1 + \tau_{GAS,t}) p_{GAS,t}^{1-\mu_{GAS}} + (1 - \gamma_{GAS,t}) p_{FA,t}^{1-\mu_{GAS}} \right]^{\frac{1}{1-\mu_{GAS}}} \quad (57)$$

CPI is the basis for the calculation of headline inflation. Core inflation excludes the direct effects of gasoline prices as well as tariffs. Therefore, core inflation is calculated from an analogue of equation (56) where its component price p_{TAx} (instead of p_{TA}) excludes the direct effects of tariffs on the price of imports:

$$CPI_t^X \equiv \left[\gamma_{A,t} p_{TAx,t}^{1-\epsilon_A} + (1 - \gamma_{A,t}) p_{N,t}^{1-\epsilon_A} \right]^{\frac{1}{1-\epsilon_A}} \quad (58)$$

The formulation of the investment good is simpler, produced with only a single-nested constant elasticity of substitution (CES) technology:

$$\begin{aligned} E_t(e)^{1-\frac{1}{\mu_E}} &= \left[(1 - \gamma_{E,t})^{\frac{1}{\epsilon_E}} N_{E,t}(e)^{1-\frac{1}{\epsilon_E}} \right. \\ &\left. + \gamma_{E,t}^{\frac{1}{\epsilon_E}} \left[\nu_{E,t}^{\frac{1}{\mu_E}} Q_{E,t}(e)^{1-\frac{1}{\mu_E}} + (1 - \nu_{E,t})^{\frac{1}{\mu_E}} M_{E,t}(e)^{1-\frac{1}{\mu_E}} \right]^{\frac{\mu_E}{\mu_E-1}} \left(1 - \frac{1}{\epsilon_E}\right) \right]^{\frac{\epsilon_E}{\epsilon_E-1}} \end{aligned} \quad (59)$$

Three intermediate inputs are used in the production of the investment good E : where a basket N_E of nontradable goods, is combined with the notional basket of tradable goods T_E (itself a combination of a basket Q_E of domestic tradable goods, and a basket M_E of imported investment goods). The elasticity of substitution between tradables and nontradables is $\epsilon_E > 0$, the elasticity of substitution between domestic and imported tradables is $\mu_E > 0$. The biases towards the use of the three inputs in the production of the investment good are, $(1 - \gamma_E)$ for the nontradable good, $\gamma_E \nu_E$ for the domestically-produced tradable good and $\gamma_E (1 - \nu_E)$ for the imported tradable good with $0 < \gamma_E, \nu_E < 1$.

Firm e takes as given the prices of the four inputs and minimizes its costs subject to the technological constraint (59). Cost minimization implies that firm e 's demands for intermediate inputs are:

$$N_{E,t}(e) = (1 - \gamma_{E,t}) \left(\frac{p_{N,t}}{p_{E,t}} \right)^{-\varepsilon_E} E_t(x) \quad (60)$$

$$Q_{E,t}(e) = \gamma_{E,t} \nu_{E,t} \left(\frac{p_{Q,t}}{p_{E,t}} \right)^{-\mu_E} \left(\frac{p_{TE,t}}{p_{E,t}} \right)^{\mu_E - \varepsilon_E} E_t(x) \quad (61)$$

$$M_{E,t}(e) = \gamma_{E,t} (1 - \nu_{E,t}) \left(\frac{p_{ME,t}}{p_{E,t}} \right)^{-\mu_E} \left(\frac{p_{TE,t}}{p_{E,t}} \right)^{\mu_E - \varepsilon_E} E_t(x) \quad (62)$$

where p_N , p_Q and p_{ME} are the relative prices of the inputs, p_{TE} is the shadow relative price of the composite basket of domestic and foreign tradables:

$$p_{TE,t} \equiv \left[\nu_{E,t} p_{Q,t}^{1-\mu_E} + (1 - \nu_{E,t}) p_{ME,t}^{1-\mu_E} \right]^{\frac{1}{1-\mu_E}} \quad (63)$$

and p_E is the relative price of investment goods. The supply of investment goods will be considered in Section 2.5.2.

2.4 Price setting by the firms

Now that we have outlined the real side of the five production sectors for the firms, we will consider the price-setting decisions faced by firms. We will consider the raw materials sectors first, then, the two nontraded sectors (nontradables and gasoline) and finally, the tradables sector.

2.4.1 The raw materials sectors

For the raw materials sectors, we will discuss the commodities sector, but the arguments below apply exactly to the oil sector as well, as firms in both oil and commodities sectors (o and s) face significant real adjustment costs, but have market power (and hence fix a price with a mark-up over real marginal cost) but are flexible in their price setting every period. since they face no nominal rigidities and prices can adjust instantaneously. We consider this an accurate reflection of the behaviour of raw materials prices in the data. The oil sector's pricing differs slightly in form from that of the commodities sector since oil is also used in the production of gasoline, and not only tradable and nontradable goods.

In the commodities sector each firm s takes into account the demand (24) for its product

and sets its nominal price by maximizing the present discounted value of real profits:

$$\max_{p_t^H(s)} \mathbf{E}_t \sum_{\tau=t}^{\infty} D_{t,\tau}^H \pi_{t,\tau}^H g_{t,\tau} [p_{Q,\tau}^H(s) - mc_{\tau}^H(s)] \left(\frac{p_{\tau}^H(s)}{p_{Q,S,\tau}^H} \right)^{-\theta_{S,t}^H} (Q_{ST,\tau}^H + Q_{SN,\tau}^H + \Sigma_R M_{S,\tau}^{R,H}) \quad (64)$$

where $D_{t,\tau}$ (with $D_{t,t} = 1$) is the appropriate discount rate, to be defined below in equation (88). As real variables are detrended and prices are deflated by the CPI, equation (64) includes $\pi_{t,\tau}$, the CPI inflation rate between time t and time τ , and $g_{t,\tau}$, the rate of growth of the global trend *TREND* between t and τ .

As domestic firms s are symmetric and charge the same equilibrium price $p(s) = p_{QS}$, the first order condition required for profit maximization can be written as:

$$p_{Q,t}(s) = \frac{\theta_{S,t}}{\theta_{S,t} - 1} mc_t(s) \quad (65)$$

where the gross markup is a negative function of the elasticity of input substitution θ_S . As the varieties of commodities produced by firms s are more alike (i.e. θ_S is a higher value since this implies a higher elasticity of substitution amongst commodities) the lower is the potential mark-up that a firm can charge over its real marginal cost.

2.4.2 The nontradables sector

Consider now profit maximization in the intermediate nontradables sector. Each firm n takes into account the demand (35) for its product and sets its nominal price by maximizing the present discounted value of real profits. There are costs of nominal price adjustment measured in terms of total profits foregone. The nominal rigidity is denoted $\Gamma_{PN,t} [p_t(n), p_{t-1}(n)]$.

The price-setting problem for the typical nontradable-goods-producing firm n is:

$$\max_{p_t(n)} \mathbf{E}_t \sum_{\tau=t}^{\infty} D_{t,\tau} \pi_{t,\tau} g_{t,\tau} [p_{\tau}(n) - mc_{\tau}(n)] \left(\frac{p_{\tau}(n)}{p_{N,\tau}} \right)^{-\theta_{N,t}} (N_{A,\tau} + N_{E,\tau} + G_{N,\tau}) (1 - \Gamma_{PN,\tau}(n)) \quad (66)$$

As firms n are symmetric and charge the same equilibrium price $p(n) = p_N$, the first order

condition can be written as:

$$\begin{aligned}
0 = & (1 - \Gamma_{PN,t}(n)) [p_t(n) (1 - \theta_{N,t}) + \theta_{N,t} m_{c_t}(n)] - [p_t(n) - m_{c_t}(n)] \frac{\partial \Gamma_{PN,t}}{\partial p_t(n)} p_t(n) \\
& - \mathbf{E}_t D_{t,t+1} \pi_{t,t+1} g_{t,t+1} [p_{t+1}(n) - m_{c_{t+1}}(n)] \frac{N_{t+1}}{N_t} \frac{\partial \Gamma_{PN,t+1}}{\partial p_t(n)} p_t(n)
\end{aligned} \tag{67}$$

Interpreting the previous equation, when prices are fully flexible ($\Gamma_{PN} = 0$), the optimization problem collapses to the standard markup rule:

$$p_t(n) = \frac{\theta_{N,t}}{\theta_{N,t} - 1} m_{c_t}(n) \tag{68}$$

where the gross markup is a negative function of the elasticity of input substitution. As the varieties of nontradable goods produced by firms n are more alike (i.e. θ_n is a higher value since this implies a higher elasticity of substitution amongst varieties of nontradable goods) the lower is the potential mark-up that a firm can charge over its real marginal cost. Deviations from markup pricing occur if firms face costs for modifying their prices in the short term. The speed of adjustment in response to shocks depends on the trade-off between current and future expected costs, making the price-setting process forward-looking, but preferably also with a lag - this is the basis of the linearised formulation of the hybrid New Keynesian Phillips' Curve, where price inflation is generally a function of its lag, its expected one-period-ahead level, and contemporaneous real marginal cost (see Galì and Gertler (1999)). Such a Phillips' curve is implied by all the relative prices in the BoC-GEM.

There are three types of adjustment costs associated with price setting that are generally used in DSGE models. There is the form originally stated by Calvo (1983), where some random share of firms are assumed to adjust their prices entirely each period. In this case, there is the concept of contract length for the prices, generally assumed to be four quarters. Similarly based on contract length is Taylor (1980), where some firms are assumed to adjust their prices fully each period. It differs from Calvo (1983) in that all firms will adjust their prices at regular, but staggered, intervals.¹⁴ Finally, a third common specification is based on Rotemberg (1982), where it is assumed that all firms partially adjust their prices each period towards the steady-state price level. We chose the Rotemberg formulation, as it allows the model to reproduce realistic nominal dynamics in an analytically tractable form; furthermore its main parameter ϕ_{PN1} (see below) can be translated into contract lengths as in the Calvo and Taylor pricing methodologies.

¹⁴Whereas in Calvo pricing, some firms could theoretical never adjust their prices (unless full indexation is assumed, which disallows price dispersion).

The original Rotemberg formulation was adopted in a model assuming zero steady-state inflation and therefore a constant price level; because the BoC-GEM contains a non-zero steady-state inflation rate (and hence a non-stationary price level) we assume that firms are attempting to stabilize the inflation rate at some combination of a constant inflation rate (i.e. the first difference of inflation) and the steady-state level of inflation (i.e. the actual versus targeted inflation gap). Achieving a constant inflation rate stabilizes price movements in the long run and allows a lag to enter the implied Phillips curve; however using this method exclusively implies prices follow a path with (excessively) long cycles before reaching the steady state. Trying to move the inflation rate directly to the steady-state level of inflation allows the entire nominal side of the model to converge smoothly without the excessive cycling implied by converging only to a stable inflation rate; but using exclusively would remove any backward-lookingness from the Phillips' curve and impart what is generally agreed in the literature to be too high a degree of perfect foresight. The nominal rigidities are formulated then as the following:

$$\Gamma_{PN,t}(n) \equiv \frac{\phi_{PN1}}{2} \left(\pi_{t-1,t} \frac{p_t(n)/p_{t-1}(n)}{\pi_{Nt-2,t-1}^{\phi_{PN2}} \Pi_{t-4,t}^{0.25(1-\phi_{PN2})}} - 1 \right)^2 \quad (69)$$

where the nominal rigidities are related to changes of the nominal price of nontradable n relative to a target that is a weighted average of last period's nontradable price inflation (weighted by ϕ_{PN2}) and the quarterly version of the year-on-year inflation target, $\Pi_{t-4,t}$ (weighted by $(1 - \phi_{PN2})$). This is the formulation for nominal rigidities used for all relative prices in the BoC-GEM.

2.4.3 The gasoline sector

Profit-maximizing behaviour in the gasoline sector, a nontraded good, is much like that in the nontradables sector. However, there are no nominal rigidities in the price of gasoline (thereby resembling price-setting in the raw materials sectors).

The price-setting problem is then characterized as:

$$\max_{p_t(g)} \mathbf{E}_t \sum_{\tau=t}^{\infty} D_{t,\tau} \pi_{t,\tau} g_{t,\tau} [p_{\tau}(g) - mc_{\tau}(g)] \left(\frac{p_{\tau}(g)}{p_{GAS,\tau}} \right)^{-\theta_{GAS,t}} GAS_{\tau} \quad (70)$$

Since domestic firms g are symmetric and charge the same equilibrium price $p(g) = p_{GAS}$,

the first order condition required for profit maximization can be written as:

$$p_t(g) = \frac{\theta_{GAS,t}}{\theta_{GAS,t} - 1} mc_t(g) \quad (71)$$

2.4.4 The tradables sector and exchange rate pass-through

Consider now the price-setting problem in the tradables sector. To the extent that the five regional blocs represent segmented markets in the global economy, each firm h has to set five prices, one in the domestic market and the other four in the export markets. Exports are invoiced (and prices are set) in the currency of the destination market.¹⁵ As we discuss export markets, once again our notation needs to make explicit the regions' indices.

Accounting for (49), the four price-setting problems of firm h in region H can then be characterized as follows:

$$\begin{aligned} & \max_{p_t^R(h^H)} \sum_R \mathbb{E}_t \sum_{\tau=t}^{\infty} D_{t,\tau}^H \pi_{t,\tau}^H g_{t,\tau} [\varepsilon_{\tau\tau}^{H,R} \bar{p}^R(h^H) - mc_{\tau}^H(h^H)] \\ & * \frac{s_{SS}^R}{s_{SS}^H} \left(\frac{\bar{p}_{\tau}^R(h^H)}{\bar{p}_{M,\tau}^{R,H}} \right)^{-\theta_{T,\tau}^H} \left(M_{A,\tau}^{R,H} + M_{E,\tau}^{R,H} \right) \left(1 - \Gamma_{PM,\tau}^{R,H}(h) \right) \end{aligned} \quad (72)$$

When $H \neq R$, $\bar{p}^R(h^H)$ is the wholesale price (i.e. before tariffs are applied by region R) of good h^H in region R , $\bar{p}_M^{R,H}$ is the wholesale price of region R 's imports of consumption and investment goods from region H , and $M_A^{R,H} + M_E^{R,H}$ are region R 's imports from region H . The term $\varepsilon^{H,R}$ is the real bilateral exchange rate between region H and region R (an increase in $\varepsilon^{H,R}$ represents a depreciation of region H 's currency against region R) and $\Gamma_{PM}^{H,R}(h^H)$ are adjustment costs related to changes of the price of good h^H in region R . These costs are the analogs of equation (69):

$$\Gamma_{PM,t}^{R,H}(h^H) \equiv \frac{\phi_{PM1}^{R,H}}{2} \left(\pi_{t-1,t}^R \frac{\bar{p}_t^R(h^H)/\bar{p}_{t-1}^R(h^H)}{(\pi_{Mt-2,t-1}^R)^{\phi_{PM2}^{R,H}} (\Pi_{t-4,t}^R)^{0.25(1-\phi_{PM2}^{R,H})}} - 1 \right)^2 \quad (73)$$

For the domestic prices of tradables $p^H(h^H)$ we still use equation (72) with $R = H$, adopting the notational conventions $\bar{p}_M^{H,H} = p_Q^H$, $M_A^{H,H} = Q_A^H$ and $M_E^{H,H} = Q_E^H$ as described in equation (39), and $\Gamma_{PM}^{H,H} = \Gamma_{PQ}^H$.

¹⁵This is more commonly know in the literature as "local currency pricing".

Profit maximization in the tradables sector yields:

$$\begin{aligned}
0 = & \left(1 - \Gamma_{PM,t}^{R,H}(h^H)\right) \left[\varepsilon_t^{H,R} \overline{p}_t^R(h^H) (1 - \theta_{T,t}^H) + \theta_{T,t}^H mc_t^H(h^H)\right] \\
& - \left[\varepsilon_t^{H,R} \overline{p}_t^R(h^H) - mc_t^H(h^H)\right] \frac{\partial \Gamma_{PM,t}^{R,H}}{\partial \overline{p}_t^R(h^H)} \overline{p}_t^R(h^H) - \mathbf{E}_t\{D_{t,t+1}^H \pi_{t,t+1}^H g_{t,t+1} \\
& * \left[\varepsilon_{t+1}^{H,R} \overline{p}_{t+1}^R(h^H) - mc_{t+1}^H(h^H)\right] \left(\frac{M_{A,t+1}^{R,H} + M_{E,t+1}^{R,H}}{M_{A,t}^{R,H} + M_{E,t}^{R,H}}\right) \frac{\partial \Gamma_{PM,t+1}^{R,H}}{\partial \overline{p}_t^R(h^H)} \overline{p}_t^R(h^H)\} \quad (74)
\end{aligned}$$

If real adjustment costs in the export market are strong (i.e. the parameter $\phi_{PM1}^{R,H}$ is relatively large), the prices of region H 's goods in the foreign markets are characterized by significant stickiness in local currency. In this case, the degree to which exchange rate movements (and other shocks to marginal costs in region H) pass through into import prices in region R is rather low. If instead the $\phi_{PM1}^{R,H}$ coefficients are zero worldwide, equation (74) collapses to the typical markup rule with full and immediate exchange rate pass-through:

$$p_t^{H,H}(h^H) = p_{Q,t}^H = \varepsilon_t^{H,R} \overline{p}_t^R(h^H) = \varepsilon_t^{H,R} \overline{p}_{M,t}^{R,H} = \frac{\theta_{T,t}}{\theta_{T,t} - 1} mc_t \quad (75)$$

2.5 The consumers' problem

Now that we have fully articulated the production side of the economy, we can address the consumers' problem in turn. In each region there is a continuum of households indexed by $j \in [0, ss]$, the same index as labour inputs. Some households have access to capital markets, some do not. The latter finance their consumption by relying exclusively on their labour incomes. We refer to the first type as 'Ricardian' or 'forward-looking'; they represent a share $(1 - s_{LC})$ of domestic households and are indexed by $j \in [0, ss(1 - s_{LC})]$. We refer to the second type as 'non-Ricardian' or 'liquidity-constrained'; they represent a share s_{LC} of domestic households and are indexed by $j \in (ss(1 - s_{LC}), ss]$. Moreover, in order to make the labour market more tractable in the structure of the model, we associate liquidity-constrained consumers with low-skilled workers, and forward-looking consumers with highly skilled labour, as in Faruquee et al. (2007b). Therefore we assume that $\psi_{L,t}$ is the elasticity substitution between the two classes of labour of the liquidity-constrained (low-skilled) households and the forward-looking (high-skilled) households.

2.5.1 The utility function

The specification of households' preferences uses the Greenwood, Hercowitz and Huffman (1988) (GHH) utility function, adjusted for habit formation and preference shocks. Often a utility function that is additively separable in consumption and leisure is used, but this is inappropriate for the GEM because growth is not zero in the steady-state, necessitating a functional form that recognizes capital and other factors will grow at the trend productivity growth rate g_t , while labour grows at some population growth rate (here assumed to be zero). This can be satisfied by using a multiplicative utility function of the King-Plosser-Rebelo form (King et al. (1988), such as in ToTEM (Murchison and Rennison (2006)). However this can prove to be an intractable functional form when using the nonlinear model.¹⁶ Therefore we continue the use of the GHH utility function as in Faruqee et al. (2007a, 2007b), which is additively separable, since both capital and labour are assumed to grow at the rate of productivity. It has an added feature that the intertemporal elasticity of substitution for labour is zero, which will be discussed below when deriving the optimal wage for consumers (Correia, Neves and Rebelo (1995)).

In the following discussion, we will consider the forward-looking household FL , but the notation can be applied equally to the liquidity constrained households LC . We have $\mathcal{W}_{FL,t}(j)$ the lifetime expected utility of household j :

$$\mathcal{W}_{FL,t}(j) \equiv \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta_{t,\tau} g_{t,\tau}^{1-\sigma} u_{\tau}(C_{FL,\tau}(j), \ell_{FL,\tau}(j)) \quad (76)$$

where the instantaneous felicity is a function of consumption demanded C and labour effort supplied ℓ :

$$u_{FL,t}(C_{FL,t}(j), \ell_{FL,t}(j)) = Z_{UFL,t} \left(1 - \frac{b_{cfl}}{g_{t-1,t}}\right) \left(\frac{1 - b_{\ell fl}}{1 - \sigma_{fl}}\right) \times \left[\frac{C_{FL,t}(j) - b_{cfl} C_{FL,j,t-1}/g_{t-1,t}}{1 - b_{cfl}/g_{t-1,t}} - \frac{Z_{VFL,t}}{1 + \zeta_{fl}} \left(\frac{\ell_{FL,t}(j) - b_{\ell fl} \ell_{FL,j,t-1}}{1 - b_{\ell fl}}\right)^{1+\zeta_{fl}}\right]^{1-\sigma_{fl}}, \quad (77)$$

In the expressions above $\beta_{t,\tau}$ is the discount rate between time t and time τ , possibly different across regions. As mentioned in Section 2.2, because of technological progress associated with home production activities (here related to the global trend), the term $g_{t,\tau}^{1-\sigma}$

¹⁶ Although to be fair, the problem of tractability does not exist in the version of ToTEM that is commonly used, since it is linearised around a steady state using a first-order Taylor approximation.

in (76) implies that the disutility of labour effort increases with the common trend.¹⁷ The parameter σ in equations (76) and (77) is the reciprocal of the elasticity of intertemporal substitution. The parameter ζ is the reciprocal of the Frisch elasticity of labour and affects the curvature of the labour disutility function. The terms Z_{UFL} and Z_{VFL} are shocks to consumption and labour respectively that are modelled as stochastic processes.

Since there is habit persistence in consumption demand, we find the term $C_{FL,j,t-1}$ in equation (77), which is the past per-capita consumption of household j 's peers (i.e. forward-looking agents).¹⁸ Habit persistence in consumption is governed by the parameter $0 < b_{cfl} < 1$. $b_{cfl} = 0$ would imply that agents only draw utility from consuming now in any given period, whereas $b_{cfl} = 1$ would imply that utility increases (decreases) as the rate of growth of consumption increases (decreases). Similarly, there is habit persistence in labour supply governed by the parameter $0 < b_{lfl} < 1$. Since habit persistence means that agents place a large weight on their past behaviour in terms of consuming and use of leisure time; this helps match the "humped-shape response" of consumption demand (and labour effort supplied) that is a stylized fact in most economies in the face of a large variety of shocks.

Households' preferences are therefore symmetric within their respective categories (i.e. forward-looking consumers and liquidity-constrained consumers), but because of the possibility of different degrees of habit formation in either consumption or leisure, households' preferences are not necessarily symmetric across categories.

2.5.2 Budget constraint for forward-looking consumers

The individual flow budget constraint for the forward-looking household $j \in [0, (1 - s_{LC})ss]$ is:

$$\begin{aligned}
B_t(j) + \varepsilon_t B_t^*(j) &\leq (1 + i_{t-1}) \frac{B_{t-1}(j)}{\pi_{t-1,t} g_{t-1,t}} + (1 + i_{t-1}^*) [1 - \Gamma_{B,t-1}] \frac{\varepsilon_t B_{t-1}^*(j)}{\pi_{t-1,t}^{US} g_{t-1,t}} \\
&+ (1 - \tau_{K,t}) r_t K_t(j) + (1 - \tau_{L,t}) w_{FL,t}(j) \ell_{FL,t}(j) (1 - \Gamma_{WFL,t}(j)) \\
&- C_{FL,t}(j) - p_{E,t} I_t(j) + \Phi_t(j) - TT_t(j)
\end{aligned} \tag{78}$$

Households hold two nominal bonds, denominated in domestic and U.S. currency, re-

¹⁷The restriction $\beta_{t,\tau} g_{t,\tau}^{1-\sigma} < 1$ is imposed to ensure that utility is bounded.

¹⁸This form is known as external habit persistence - an agent j informs his decision on consumption $C_{FL,t}(j)$ based on the economy-wide measure of consumption last period $C_{FL,t-1}$, instead of solely his own measure of consumption $C_{FL,t-1}(j)$. When habit is formed using $C_{FL,t-1}(j)$ this is known as internal habit persistence, and less analytically and computationally tractable than the external form (see Laxton and Pesenti (2003)).

spectively. $B_t(j)$ is the holdings of the domestic (government-issued) bond by household j , expressed in terms of domestic consumption units, $B_t^*(j)$ is holdings of the international bond, expressed in terms of U.S. consumption units, and ε_t is the CPI-based real exchange rate, expressed as the price of one U.S. consumption basket in terms of domestic consumption.¹⁹

Financial assets, financial intermediation, UIRP and the link with government debt

The short-term nominal rates i_t and i_t^* are paid at the beginning of period $t + 1$ and are known at time t . The two rates are directly controlled by their respective national governments. Only the U.S.-currency bond is traded internationally: the U.S. bond is in zero net supply worldwide, while the domestic bond is issued by the domestic government.²⁰ It follows that the net financial wealth of forward-looking household j at time t is:

$$F_t(j) \equiv (1 + i_{t-1}^*) [1 - \Gamma_{B,t-1}] \frac{\varepsilon_t B_{t-1}^*(j)}{\pi_{t-1,t}^{US} g_{t-1,t}} \quad (79)$$

A financial friction Γ_B is introduced to guarantee that the five regions' net foreign asset positions follow a stationary process, allowing their economies to converge asymptotically to a well-defined steady state (Schmitt-Grohé and Uribe (2003)). It also contains a differential between regions' rates of time preference β . Households that take a position in the international bond market must deal with financial intermediaries who charge a transaction fee Γ_B on sales/purchases of the international bond.²¹ This transaction cost is a function of the average net foreign asset position of the whole economy. Specifically, we adopt the following functional form

$$1 - \Gamma_{B,t} = \left(1 - \phi_{B1} \frac{\exp(\phi_{B2} [\varepsilon_t B_t^*/GDP_t - b_{DES,t}^*]) - 1}{\exp(\phi_{B2} [\varepsilon_t B_t^*/GDP_t - b_{DES,t}^*]) + 1} - Z_{B,t} \right) \frac{1}{\exp(\phi_{B3} [\Delta_{t,t+1}/\Delta_{t-1,t} - (\pi/\pi^{US})^{0.25}])} \frac{\beta_t^{US}}{\beta_t} \quad (80)$$

where $0 \leq \phi_{B1} \leq 1$, $\phi_{B2} > 0$, $0 \leq \phi_{B3} \leq 1$, and $\varepsilon_t B_t^* \equiv (1/ss) \varepsilon_t \int_0^{ss(1-s_{LC})} B^*(j) dj$, is the per-capita net foreign asset position of the region. The term b_{DES}^* is the 'desired' net asset

¹⁹It is understood that ε is shorthand for $\varepsilon^{H,US}$, where H denotes the country under consideration.

²⁰If the country under consideration is the US, $\varepsilon = 1$ and $i = i^*$.

²¹In our model it is assumed that all intermediation firms are owned by a region's residents, and that their revenue is rebated to domestic households in a lump-sum fashion - part of Φ in equation (78). There are no intermediation costs for US residents entering the international bond market - that is, there is no difference between onshore and offshore US interest rates.

position of the region expressed as a ratio of GDP .²² This variable measures the degree of international exposure that financial intermediaries consider appropriate for the economy, based on their assessment of the economic outlook.

To understand the role played by the financial intermediation cost Γ_B , suppose first that b_{DES}^* , Z_B and ϕ_{B3} equal zero and that the rates of time preference are the same across regions ($\beta^{US} = \beta$). In this case, when the net foreign asset position of the region is equal to its ‘desired’ level of zero, it must be the case that $\Gamma_B = 0$ and the return on the international bond is equal to $1 + i^*$. If the region is a net creditor worldwide then Γ_B rises above zero, implying that the region’s households lose an increasing fraction of their international bond returns to financial intermediaries. When holdings of the international bond go to infinity, the return on the international bond approaches $(1 + i^*)(1 - \phi_{B1})$. By the same token, if the region is a net debtor worldwide then Γ_B falls from zero to $-\phi_{B1}$, implying that households pay an increasing intermediation premium on their international debt. When net borrowing goes to infinity, the cost of borrowing approaches $(1 + i^*)(1 + \phi_{B1})$. The parameter ϕ_{B2} controls the flatness of the Γ_B function: if $\phi_{B2} = 0$ then $\Gamma_B = 0$ regardless of the net foreign asset position; if ϕ_{B2} tends to infinity then $1 - \Gamma_B = (1 - \phi_{B1})$ for any arbitrarily small net lending position, and $1 - \Gamma_B = (1 + \phi_{B1})$ for any arbitrarily small net borrowing position.

Consider now the other components of equation (80). By including an international differential for rates of time preference $\frac{\beta_t^{US}}{\beta_t}$, there can be differences across regions in their real interest rates (defined below in equation (92)) even in the long run. The variable $Z_{B,t}$ is a shock that follows a stochastic process that in our framework of international financial intermediation plays the same role that ‘uncovered interest parity shocks’ in other open-economy models.²³

The term b_{DES}^* can be non-zero (either positive or negative) and the above discussion will still be entirely valid. The desired net asset position in region H is characterized as:

$$b_{DES,t}^{*H} = b_{NEUT}^{*H} - \phi_{F1}^H \frac{B_t^H}{GDP_t^H} + \sum_{R \neq H} \phi_{F2}^{R,H} \phi_{F1}^R \frac{B_t^R}{GDP_t^R} \quad (81)$$

where b_{NEUT}^{*H} is a region-specific (possibly time-varying) constant, adjusted to account for changes in the debt-to-GDP ratios in either the domestic economy (with a weight of $0 < \phi_{F1}^H < 1$ on $\frac{B^H}{GDP^H}$) or the rest of the world (with a weight of $0 < \phi_{F2}^{R,H} < 1$ on $\phi_{F1}^R \frac{B^R}{GDP^R}$).

²²The concept of GDP in our model will be defined and discussed below in equation (123).

²³Fluctuations in Z_B cannot be large enough to push Γ_B or else this implies a negative financial intermediation cost (bond-holders receive a fee for transacting in the market).

This specification provides a plausible link between domestic government debt and net foreign asset positions. If the targeted debt-to-GDP ratio increased in the United States, investors in the rest of the world would require a higher return on U.S. securities, leading to a higher share of U.S. assets in their portfolios or a reduction of net borrowing from the U.S.. If however the target debt increased in the home region as well, the U.S. premium would fall somewhat.²⁴

Finally in equation (80) there is the term $\Delta_{t,t+1}/\Delta_{t-1,t} - (\pi/\pi^{US})^{0.25}$ associated with the parameter ϕ_{B3} . It attempts to capture the forward premium puzzle - the empirical fact that there is a risk premium on exchange rate transactions that is negatively correlated with expected future depreciations (Duarte and Stockman (2005)). It implies that domestic investors will accept a lower return on their holdings of the international bond relative to their holdings of domestic debt, if the future real exchange rate is expected to depreciate in consecutive periods (and is therefore easier to predict). In such an instance they expect that in domestic currency terms that their holdings of the international bond will increase in value simply from a shift in the bilateral real exchange rate, hence a lower return from the real interest rate is acceptable (Adolfson et al. (2005)).

When we derive the uncovered interest rate parity condition below as equation (90), we will see that the term $1 - \Gamma_B$ will enter; it will serve as the main channel for the effects of equation (80). It implies that the standard UIRP condition will be adjusted for risk (i.e. financial intermediation costs) and its dynamics will be modified by the inclusion of a lag of the exchange rate (from the representation of the forward premium puzzle derived from Adolfson et al. (2005)). To better capture these extra features, this equation equating the future one-period change in the exchange rate and the forward sum of the interest rate differentials *vis-à-vis* the U.S. short-term interest rate is referred to as the modified risk-adjusted UIRP condition.

Physical capital Households accumulate physical capital $K(j)$ which they rent to domestic firms at the after-tax rental rate of $r(1 - \tau_K)$. This is the aggregate of the capital stocks

²⁴Our approach should be viewed only as a crude approximation to the actual determinants of cross-country spreads and interest rate premia in response to macroeconomic imbalances. There are other models which endogenize this feature, usually in a Blanchard-Weil framework (see, for example, Botman et al. (2006)) but many other features present in the GEM are not currently possible under those types of models, hence our use of a reduced form approximation.

from all sectors of production (see equation (113) below). The law of motion of capital is:

$$K_{t+1}(j)g_{t,t+1} = (1 - \delta) K_t(j) + \Gamma_{I,t}K_t(j) \quad 0 < \delta \leq 1 \quad (82)$$

where δ is the region-specific depreciation rate of capital. To simulate realistic investment flows, capital accumulation is subject to adjustment costs. Therefore capital is represented by $\Gamma_{I,t}K_t(j)$, where $\Gamma_I(\cdot)$ is quadratic, with the properties of being an increasing, concave, and twice-continuously differentiable function of the investment/capital ratio $I_t(j)/K_t(j)$ that ensure no adjustment costs in steady state: $\Gamma_I(\delta + g - 1) = \delta + g - 1$ and $\Gamma'_I(\delta + g - 1) = 1$. The form of the real adjustment cost $\Gamma_I(j)$ is:

$$\Gamma_{I,t}(j) \equiv \frac{I_t(j)}{K_t(j)} - \frac{\phi_{I1}}{2} \left(\frac{I_t(j)}{K_t(j)} - (\delta + g - 1) \right)^2 - \frac{\phi_{I2}}{2} \left(\frac{I_t(j)}{K_t(j)} - \frac{I_{t-1}}{K_{t-1}} \right)^2 \quad (83)$$

where $\phi_{I1}, \phi_{I2} \geq 0$.

Labour income Each forward-looking household j is the monopolistic supplier of a specific labour input and sets the nominal wage for its labour variety j . Labour incomes are taxed at the rate τ_L . Wages are assumed to adjust sluggishly in the short-run, either because of contracts, or a general resistance to a change in the wage level by workers. The adjustment cost is denoted $\Gamma_{WFL,t}$ and its specification is the analogue of the nominal rigidities as found in equation (69) above:

$$\Gamma_{WFL,t}(j) \equiv \frac{\phi_{WFL1}}{2} \left(\frac{\pi_{WFL,t-1,t}}{\pi_{WFL,t-2,t-1}^{\phi_{WFL2}} (\Pi_{t-4,t}^{0.25} g_{t-1,t})^{(1-\phi_{WFL2})}} - 1 \right)^2 \quad (84)$$

where

$$\pi_{WFL,t-1,t} = \frac{w_{FL,t}(j)}{w_{FL,t-1}(j)} \pi_{t-1,t} g_{t-1,t} \quad (85)$$

and necessitating the presence of the growth term $g_{t-1,t}$, since real wages are nominal wages detrended by both inflation and productivity.

Other features of the budget constraint Forward-looking households own all domestic firms and there is no international trade in claims on firms' profits. The variable Φ includes all dividends accruing to shareholders, plus all revenue from nominal and real adjustment rebated as a lump sum to all forward-looking households, plus revenue from financial intermediation

which is assumed to be provided by domestic firms exclusively.

Finally, agents pay lump-sum (non-distortionary) taxes net of transfers $TT_t(j)$.

2.5.3 Consumer optimization for forward-looking households

The representative forward-looking household chooses bond holdings, capital and consumption paths, and sets wages to maximize its expected lifetime utility (76) subject to (78) and (82), taking into account equations (113) and (114) from Section 2.7 on market clearing.

For expositional convenience, it is worthwhile to write explicitly the maximization problem of agent $j \in [0, (1 - s_{LC})ss]$ in terms of the following Lagrangian:

$$\begin{aligned}
& \max_{\{C_{FL,\tau}(j), I_\tau(j), B_\tau(j), B_\tau^*(j), K_{\tau+1}(j), w_{FL,\tau}(j)\}_{\tau=t}^\infty} \mathbf{E}_t \beta_{t,\tau} g_{t,\tau}^{1-\sigma} \left\{ u \left(C_{FL,\tau}(j), w_{FL,\tau}^{-\psi_{L,\tau}}(j) w_\tau^{\psi_{L,\tau}} \ell_{FL,\tau}, \right) \right. \\
& + \mu_{FL,\tau}(j) \left(-B_\tau(j) - \varepsilon_\tau B_\tau^*(j) + \frac{(1 + i_{\tau-1}) B_{\tau-1}(j)}{\pi_{\tau-1,\tau} g_{\tau-1,\tau}} + \frac{(1 + i_{\tau-1}^*)(1 - \Gamma_{B,\tau-1}) \varepsilon_\tau B_{\tau-1}^*(j)}{\pi_{\tau-1,\tau}^{US} g_{\tau-1,t}} \right. \\
& + (1 - \tau_{K,\tau}) r_\tau K_\tau(j) + (1 - \tau_{L,\tau}) w_{FL,\tau}(j)^{1-\psi_{L,\tau}} w_\tau^{\psi_{L,\tau}} \ell_\tau (1 - \Gamma_{WFL,\tau}[w_{FL,\tau}(j), w_{FL,\tau-1}(j)]) \\
& \left. \left. - C_{FL,\tau}(j) - p_{E,\tau} I_\tau(j) + \Phi_\tau(j) - TT_\tau(j) \right) \right. \\
& \left. + \lambda_{FL,\tau}(j) \left(-K_{\tau+1}(j) g_{\tau,\tau+1} + (1 - \delta) K_\tau(j) + \Gamma_{I,\tau} [I_\tau(j)/K_\tau(j)] K_\tau(j) \right) \right\} \quad (86)
\end{aligned}$$

and μ and λ are the multipliers associated with, respectively, the budget constraint and capital accumulation.

The first order conditions with respect to $C_{FL,t}(j)$ and $I_t(j)$ yield:

$$\mu_{FL,t}(j) = \partial u_{FL,t}(j) / \partial C_{FL,t}(j) = \lambda_{FL,t}(j) \Gamma'_{I,t}(j) / p_{E,t} \quad (87)$$

In a symmetric setup, $\partial u_{FL,t}(j) / \partial C_{FL,t}(j)$ is the same across forward-looking agents j . Their stochastic discount rate and pricing kernel is therefore the variable $D_{t,\tau}$, which is defined as:

$$D_{t,\tau} \equiv \beta_{t,\tau} g_{t,\tau}^{1-\sigma} \frac{\mu_{FL,\tau}}{\mu_{FL,t}} \frac{1}{\pi_{t,\tau}} \frac{1}{g_{t,\tau}} \quad (88)$$

Accounting for the above expressions, the first order conditions with respect to $B_t(j)$ and

$B_t^*(j)$ are, respectively:

$$1 = (1 + i_t) \mathbf{E}_t D_{t,t+1} \quad (89)$$

$$1 = (1 + i_t^*) (1 - \Gamma_{B,t}) \mathbf{E}_t (D_{t,t+1} \Delta_{t,t+1}) \quad (90)$$

where Δ denotes the rate of nominal exchange rate depreciation against the U.S. dollar:

$$\Delta_{t,\tau} = \frac{\varepsilon_\tau \pi_{t,\tau}}{\varepsilon_t \pi_{t,t}^*} \quad (91)$$

Combining equations (89) and (90) yields the modified risk-adjusted uncovered interest rate parity, recalling that the return on international bond holdings is augmented to account for the costs of intermediation Γ_B . In the steady state, the interest rate differential $(1 + i) / [(1 + i^*) (1 - \Gamma_B)]$ is equal to the steady-state nominal depreciation rate of the currency *vis-à-vis* the U.S. dollar, and relative purchasing power parity holds.

In the steady state equations(88) and (89) imply

$$\frac{(1 + i)}{\pi} = \frac{g^\sigma}{\beta} \quad (92)$$

such that the 'natural' interest rate of the economy is equivalent to $(1 + i) / \pi$, the real interest rate (nominal interest rate divided by the gross steady-state quarterly inflation rate π) and defined as g^σ / β , a function of the gross steady-state quarterly rate of growth of the world economy g and the rate of time preference (the inverse of the discount factor β).²⁵

The first order condition with respect to $K_{t+1}(j)$ is:

$$\begin{aligned} \frac{p_{E,t}}{\Gamma'_{I,t}(j)} \mathbf{E}_t g_{t,t+1} &= \mathbf{E}_t \{ D_{t,t+1} \pi_{t,t+1} g_{t,t+1} ((1 - \tau_{K,t+1}) r_{t+1} \\ &+ \frac{p_{E,t+1}}{\Gamma'_{I,t+1}(j)} [1 - \delta + \Gamma_{I,t+1}(j) - \Gamma'_{I,t+1}(j) \frac{I_{t+1}(j)}{K_{t+1}(j)}]) \} \end{aligned} \quad (93)$$

Expression (93) links capital accumulation to the behavior of the after-tax price of capital $(1 - \tau_K) r$. In the steady state $1 + (1 - \tau_K) r / p_E$ is equal to the sum of the natural real rate of interest g^σ / β and the rate of capital depreciation δ .

Finally, the first order condition with respect to forward-looking agents' real wage $w_{FL}(j)$

²⁵International differences in natural rates of interest can arise from asymmetric rates of time preference across regions. They are accounted for in the definition of the risk premium Γ_B in equation (80).

determines the wage dynamics specific to the forward-looking households since $w_{FL}(j)$ is not the same as the average wage $w(j)$ of the economy:

$$\begin{aligned}
-\psi_{FL,t} \frac{u_{\ell,t}(j)}{u_{C,t}(j)} \frac{1}{w_{FL,t}(j)} &= (\psi_{FL,t} - 1) [1 - \Gamma_{WFL,t}(j)] (1 - \tau_{L,t}) + \frac{\partial \Gamma_{WFL,t}(j)}{\partial w_{FL,t}(j)} w_{FL,t}(j) (1 - \tau_{L,t}) \\
+ \mathbf{E}_t D_{t,t+1} \pi_{t,t+1} g_{t,t+1} &\frac{(w_{FL,t+1}(j)/w_{t+1})^{-\psi_{L,t+1}}}{(w_{FL,t}(j)/w_t)^{-\psi_{L,t}}} \frac{w_{FL,t+1}(j)}{w_{FL,t}(j)} \frac{\ell_{FL,t+1}}{\ell_{FL,t}} \frac{\partial \Gamma_{WFL,t+1}(j)}{\partial w_t(j)} w_{FL,t}(j) (1 - \tau_{L,t+1})
\end{aligned} \tag{94}$$

In a steady state where nominal rigidities have fully resolved, the real wage $w_{FL}(j)$ is equal to the marginal rate of substitution $MRS_{FL}(j)$ between consumption and leisure, augmented by the markup (which reflects the monopoly power of forward-looking agents in the labour market):

$$w_{FL,t}(j) = \frac{\psi_{FL,t}}{(\psi_{FL,t} - 1)} \frac{-u_{\ell FL,t}(j)}{u_{CFL,t}(j)} \equiv \frac{\psi_{FL,t}}{(\psi_{FL,t} - 1)} MRS_{FL,t}(j) \tag{95}$$

where an agent j 's marginal rate of substitution between consumption and leisure $MRS_{FL,t}(j)$ is solely a function of the labour supplied by forward-looking agents, and is invariant to the outcome of the consumption-savings decision - there is no trade-off in their wage decision between consumption and leisure. This is a property of the GHH utility function (Greenwood, Hercowitz and Huffman. (1988)) that implies labour effort will exhibit strong procyclical movements.

2.5.4 Consumer optimization for liquidity-constrained households

Liquidity-constrained households have no access to capital markets. They can optimize their labour supply solely based on their labour income. Similar to forward-looking households, they can optimally set their wages to exploit their market power. The maximization problem of agent $j \in ((1 - s_{LC}) ss, ss]$ can be written as follows:

$$\begin{aligned}
\max_{C_t(j), w_t(j)} &u_t (C_{LC,t}(j), \ell_{LC,t}(j)) + \mu_t(j) (-C_{LC,t}(j) - TT_t(j) \\
&+ (1 - \tau_{L,t}) w_{LC,t}(j)^{1-\psi_{L,t}} w_t^{\psi_{L,t}} \ell_{LC,t} (1 - \Gamma_{WLC,t}[w_{LC,t}(j), w_{LC,t-1}(j)]))
\end{aligned} \tag{96}$$

Their resulting level of consumption is:

$$C_{LC,t}(j) = (1 - \tau_{L,t}) w_{LC,t}(j) \ell_{LC,t}(j) \tag{97}$$

The first order conditions with respect to liquidity-constrained agents consumption $C_{LC}(j)$ and their real wage $w_{LC}(j)$ determines the partial adjustment of wages:

$$-\psi_{LC,t} \frac{u_{\ell LC,t}(j)}{u_{C LC,t}(j)} \frac{1}{w_{LC,t}(j)} = (1 - \tau_{L,t}) [(\psi_{LC,t} - 1) (1 - \Gamma_{WLC,t}(j)) + \frac{\partial \Gamma_{WLC,t}(j)}{\partial w_{LC,t}(j)} w_{LC,t}(j)] \quad (98)$$

and their real wage $w_{LC}(j)$ is a function of a unique marginal rate of substitution $MRS_{LC}(j)$ between consumption and leisure, augmented by a markup, in a manner similar to the forward-looking agents.

2.5.5 Aggregating across categories of consumers

In order to talk about economy-wide measures of labour and wages, we need to aggregate across categories of consumers, of which we have two - liquidity-constrained and forward-looking.

So equation (99) determines the aggregate wage rate w :

$$w_t^{1-\psi_{L,t}} = s_{LC} w_{LC,t}^{1-\psi_{L,t}} + (1 - s_{LC}) w_{FL,t}^{1-\psi_{L,t}} \quad (99)$$

where ψ_L is the degree of substitutability between lower-skilled and higher-skilled workers.

The aggregate labour supply ℓ function is implied by the labour supply of forward-looking agents ℓ_{FL} and liquidity-constrained agents ℓ_{LC} :

$$\ell_t^{1-\frac{1}{\psi_{L,t}}} = (1 - s_{LC})^{\frac{1}{\psi_{L,t}}} \ell_{FL,t}^{1-\frac{1}{\psi_{L,t}}} + s_{LC}^{\frac{1}{\psi_{L,t}}} \ell_{LC,t}^{1-\frac{1}{\psi_{L,t}}} \quad (100)$$

2.6 Government

The government is the third type of agent in the model, and it fulfills two roles, as the fiscal agent that collects and distributes tax revenues, and as the monetary authority that provides (at a minimum) a nominal anchor for the domestic economy.

2.6.1 Fiscal agent

Government expenditures, while part of GDP, do not form part of the productive capital stock, nor do they enter the consumers' utility functions. Public spending falls into three categories. There is government consumption G_C , government investment G_I , and G_N

denotes public purchases of intermediate nontradables. In the data, G_C can be considered as the purchase of goods, G_I is government investment in fixed capital and G_N is government spending on wages and services. There are seven sources of (net) tax revenue: a distortionary tax on capital income τ_K , a distortionary tax on labour income τ_L , lump-sum taxes TT net of transfers to households, tariffs τ_{TRF} imposed on imported goods by region H on region R , royalties τ_{ROYAL} from the extraction of crude oil reserves, a distortionary *ad-valorem* tax on oil τ_{OIL} used in the production of tradable and nontradable goods, and a distortionary *ad-valorem* tax on gasoline τ_{GAS} .²⁶ The government finances the excess of public expenditure over net taxes by issuing debt denominated in nominal currency, denoted B in per-capita terms. All national debt is held exclusively by domestic (forward-looking) agents. The budget constraint of the government is:

$$B_t \geq (1 + i_{t-1}) \frac{B_{t-1}}{\pi_{t-1,t} g_{t-1,t}} + G_t - G_{REV,t} \quad (101)$$

where:

$$G_t = G_{C,t} + p_{E,t} G_{I,t} + p_{N,t} G_{N,t} \quad (102)$$

and government revenues are defined as:

$$G_{REV,t} = \frac{1}{ss} \left(\int_0^{ss} TT_t(j) dj + \tau_{K,t} r_t \int_0^{ss(1-s_{LC})} K_t(j) dj + \tau_{L,t} \int_0^{ss} w_t(j) \ell_t(j) dj \right) + G_{TRF,t} + G_{OIL,t} + G_{ROYAL,t} + G_{GAS,t} \quad (103)$$

which have the following component revenues:

$$G_{TRF,t}^H = \sum_{R \neq H} \frac{1}{ss^H} \int_0^{ss^H} \tau_{TRF,t}^{H,R} p_{M,t}^{H,R} [M_t^{H,R}(x^H) + M_t^{H,R}(e^H)] \quad (104)$$

$$G_{OIL,t} = \frac{1}{ss} \int_0^{ss} \tau_{OIL,t} [p_{OT,t} O_t(h) + p_{ON,t} O_t(n)] \quad (105)$$

$$G_{ROYAL,t} = \frac{1}{ss} \int_0^{ss} \tau_{ROYAL,t} p_{OIL,t} OIL_t(s) \quad (106)$$

$$G_{GAS,t} = \frac{1}{ss} \int_0^{ss} \tau_{GAS,t} p_{GAS,t} GAS_t(g) \quad (107)$$

²⁶ An additional tax not yet in the model is a value-added tax (VAT) such as the Goods and Services Tax (GST) in Canada. Such a tax rate will be introduced in the near future.

We can define the average tax rate for the economy τ as:

$$\tau_t \equiv G_{REV,t}/GDP_t \quad (108)$$

The deficit-to-GDP ratio is:

$$\frac{DEF_t}{GDP_t} = \left(B_t - \frac{B_{t-1}}{\pi_{t-1,t}g_{t-1,t}} \right) / GDP_t \quad (109)$$

From (101), in steady state we obtain:

$$\frac{B}{GDP} = \frac{\pi g}{\pi g - (1 + i)} \left(\frac{G}{GDP} - \tau \right) = \frac{\pi g}{\pi g - 1} \frac{DEF}{GDP} \quad (110)$$

The prior three equations define the relations between debt-to-GDP, average tax rate, and deficit-to-GDP ratio that are sustainable in the long term. In what follows we treat the long-run debt-to-GDP ratio as a policy parameter set by the government, and let τ and DEF/GDP be determined by (110).

It is assumed that the government controls all of the component tax rates directly, with the exception of the labour tax τ_L . The labour tax is the residual tax rate that allows the aggregate tax rate τ to respect a fiscal rule based on the stable long-run debt-to-GDP ratio in equation (110). The fiscal rule for τ is specified as:

$$\begin{aligned} \tau_t = & (\tau_{t-1} + \tau_t + \mathbf{E}_t \tau_{t+1})/3 + \phi_{\tau 1} \left(\frac{B_t}{GDP_t} - \phi_{\tau 2} B_{TAR,t} - (1 - \phi_{\tau 2}) \frac{B_{t-1}}{GDP_{t-1}} \right) \\ & + \phi_{\tau 3} \left(\frac{DEF_t}{GDP_t} - \frac{DEF}{GDP} \right) \end{aligned} \quad (111)$$

where B_{TAR} is the debt-to-GDP target, which is specified as a stochastic process. Therefore the aggregate tax rate is a smoothed function of past and expected future tax rates, adjusted upward when the current debt-to-GDP ratio ($\phi_{\tau 1}$) is different from the average of its current target and its past observed level ($\phi_{\tau 2}$) or when the current deficit-to-GDP ratio differs from its sustainable steady-state level ($\phi_{\tau 3}$). Having $\phi_{\tau 3}$ greater than zero smooths the short-run development of the deficit-to-GDP ratio in face of any movement in the debt-to-income ratio.

2.6.2 Monetary authority

The government in its role as the monetary authority is assumed to define an objective for its monetary policy, and it controls the short-term nominal interest rate i_t as its instrument.

The monetary authority can then specify some target to hit using its instrument. In the BoC-GEM, monetary policy is specified usually as an inflation-forecast-based rule. That is, we use an annualized interest rate rule of the form:²⁷

$$(1 + i_t)^4 = (1 + i_{t-1})^{4\omega_i} (1 + i_t^{neut})^{4(1-\omega_i)} \mathbf{E}_t (\pi_{t-1,t+3}^X - \Pi_{t-1,t+3})^{\omega_1} (GDP_t/GDP_{POT,t})^{\omega_2} \quad (112)$$

where the current interest rate i_t is a function of the average of the lagged rate i_{t-1} and the current ‘neutral’ interest rate i_t^{neut} ²⁸ as well as

- a weight of ω_1 on $\mathbf{E}_t (\pi_{t-1,t+3}^X - \Pi_{t-1,t+3})$, the expected year-on-year core inflation gap three quarters in the future, to return the economy to its target inflation rate by looking at the core year-on-year rate of inflation $\pi_{t-4,t}^X$;
- a weight of ω_2 on $GDP_t/GDP_{POT,t}$, the output gap (output (GDP) divided by potential output(GDP_{POT})), to return the economy to its potential level of output in the steady state. GDP can be defined as either the model definition, or a definition consistent with measured GDP.²⁹ Potential output GDP_{POT} is the rate of output that prevails under the current capital stock and the steady-state level of labour inputs and technological process.³⁰

Note that the inflation gap mentioned here is not total CPI inflation (derived from CPI in equation (57))but core inflation (derived from CPI^X in equation(58)) which excludes the effects of fuel prices (i.e. gasoline) and indirect taxes (in this version of the BoC-GEM, only tariffs).

The rule in equation (112) can be modified to include policy responses to a set of other variables (such as the nominal exchange rate) expressed as deviations from their targets. For this application of the model, such a modification is indeed put into practice. In the

²⁷Recall that $\Pi_{t-\tau,t-\tau+4}$ is the year-on-year gross CPI inflation target at time t for the four-quarter period between $t - \tau$ and $t - \tau + 4$.

²⁸The ‘neutral’ rate is the interest rate at which the targeted variables are stabilized at their target values, meaning the real interest rate is a constant related to the growth rate of the economy and the rate of time preference (see equation (92))

²⁹For the measured concept of GDP (as referred to as "National Accounts real GDP") see Appendix B. The measured concept of GDP is our preference for the sake of realism, since data are not measured accurately with continually shifting relative prices across time, as in the model.

³⁰This is not to be confused with any concept of the flexible-price output gap (see Neiss and Nelson (2003) or Woodford (2004) for two differing examples). Instead, this is an extension of the older idea of potential output used at the Bank of Canada, as expressed in Butler (1996) and still used in the Bank of Canada’s Monetary Policy Report.

case of Emerging Asia (AS), we model an exchange rate targeting regime by introducing the component $\omega_3^{AS}\Delta_t^{AS}$ in (112), where Δ^{AS} is defined by equation (91). We choose a very high value of ω_3^{AS} so that the AS currency moves in tandem with the U.S. dollar - the bilateral nominal exchange rate is 'pegged'. At any rate, monetary policy only has effects in the short to medium term, and after all the targets have been achieved the monetary policy reaction function reduces to the neutral rate of interest.

2.7 Market clearing

The model is closed by imposing the following resource constraints and market clearing conditions.

For each region H , the domestic resource constraints for capital and labour are, respectively:

$$\begin{aligned} \int_0^{ss^H(1-s_{LC}^H)} K_t^H(j^H) dj^H &\geq \int_0^{ss^H} K_t^H(n^H) dn^H + \int_0^{ss^H} K_t^H(h^H) dh^H \\ + \int_0^{ss^H} K_t^H(s^H) ds^H &+ \int_0^{ss^H} K_t^H(o^H) do^H + \int_0^{ss^H} K_t^H(g^H) dg^H \end{aligned} \quad (113)$$

and:

$$\begin{aligned} \ell_t^H(j^H) &\geq \int_0^{ss^H} \ell_t^H(n^H, j^H) dn^H + \int_0^{ss^H} \ell_t^H(h^H, j^H) dh^H \\ + \int_0^{ss^H} \ell_t^H(s^H, j^H) ds^H &+ \int_0^{ss^H} \ell_t^H(o^H, j^H) do^H + \int_0^{ss^H} \ell_t^H(g^H, j^H) dg^H \end{aligned} \quad (114)$$

where these constraints imply that labour and capital can move freely across sectors, subject to any short-run real adjustment costs they face, such as in the production of commodities or oil.³¹

The resource constraint for the nontradable good n^H is:

$$N_t^H(n^H) \geq \int_0^{ss^H} N_{A,t}^H(n^H, x^H) dx^H + \int_0^{ss^H} N_{E,t}^H(n^H, e^H) de^H + G_{N,t}^H(n^H) \quad (115)$$

³¹See equation (2) for the case of commodities.

while the tradable h^H can be used by domestic firms or imported by foreign firms:

$$\begin{aligned}
T_t(h^H) &\geq \int_0^{ss^H} Q_{A,t}(h^H, x^H) dx^H + \int_0^{ss^H} Q_{E,t}(h^H, e^H) de^H \\
&+ \sum_{R \neq H} \left(\int_0^{ss^R} M_{A,t}^{R,H}(h^H, x^R) dx^R + \int_0^{ss^R} M_{E,t}^{R,H}(h^H, e^R) de^R \right) \quad (116)
\end{aligned}$$

The same follows for the commodities and oil goods (s^H and o^H respectively). For example, for commodities:

$$\begin{aligned}
S_t(s^H) &\geq \int_0^{ss^H} Q_{SN,t}(s^H, n^H) dn^H + \int_0^{ss^H} Q_{ST,t}(s^H, h^H) dh^H \\
&+ \sum_{R \neq H} \left(\int_0^{ss^R} M_{SN,t}^{R,H}(s^H, n^R) dn^R + \int_0^{ss^R} M_{ST,t}^{R,H}(s^H, h^R) dh^R \right) \quad (117)
\end{aligned}$$

The final good A can be used for private (by both liquidity-constrained and forward-looking households) or public consumption:

$$\int_0^{ss^H} A_t^H(x^H) dx^H \geq \int_0^{ss^H(1-s_{LC}^H)} C_{FL,t}^H(j^H) dj^H + \int_{ss^H(1-s_{LC}^H)}^{ss^H} C_{LC,t}^H(j^H) dj^H + ss^H G_{C,t}^H \quad (118)$$

and similarly for the investment good E :

$$\int_0^{ss^H} E_t^H(e^H) de^H \geq \int_0^{(1-s_{LC}^H)ss^H} I_t^H(j^H) dj^H + ss^H G_{I,t}^H \quad (119)$$

All profits, adjustment costs and intermediation revenues are assumed to accrue to forward-looking households as $\Phi_t(j)$.

Market clearing in the asset market requires:

$$\int_0^{ss^H(1-s_{LC}^H)} B_t^H(j^H) dj^H = ss^H B_t^H \quad (120)$$

for the each of the five regions' government bond markets, and:

$$\sum_R \int_0^{ss^R(1-s_{LC}^R)} B_t^{*R}(j^R) dj^R = 0. \quad (121)$$

for the international bond market (net foreign assets). Finally, aggregating the budget constraints across private and public agents after imposing the appropriate transversality conditions we obtain the law of motion for financial wealth, where the present value of next period's financial wealth is equal to the sum of this period's financial wealth, plus the financial transactions costs incurred by the holdings of U.S.-dollar-denominated international bonds, plus the domestic production of the economy, plus the net export positions of commodities ($TBAL_S$) and oil ($TBAL_O$) less this period's consumption, investment and government expenditures:

$$\begin{aligned}
\mathbf{E}_t D_{t,t+1}^H \pi_{t,t+1}^H g_{t,t+1} F_{t+1}^H &= F_t^H + \Gamma_{B,t-1}^H \frac{(1 + i_{t-1}^*) \varepsilon_t^{H,US} B_{t-1}^{*H}}{\pi_{t-1,t}^{US} g_{t-1,t}} \\
&+ p_{N,t}^H N_t^H + p_{T,t}^H T_t^H + (1 + \tau_{GAS,t}^H) p_{GAS,t}^H GAS_t^H \\
&+ TBAL_{S,t}^H + TBAL_{O,t}^H - C_t^H - p_{E,t}^H I_t^H - G_t^H \quad (122)
\end{aligned}$$

2.8 Definition of the Gross Domestic Product

The Gross Domestic Product (in consumption units - i.e. deflated by the Consumer Price Index) can be stated as either the sum of all goods consumed domestically once the net flows of final goods (EX and IM) are accounted for, or all goods produced domestically once net cross-border flows of all intermediate goods ($TBAL_S$ and $TBAL_O$) are accounted for:

$$\begin{aligned}
GDP_t^H &= A_t^H + p_{E,t}^H E_t^H + p_{N,t}^H G_{N,t}^H + EX_t^H - IM_t^H \\
&= p_{N,t}^H N_t^H + p_{T,t}^H T_t^H + (1 + \tau_{GAS,t}^H) p_{GAS,t}^H GAS_t^H + TBAL_{S,t}^H + TBAL_{O,t}^H \quad (123)
\end{aligned}$$

where total exports EX are:

$$EX_t^H = p_{T,t}^H T_t^H + p_{S,t}^H S_t^H + p_{O,t}^H O_t^H - p_{Q,t}^H (Q_{A,t}^H + Q_{E,t}^H) - p_{QS,t}^H Q_{S,t}^H - p_{QO,t}^H Q_{O,t}^H \quad (124)$$

total imports IM are:

$$IM_t^H = \sum_{R \neq H} p_{M,t}^{H,R} (M_{A,t}^{H,R} + M_{E,t}^{H,R}) + p_{MS,t}^{H,R} M_{S,t}^{H,R} + p_{MO,t}^{H,R} M_{O,t}^{H,R} \quad (125)$$

the trade balance in commodities $TBAL_S$ is:

$$TBAL_{S,t}^H = p_{S,t}^H S_t^H - p_{QS,t}^H Q_{S,t}^H - \sum_{R \neq H} p_{MS,t}^{H,R} M_{S,t}^{H,R} \quad (126)$$

and the trade balance in oil $TBAL_O$ is similarly stated.

Finally, Appendix B deals with the conventional measures of volumes typically found in the National Accounts and issues related to that topic.

3. Calibration of the model

Given the large and complex nature of the BoC-GEM it is readily apparent that at this point in time, full estimation of the parameters of the model is not possible. Therefore, the model must be calibrated and we have relied on previous work on the GEM (such as Laxton and Pesenti (2003), Bayoumi, Laxton and Pesenti (2004), Faruqee et al. (2007a,b)) to guide our calibration work. Work done in tandem with this technical report has also provided useful insights for the calibration - see Coletti, Lalonde and Muir (2007) and Elekdag et al. (2007). We also rely on previously published work for particular countries, namely for Canada (Murchison and Rennison (2006) - the reference work for the Bank of Canada projection and policy analysis model for Canada, ToTEM; Perrier (2005)), the euro Area (Coenen, McAdam and Straub (2007) - refers to the NAWM (New Area-Wide Model), a DSGE model; de Walque, Smets and Wouters (2005) and Smets and Wouters (2005) for a Bayesian-estimated DSGE model of both the U.S. and the European economies) and the United States (Brayton et al. (1997) - a published reference for the Federal Reserve's Board of Governor's PAC model of the United States, FRB/US;³² Erceg, Guerrieri and Gust (2005a, 2005b) - the references for the SIGMA DSGE model used at the Federal Reserve's Board of Governors; Gosselin and Lalonde (2005) - the source for the Bank of Canada's PAC model of the U.S. economy, MUSE; Juillard et al. (2006) - a small Bayesian-estimated DSGE model of the U.S. economy). Other references will be made throughout the remainder of this section on calibration.

This section is divided into 7 parts. Sections 3.1 and 3.2 present the steady-state parameters and ratios. Section 3.3 looks at the calibration of the international linkages of the model (both the steady state and their dynamics). Sections 3.4 and 3.5 deal with the steady-state and dynamic calibrations of the oil and commodities sectors respectively. Section 3.6 presents the real adjustment costs and the nominal rigidities in the dynamic model of the BoC-GEM. Finally, Section 3.7 presents the calibration of the fiscal and monetary policy rules.

³²"PAC" stands for "polynomial adjustment cost". See the seminal work by Tinsley (1993) for more on the methodology.

3.1 Key steady-state parameters

In general, the key parameters of the steady state are the same across the five regions of the world (see Table 2). At steady state, in all the regions of the world, the growth rate of GDP per capita is equal to 1.9 per cent per annum and the real interest rate is equal to 3 per cent. Other key parameters are associated with the consumers' utility function and their utility maximization problem of which there are two - one for forward-looking agents (equation (86) above) and one for liquidity-constrained consumers (equation (96)). The intertemporal elasticity of substitution in consumption is set to 0.7 for all the regions. The chosen value is in the mid-range of the values used in the DSGE literature (between 0.5 and 1.0). This parameter drives the amplitude of the effect of the interest rate on consumption. This value of the elasticity of intertemporal substitution combined with habit persistence in consumption equal to 0.85 generates the expected gradual humped-shape response of consumption to shocks. The parameter for habit persistence in consumption is relatively high compared to the literature. However, Juillard et al. (2006) estimate a DSGE model of the United States similar in structure to the GEM and obtain a result close to our calibration (i.e. 0.83 vs. 0.85) using a similar intertemporal elasticity of substitution in consumption (0.8 vs. 0.7). With the exception of emerging Asia, we set the share of liquidity-constrained consumers to 20 per cent. This value combined with the existing link between the government debt and the net foreign assets³³ allows the model to generate a reasonable impact of debt shock on output. Because of the limited access to credit in newly industrializing economies, we set the share of liquidity-constrained consumers in emerging Asia to 50 per cent, as in Faruquee et al. (2007a). Like Erceg, Guerrieri and Gust (2005b), we set the Frisch elasticity of labour supply to 0.2 for all regions, which is consistent with results obtained in microeconomic studies (between 0.05 and 0.35). The labour habit persistence parameter is 0.75 - a value almost identical to the estimation result of Juillard et al. (2006). Like Erceg, Guerrieri and Gust (2005b) and Murchison and Rennison (2006), we set the elasticity of substitution between domestically produced tradable goods and imported tradable goods at 1.5 which is lower than the values assumed in previous published work using the GEM (i.e. 2.5).

All the production functions of the model use a constant-elasticity-of-substitution (CES) technology (see Table 4). For all the regions, the elasticity of substitution between factors of production of the tradable, nontradable and gasoline sectors is 0.7. This value is below one, which would correspond to a Cobb-Douglas production function.³⁴ For the sectors which

³³The link between the government debt and the net foreign assets is modelised via a simple rule which is calibrated according to results obtained with a overlapping generation version of GEM called GFEM.

³⁴Perrier (2005) shows that the Cobb-Douglas assumption is rejected by the data for Canada.

include a fixed factor of production (oil and commodities) we assume even less substitution between the remaining factors of production, setting the elasticity of substitution to 0.6. In the short and medium terms the effective elasticities of substitution in the oil, gasoline and commodities sectors are even lower because of short-term real adjustment costs in the factors of production. The calibration of these adjustment costs is addressed below in Section 3.5. For all the regions, we assume that the tradable sector is more capital intensive than the nontradable sector. With the exception of emerging Asia, the calibration implies a capital-to-output ratio between 1.8 and 1.9 (emerging Asia: 2.3).

The calibration of the price mark-up over marginal cost in the tradable and nontradable sectors is based on the estimates obtained by Martins, Scarpetta and Pilat (1996) for prices, and by Jean and Nicoletti (2002) for wages.³⁵ According to the calibration presented in Table 8, the level of competitiveness is higher in the United States and in emerging Asia than in the three other regions.

3.2 Composition of aggregate demand

Table 6 outlines the calibration of the composition of aggregate demand for the five regions. At steady state, the United States and Canada are the only regions that have negative net-foreign-asset-to-GDP positions. This ratio is set to -50 per cent of GDP for the United States and -7.5 per cent of GDP for Canada. Because of their negative net foreign asset positions, the United States and Canada must generate a small trade surplus in the long run. On the other hand, at the steady state, AS, RC and CX must exhibit a small trade deficit to maintain their positive net-foreign-asset-to-GDP ratios (derived from the data underlying Lane and Milesi-Ferretti (2006)).

In the United States, the investment-to-GDP and the government-expenditure-to-GDP ratios are set to 16.0 per cent and 17.2 per cent respectively which yields a consumption-to-GDP ratio of 66.3 per cent of GDP. These ratios are very similar to those found in Juillard et al. (2006). They are also compatible with Smets and Wouters (2005). In Canada, the investment-to-GDP ratio is set to 16.6 per cent and the ratio of government expenditures relative to GDP is 26.0 per cent. Therefore, the consumption-to-GDP ratio is 57.3 per cent of GDP - much lower than in the United States.

For the other regions, the calibration of the decomposition of aggregate demand follows

³⁵Even though the study done by Martins et al. (1996) is over ten years old it is the only study that provides a consistent calibration of the mark-ups across most of the regions of the world.

Faruqee et al. (2007a). It is interesting to note that emerging Asia has the largest investment share (29.9 per cent), while RC has the next-highest government-expenditure-to-GDP ratio (23.2 per cent) because it includes the European Union.

3.3 International linkages

Figures 4 to 6 illustrate the calibration of all the bilateral trade flows between the regions in all types of tradable goods (i.e. consumption, investment, oil and commodities), the oil sector, and the commodities sector. The calibration of this trade matrix is central to the properties of the model, especially for the steady-state movements and dynamics of the different bilateral exchange rates, as well as the spillover effects of any shock in one region to another. The calibration of the trade links is based on the current trends in trading patterns observed in the COMTRADE database, maintained by the United Nations. As expected, 82 per cent of Canadian exports are sent to the United States. For the United States, its three largest trading partners are Canada, emerging Asia (including China) and the commodity exporter bloc (which includes Mexico). Emerging Asia exports worldwide, but most notably to the United States and RC (where Japan depends heavily on their commodities exports). Also note the patterns for the aggregate trade volumes - Canada, a small open economy, has the highest trade volumes, with an export-to-GDP ratio of 37 per cent, while RC (which includes the largely self-contained trading bloc of the European Union) is the smallest, with an export-to-GDP ratio of 9 per cent.

The adjustment in the trade sector of the economies is largely governed by the speed of adjustment of the net foreign asset to GDP ratio to its desired ratio. This is the determinant for the financial intermediation costs (equations (80)) that form part of the modified risk-adjusted UIRP condition (equation (90)). While we do not have any reliable econometric estimates of the speed of adjustment of the NFA-to-GDP ratio, we set the two necessary parameters such that the net-foreign-assets-to-GDP ratio converges to its desired level (i.e. the NFA gap) within 15 to 20 years after a shock to the desired level. This speed of adjustment is a compromise. Faster convergence of the NFA gap implies the bilateral U.S. dollar exchange rate deviates too strongly from the standard uncovered interest rate parity condition even in the short run. However, a speed of adjustment that is too low eliminates, in practice, the stock-flow dynamics between the current account and the net foreign asset position, and creates extremely long-lived gaps throughout each economy. This would generate a disequilibrium in the current account for an implausibly long period of time. Given the current context of global imbalances, this result would be an undesirable property

for the model.

3.4 The oil sector

3.4.1 *The supply of oil*

The CES production function of the oil sector depends on capital, labour and crude oil reserves. Therefore, real marginal cost in the oil sector depends on the real wage, the real rental price of capital and the real price of crude oil reserves. Figure 7 illustrates how the market for crude oil reserves behaves in terms of demand and supply curves. The supply curve is vertical because the reserves are assumed to be a fixed factor of production (relative to GDP). All else being equal, and for a given demand curve, a region with a larger endowment of reserves, like the commodity-exporting region (CX), will have a lower real price of crude oil reserves and a lower marginal cost than a region with fewer reserves, such as the United States. This is even more important if the share of crude oil reserves in the production of oil is large relative to capital and labour. Table 3 presents the distribution of crude oil reserves across the world based on data published by the U.S. Department of Energy. Overall, the calibration is consistent with the data. In order to better align the price of oil across the regions we make some minor adjustments to slightly reduce the dispersion of marginal costs across the different regions. Nevertheless, the commodity exporter (CX) holds 75 per cent of the world oil reserves.

Because the reserves of crude oil are modelled as a fixed factor of production in equation (25), the more intensively a region's production of oil uses its reserves of crude oil, the more vertical the supply curve (as in Figure 7). In reality, the crude oil reserves are partly endogenous and are a function of the price of oil. The Athabasca tar sands in western Canada are a good example. As oil prices move higher, more of the tar sands fields can be profitable in the long run, therefore the producers increase the level of active crude oil reserves for Canada. Higher oil prices also induce more exploration, with the (usual) consequence that new reserves of crude oil will be found and become active.³⁶

Regions where oil production depends more on the amount of capital (such as Canada, which depends on tar sands and offshore oil fields) have a supply curve that is less vertical because capital is an endogenous and variable factor of production. However, the level of

³⁶Recall that the level of crude oil reserves, the fixed factor *OIL*, is exogenous to the model. However, since it is formulated as a stochastic process, an exogenously-specified permanent shock can approximate this behaviour when and where it is necessary.

capital used to exploit the tar sands and offshore fields is very slow to adjust because of the complexity of the capital required to extract oil from those two types of oil fields. Therefore, we introduce strong real adjustment costs in the short run that greatly reduce the effective elasticity of substitution between factors of production in the oil sector during the first five years. Section 3.5 presents the calibration of these real adjustment costs.

The calibration of the relative shares of capital and crude oil reserves in the oil sector's production function (equation(25)) greatly affects the slope of the supply curve of oil. The bias parameters of the oil production function of each of the regions are calibrated according to the following hypotheses:

- The OPEC production process is simpler and more reserve-intensive than offshore oil fields and tar sands. For instance Norway, the Gulf of Mexico (28 per cent of U.S. production), and the Gulf of St. Lawrence offshore oil fields and the Athabasca tar sand fields (both in Canada) are much more capital intensive than the oil-well fields of the Arabian Peninsula. The Norges Bank assumes that reserves, capital and labour represent respectively 50 per cent, 40 per cent and 10 per cent of the costs of production for the Norwegian state oil producer, Statoil. At the opposite end of the spectrum, Hunt (2005) assumes that reserves represent 96 per cent of OPEC costs of production (essentially calibrated on Saudi Arabia).
- Roughly half of Canadian oil production relies on the Athabasca tar sands. Therefore we made the assumption that Canada has the most capital-intensive oil production process of the five regions.
- For every region, oil production is more reserve intensive than it is intensive in variable factors of production (i.e. capital and labour).

Table 4 shows the calibration of the bias towards each of the factors of production of the oil sector for the five regions. The bias parameters are calibrated on the assumption that for the tar sands and offshore fields the capital share is between 40 per cent and 50 per cent. For the oil well fields, capital represents only between 5 per cent and 10 per cent of the production cost. We also assume that the labour share is small for all the regions.

Canada is the region where the marginal cost depends the most on the rental price of capital, whereas for CX, its marginal cost depends almost exclusively on the price of crude oil reserves. Because CX has 75 per cent of world crude oil reserves and its production is not as

capital intensive, it has the lowest marginal cost. This is consistent with anecdotal evidence that in Saudi Arabia the marginal cost of producing a barrel of oil is between US\$2 and US\$3. In order to account for the wedge between marginal cost in CX and the observed price of oil, we make the assumption that firms in CX have strong market power. This assumption allows us to have an oil price that is similar across regions in spite of the fact that CX has a marginal cost that is much lower than the other regions. Furthermore, in order to generate an oil price that moves uniformly across all the regions, we assume a very high elasticity of substitution of 10 between the demand for domestically-produced and imported oil in all regions.

3.4.2 The demand for oil and gasoline

The demand for oil comes from three sources: the refineries that produce gasoline, the firms that produce nontradable goods and the firms that produce tradable goods. These firms can buy oil on the domestic market or import it from abroad. The trade flows of oil are calibrated based on the COMTRADE U.N. database in 2003. Figure 5 shows that Canada and the commodity-exporting region CX are the only net exporters of oil at 3.6 per cent and 8.0 per cent of GDP respectively.

In order to calibrate the demand for oil for the refineries producing gasoline, we need to know the demand generated by consumers. Recall that the consumption basket is composed of gasoline and tradable and nontradable goods. Since consumption is represented by a nested CES aggregator we need to set the values of the bias parameter towards gasoline in the consumption aggregator to fit the share of gasoline in consumption. To do so, we rely on the weight of fuel in the CPI index (see Table 9). For CX and AS, data are not readily available so we make the assumption that the gasoline share of consumption is lower than in the industrialized countries. So when we calibrate the aggregate consumption equation (50), we set the elasticity of substitution between tradable and nontradable goods to 0.5 as previous work with GEM (Faruquee (2007a)). We also assume a lower elasticity of substitution between gasoline and the rest of consumption (i.e. 0.3).

On the supply side of the gasoline sector (equation (34)), we assume a similar technology of production of gasoline around the world. Therefore, for the five regions, we set the oil, capital and labour shares of production to around 60 per cent, 25 per cent and 15 per cent respectively.

We also need to decide on the distribution of oil between the gasoline refineries and the

tradable and nontradable sectors. We find this by first calculating the ratio of oil and natural gas production in GDP for the five regions using data from the U.S. Department of Energy in 2003. These data combined with the net export positions gives the ratio of domestic oil demand to GDP. From this ratio we subtract the demand for oil by gasoline refineries, and then assume that the tradable sector is more oil intensive than the nontradable sector. In general we find that 60 per cent of oil demand is from the gasoline refineries (see Tables 6 and 7).

3.5 The commodities sector

The calibration of the commodities sector is analogous to that of the oil sector. However, data availability issues and the fact that this sector is actually a basket of many different commodities make the calibration less precise. We assume commodities includes the products of the agricultural, fisheries, forestry and mining sectors of the economy, as well as some fuel sources such as coal. The major source of data for the share of the commodities sector in total production is the COMTRADE U.N. database. This data combined with production data for some countries allows us to calibrate the bias parameters of commodities in the production of tradable and nontradable goods. Turning to the supply of commodities, we make the assumption that the share of land, capital and labour are respectively around 55 per cent, 25 per cent and 20 per cent for all the regions.

3.6 Rigidities and adjustment costs

3.6.1 Real adjustment costs in capital, investment and imports

As explained in Section 2.5.2, capital and investment are subject to quadratic adjustment costs. Like Faruqee et al. (2007a) and Juillard et al. (2006), for all the regions but RC, we assume that the adjustment costs related to a change in the level of capital are relatively small whereas those related to the change in the level of investment are large, around 100. The latter value is larger than the value of 78 estimated by Juillard et al. (2006) but in the context of our model it was necessary to assume greater adjustment costs to better match the properties of the ToTEM, MUSE and FRB/US models. Based on the estimation results obtained by de Walque, Smets and Wouters (2005), for the euro area we set the values of the investment adjustment cost in RC (which includes the European Union) at 160 instead of 100. For all the regions we set the real adjustment cost parameters associated to the share of imported consumption and investment goods to 0.95, as in Faruqee et al. (2007a,b).

3.6.2 Real adjustment costs in the oil, gasoline and commodities sectors

In the oil sector, we assume strong real adjustment costs related to changing the capital share and labour share of production. This is particularly the case for offshore oil fields and the Athabasca tar sands in Canada. We calibrate these real adjustment costs based on the following principles;

- In general, several years are necessary for a new oil-producing facility to be running at capacity. Therefore, during the initial years following a permanent increase in the demand for oil, oil production should barely increase (i.e. the short-run supply curve is almost vertical).
- For the regions that rely heavily on tar sands (Canada) and offshore oil fields (Norway, U.S. and China) we assume that the adjustment costs are greater.

For the gasoline sector we assume the same sort of strong real adjustment costs on capital and labour, while assuming that the technology for gasoline refineries is similar across regions. In the commodities sector we assume that the production process is somewhat more flexible than in the oil sector. This assumption allows the BoC-GEM to generate a smaller reaction of commodities prices to world demand fluctuations than for oil prices, as implied by the data (Lalonde, Zhu and Demers (2004)). Table 10 presents the calibration of the parameters that define the amplitude of the real adjustment costs in the oil, gasoline and commodities sectors.

Similar to the supply side of the oil market, we assume that it is costly for firms to adjust the share of oil and commodities used in the production of tradable and nontradable goods. These real adjustment costs dampen the reaction of the demand for oil and commodities during the first two years for commodities, following a permanent supply shock, decreasing the price elasticity of the short- and medium-run demand curves for oil and commodities. We assume that these real adjustment cost parameters are equal to 300 for oil demand and 200 for commodities demand.

3.6.3 Nominal rigidities

In the BoC-GEM we assume that it is costly to adjust the level and the first difference of inflation and the growth rate of nominal wages relative to their steady states. Juillard et

al. (2006) arrives at a value of 700 for the nominal rigidities' parameters.³⁷ Moreover, these values map well the properties of ToTEM (based on Calvo (1983) pricing), MUSE and FRB/US (both based on polynomial adjustment costs for pricing - see Tinsley (1993)). Therefore, for all the regions but RC, we set the nominal rigidities' parameters to 700 (see Table 11). In RC, which is represented mainly by the European Union and Japan, we assume somewhat larger nominal rigidities than elsewhere, an assumption consistent with the estimation results obtained by de Walque, Smets and Wouters (2005).

We also assume that the nominal rigidities are larger in the labour market than in the goods market (usually 800). Finally, to have plausible properties concerning the exchange rate pass-through in the short run, we need very strong nominal rigidities on import prices (around 4000).³⁸ Murchison and Rennison (2006) make a similar assumption.

3.7 Fiscal and monetary policy rules

Finally, in Table 12, we characterize the behaviour of the government, which acts as both the fiscal agent and the monetary authority.

As the fiscal agent, the governments of all five regions are assumed to target an explicit level of government debt, as found in Table 6 using the tax-targeting rule of equation (111), which determines the labour income tax rate. The smoothing parameter required to impart the degree of sluggishness found in ToTEM (for CA), MUSE (for US) or NAWM (for the euro area as part of RC) is 0.005. The weight in the government-debt-to-GDP ratio gap on the debt-to-GDP target (as opposed to the lagged debt-to-GDP ratio) is unity for all regions.³⁹ This parameterization insures that the tax rate will change at a speed that will guarantee that the observed debt-to-GDP ratio returns to its targeted level at a horizon that is similar to the estimated tax rate rule found in MUSE (Gosselin and Lalonde (2005)).

For monetary policy, we assume emerging Asia follows a fixed nominal exchange rate *vis-à-vis* U.S. dollar.⁴⁰ Therefore, the Asian monetary authority effectively imports its monetary

³⁷Each 100 increment in the nominal rigidity is roughly equivalent to a one-quarter-long contract in the Calvo-contracting framework (Pesenti (2007)). So a nominal rigidity of 700, for example, would result in a contract length of roughly seven quarters.

³⁸This allows the model dynamics to behave as if the law of one price does not hold in the short-run. In order to break the law of one price in the long-run, and to disentangle the purpose of nominal rigidities on import prices, we could introduce a distribution sector in future work, as in Corsetti and Pesenti (2001) and as found in the original two-country version of the GEM of Laxton and Pesenti (2003).

³⁹Future work on fiscal issues will probably lead a differentiation of these parameters across the regional blocs.

⁴⁰It is important to note that the model does not account for the sterilisation of capital flows that often

policy from the United States. The nominal interest rate in AS moves such that the modified risk-adjusted UIRP condition always maintains a constant interest rate differential between AS and US for a given nominal exchange rate.

The other four regions' monetary authorities target year-on-year core inflation, defined using the price index of equation (58). We can achieve this using several different parameterizations. One candidate parameterization is from estimated rules for the United States such as English, Nelson and Sack. (2003) or Erceg, Guerrieri and Gust (2005a) where the smoothing parameter is around 0.75 and the weight on the forward-looking inflation gap is around 0.65. Another option is to use the optimal rule found in ToTEM, which is more consistent with the optimal rule literature (Cayen, Corbett and Perrier. (2006)). In this case the smoothing parameter is set to a high value of 0.95 and the forward-looking inflation gap parameter is 0.90. The latter option gives more reasonable properties for structural and permanent shocks in the BoC-GEM, and is the basis for the model used in the simulations presented in sections 4 and 5.

4. Model properties for Canadian and U.S. domestic shocks

In this section we look at the basic model properties of the BoC-GEM for the two major regions of interest - Canada and the United States. For both regions we look at a temporary rise in consumption (a "demand shock"), and a temporary 100 basis point increase in interest rates (a "monetary policy shock").

4.1 Domestic Canadian shocks

We will first focus on the response of the Canadian economy to domestic shocks and compare the responses of the BoC-GEM with those of ToTEM (the Bank of Canada's new one-country small-open-economy DSGE model for forecasting and policy analysis).

4.1.1 A temporary shock to Canadian consumption

We trigger a temporary increase of Canadian consumption through a rise in the marginal utility of consumption (see Figure 8). The size of the shock is calibrated such that after two quarters, GDP will increase 0.5 per cent relative to control. This shock increases the demand

accompanies the maintenance of fixed (or strongly managed) exchange rate regimes in emerging Asia.

for gasoline, tradable goods and nontradable goods for roughly two years. It is interesting to note that part of the increase of consumption is financed from abroad, so imports rise by one per cent relative to control. Canadian firms meet the increase in demand by increasing their demand for labour, capital, oil and commodities. This increases the real wage, the rental price of capital, the price of oil and the price of commodities. Therefore, we see a rise in real marginal cost and consequently inflation. The peak response of year-on-year core inflation is 0.07 percentage points, four quarters after the shock, which is two quarters after the peak response of GDP. The monetary policy authority raises the nominal interest rate by 16 basis points in response to the increased inflationary pressures which creates a 0.2 percentage point appreciation of the real effective exchange rate. The increase in the interest rate and the appreciation of the Canadian dollar help return the economy to its steady-state equilibrium. With the exception of the real exchange rate, the amplitude and the timing of all of these responses are very similar to responses in ToTEM. In ToTEM, the appreciation of the exchange rate is larger than in the BoC-GEM. This difference is due to the fact that in the BoC-GEM, financial intermediation costs play a larger role than in ToTEM where exchange rate dynamics are almost exclusively determined by a modified UIRP condition with a very small role for adjustment of the country-specific risk premium.⁴¹

4.1.2 A temporary shock to the Canadian interest rate

The purpose of this shock is to demonstrate the monetary policy transmission mechanism in the economy. The shock is a temporary increase of 100 basis points in the Canadian short-term interest rate. Inertia in monetary policy insures that interest rates stay above control for around two years. The shock increases the rental price of capital and therefore reduces investment. Forward-looking consumers increase their saving and reduce their consumption. The increase in the interest rate induces a 1.7 per cent appreciation of the real effective exchange rate which increases the price of Canadian goods abroad and decreases the price of foreign tradable goods in Canada, thereby reducing demand for Canadian goods abroad and increases in Canadian imports. Overall, GDP drops by 0.34 percentage points, reaching its trough after four quarters. The reduction in domestic demand induces firms to reduce their demand for the variable factors of production. The real wage falls (as does the real rental price of capital in the medium term), and, by extension, so does real marginal cost. Consequently, year-on-year core inflation decreases by 0.26 percentage points four to five quarters after the initial impact of the shock. The magnitudes and timing of the responses are similar to that of ToTEM.

⁴¹This is ToTEM's analogue of the financial intermediation costs found in the GEM.

4.2 Domestic U.S. shocks

Now we will look domestic at U.S. shocks and their impact on the world economy, with an emphasis on Canada. We also compare the responses of the U.S. economy in the BoC-GEM with those from MUSE, the Bank of Canada's forecasting model of the U.S. economy.

4.2.1 A temporary shock to U.S. consumption

The shock, occurring in the United States, is a rise in the marginal utility of consumption, causing a temporary increase in domestic consumption, shown in Figures 10 and 11. As in the Canadian case, the size of the shock is calibrated to create a peak response of around 0.5 per cent of GDP after two quarters. This shock increases the demand for gasoline, tradable and nontradable goods for roughly two years. Year-on-year core inflation increases by 0.16 percentage points (after four quarters). The U.S. monetary authority raises interest rates by 32 basis points which generates an appreciation of the real effective exchange rate of 0.37 percentage points. The amplitude and the timing of the responses are similar to that in MUSE.

Since the United States is a large economy and is a key player in world trade, any U.S. shock has spillover effects on the other four regions of the world. This occurs through four channels. We consider the role of these four channels in the Canadian economy in detail.

- U.S. demand for foreign consumption and investment goods increases, therefore Canadian exports increase (i.e. income effect).
- The depreciation of the Canadian real effective exchange rate, which reduces the price of Canadian goods and increase Canadian exports (i.e. price effect). It has the opposite effect on imports.
- The U.S. share of world demand for oil and commodities is large enough to increase the real prices of oil and of commodities by 2.0 per cent and 0.4 per cent respectively. Given that Canada is a net exporter of these goods, it implies a positive terms-of-trade shock and a positive wealth effect, thereby increasing Canadian consumption.
- The increase in domestic demand drives up real marginal cost, while the depreciation of the exchange rate increases real import prices. Therefore, year-on-year core inflation rate rises by 0.05 percentage points and the monetary authority increases the nominal interest rate by 12 basis points, helping to re-equilibrate the economy.

All these effects increase Canadian economic activity, with the exception of the response of U.S. monetary policy. The peak response of Canadian GDP is 0.17 percentage points after three quarters - one quarter after the peak response of U.S. GDP. The rise in Canadian GDP is almost entirely due to an increase of Canadian exports to the United States of roughly 0.5 per cent relative to control. Similar effects occur in RC, CX and AS, but the effects are much smaller than those in Canada. As seen in section 3.3, of the four regions, Canada depends the most on U.S. trade. Consequently, because of the modified risk-adjusted UIRP condition, the Canadian dollar is actually appreciating relative to the RC and CX currencies.

4.2.2 A temporary shock to the U.S. interest rate

In Figures 12 and 13 we have an exogenous 100 basis point increase in the U.S. interest rate, which then stays above control for seven quarters. U.S. GDP gradually drops by 0.40 percentage points after four quarters. The U.S. economy is slightly more interest-rate sensitive than the Canadian economy. This is mainly because the GDP share of government expenditures is larger in Canada and these expenditures are less interest-rate sensitive. The shock reduces the demand for domestic and imported goods (which falls by 0.8 percentage points). The increase in the interest rate causes an appreciation of the U.S. real effective exchange rate of 0.8 per cent after three quarters. Relative to the Canadian dollar the U.S. currency appreciates 1.5 per cent in real terms. U.S. year-on-year core inflation decreases by 0.33 percentage points. These responses are almost identical to those of MUSE.

As in the U.S. consumption shock, we see spill-over effects around the world. The four channels behave for Canada in the following way:

- the drop of U.S. demand for foreign consumption and investment goods reduces Canadian exports (i.e. income effect).
- the depreciation of the Canadian dollar reduces the price of Canadian goods and increases Canadian exports (i.e. price substitution effect).
- the U.S. share of world demand for oil and commodities is large enough to reduce the real prices of oil and of commodities by 2.0 per cent and 1.1 per cent respectively. Given that Canada is a net exporter of these goods, there is a negative terms-of-trade shock and a negative wealth effect, reducing Canadian consumption.
- the fall in the demand of factors of production decreases marginal cost, while the depreciation of the real exchange rate increases real import prices. So the year-on-year

core inflation rate barely reacts (since the two effects are offsetting), which requires little action on the part of the monetary authority.

Three of the four effects are negative for Canadian economic activity while the exchange rate effect is positive. In Canada, the first effect (the income effect) is very important because the United States absorbs 82 per cent of Canadian exports. This effect combined with the fall of oil and commodities prices dominate the exchange rate effect so that Canadian exports and GDP drops by 0.39 and 0.10 percentage points respectively. Therefore, the impact of the U.S. interest rate shock on Canadian GDP is about one quarter of that on U.S. GDP.

In RC (which is mainly the European Union and Japan) GDP increases slightly because the effect of the depreciation of their currency dominates the income effect. The income effect is less important because the U.S. share of RC exports is only 40 per cent compared to 82 per cent for Canada. Furthermore, RC is a net importer of oil and commodities. Therefore, shocks which decrease the prices of oil and commodities imply positive terms-of-trade and wealth effects thereby stimulating consumption - the opposite of their effects in Canada.

In emerging Asia, because of their fixed nominal exchange rate relative to the United States, they are importing U.S. monetary policy. Therefore the nominal interest rate increases by the same amount as in the United States. Moreover, the effects of the shock are similar to those found in the United States.

5. Examples of applications of the model

This section presents some applications that require either a global model, a multi-sector model, or both. We tackle four issues. First, we explore the importance of the multi-sector dimension to understand the link between productivity and the exchange rate (e.g. the Balassa-Samuelson effect). Second, in the context of the recent rise in the price of oil, we focus on the responses of the oil and commodities sectors to demand and supply shocks. Third, we use the model to analyze the impact of the economic emergence of China and India on imported goods, oil and commodities prices. Fourth, in the context of the recent emergence of the global imbalances, we analyze the shocks that could trigger a disorderly adjustment of global imbalances - a loss of investor confidence in U.S. dollar assets and a rise in protectionism (as in Faruquee et al. (2007a) and Faruquee et al. (2007b) respectively) - and an example of policies that could contribute to the resolution of global imbalances - a fiscal consolidation in the United States. Table 1 summarizes the key findings of the simulations.

Table 1: Summary of the Simulations Using BoC-GEM

<i>Simulations</i>	<i>Key findings</i>
5.1 - Permanent productivity shocks in the United States	Illustrates the Balassa-Samuelson effect.
Permanent increase of productivity in the traded and nontraded sectors	Depreciation of the U.S. dollar.
Permanent increase of productivity in the traded sectors only	Appreciation of the U.S. dollar (i.e. Balassa-Samuelson effect).
5.2 - Demand and supply shocks to the oil and commodities sectors	Outlines the properties of two sectors unique to BoC-GEM (relative to other versions of GEM).
Temporary demand shock (1% world consumption shock)	Temporary increase in the real prices of oil (13%) and commodities (3%).
Permanent demand shock (permanent increase of productivity in the commodity importing regions)	Peak increase of 9% and 2.8% of the real prices of oil and commodities respectively. Some long-run adjustment in the supply of oil and commodities.
Permanent reduction in the supply of oil in the commodity exporter	Real price of oil peaks at +20%. In Canada, a positive wealth effect and increase of consumption; opposite for the United States. Drop of 0.3% of world GDP.
5.3 - Impact of emerging Asia on world prices (via a permanent increase in productivity)	Drop of consumption and investment prices worldwide. Increase of oil and commodity prices creates a positive wealth effect in Canada.
5.4 - Shocks related to global imbalances	Explores some topics of current interest in a global context.
Loss of appetite for U.S. assets and a U.S. risk premium shock	Increase of U.S. saving; large drop of U.S. GDP, large depreciation of U.S. dollar and notable improvement in the U.S. current account.
Impact of an increase of protectionism on the U.S. and Canadian economies	
Permanent increase of tariffs worldwide (including a collapse of NAFTA)	Large permanent loss of output worldwide.
Permanent increase of tariffs worldwide except. NAFTA is maintained	Large permanent loss of output worldwide except in Canada where the fall of GDP is relatively small.
Impact of a U.S. fiscal consolidation (permanent 50% reduction of the U.S. federal debt)	U.S. current-account-to-GDP ratio improves by 0.7 percentage points.

5.1 Permanent productivity shocks in the United States and the Balassa-Samuelson effect

In this sub-section we analyze the importance of the multi-sector aspect of the model for the link between productivity and the exchange rate. In a one-good model, an increase of domestic productivity increases supply, reduces the prices of domestic goods and this induces a depreciation of the real effective exchange rate. This depreciation is necessary to sell abroad the additional supply of goods from the increase in domestic production. When the economy is multi-sector, the impact of a productivity shock on the real effective exchange rate depends on its source. If the productivity shock occurs in all the sectors of the domestic economy then we get the same result as in a one good model (a depreciation of the real exchange rate). In contrast, if the productivity shock is specific to the tradable sector of the domestic economy, then we expect the opposite effect in long run - an appreciation of the real effective exchange rate. This effect is known as the Balassa-Samuelson effect (Balassa (1964), Samuelson (1964)).

The Balassa-Samuelson effect works as follows. If labour is mobile across sectors (but not mobile across countries) a positive shock to the level of productivity in the tradable sector will raise the real wage in all sectors of the domestic economy, including the nontradable sector. Consequently, the shock creates an increase in marginal cost and hence, the price of the nontradable good. On one hand, the shock creates a drop in the price of tradable goods while, on the other hand, the shock induces an increase in the price of nontradables. If the elasticity of substitution between domestically-produced and imported tradable goods is large enough and if the nontradable sector is more labour intensive than the tradable sector (as is normally the case in the data, and is the case in the BoC-GEM), then the increase in the nontradable good price should dominate and the real effective exchange rate should appreciate.

Figure 14 compares the responses of the U.S. economy to two different domestic shocks - a permanent increase in productivity in all the sectors of the economy (with the exception of the oil sector) and a permanent increase in the productivity of the traded sectors (tradable goods and commodities).⁴² The two sets of shocks are calibrated to have a similar output response. Both shocks reduce real marginal cost and the core inflation rate. Thus, the U.S. monetary

⁴²We exclude the oil sector from the productivity shock to the traded sectors in order to clarify the results of the shocks. A change in productivity in the oil sector will induce large, temporary, swings in oil prices (because of the very strong real adjustment costs in the short run) that will obscure and unnecessarily complicate the resulting conclusions made in this sub-section.

authority decreases the nominal interest rate which, through the modified risk-adjusted UIRP condition, creates an initial depreciation of the U.S. dollar in both shocks. This is in spite of the Balassa-Samuelson effect for the shock exclusively in the traded goods sectors. Nevertheless, after a delay of about two years, the Balassa-Samuelson effect dominates and as expected a permanent increase of the domestic productivity specific to the traded sectors induces a permanent appreciation of the domestic currency, while an increase of productivity in the entire economy creates a depreciation of the real effective exchange rate at every time horizon.

These results are mirrored in Figure 15 for Canada. Over the medium term, we see that its bilateral exchange with the United States appreciates by about 0.4 per cent when the U.S. productivity shock is economy wide, but it depreciates 0.4 per cent when the shock is only in U.S. traded goods. Its real effective exchange rate moves by less (around 0.2 per cent) in both cases. Because the United States is wealthier, it demands more Canadian exports in both shocks, but less under the U.S.-traded-goods-only shock, since the appreciating bilateral exchange rate in Canada serves to dampen its export demand. Overall, Canadian GDP increases but at different amounts because of the different responses of the exchange rate (over the medium term, 0.20 per cent in the U.S.-traded-goods-only productivity shock and about 0.15 per cent in the other case).

5.2 The oil and commodities sectors: demand and supply shocks

This section presents the responses of the oil and commodities sectors to demand and supply shocks. We use three different scenarios that exploit the links between the world economy and the prices of oil, gasoline and commodities to illuminate the response of the model to an increase of the demand for oil and commodities. The first scenario is a temporary worldwide increase in consumption. It can be viewed as a world demand shock that creates a temporary increase of the demand for oil, gasoline and commodities. The second scenario is a permanent increase of tradable and nontradable productivity in the commodity-importing regions (US, RC and AS) that generates a permanent increase in the demand for oil, gasoline and commodities. Finally, we present the response of the model to a permanent reduction of the supply of oil of the commodity-exporting region (CX) and examine the effects on Canada, where the supply of crude oil reserves is not subject to an exogenous shock.

5.2.1 A temporary shock to world consumption

First we will consider a temporary increase in the level of consumption worldwide, as in Figure 16. The size of the shock is the same across all the regions and is calibrated to create an initial increase of about one percentage point of GDP in each region, increasing the demand for the components of consumption goods (gasoline, tradable goods and nontradable goods) for roughly two years. Because tradable and nontradable goods are produced using oil and commodities, the demand for raw materials also increase. Oil demand increases even more than that of commodities because the rise in the demand for gasoline means refineries need more oil for their production.

The increase demand for oil and commodities causes an upward movement along their supply curves, each of which has a very steep slope in the short run. In the case of oil, during the first two years, the slope of the short-run supply curve is almost vertical. Recall that the steep slopes are explained by the following factors:

- The amount of crude oil reserves and land are fixed. The more that the production of oil or commodities depends on its fixed factor, the less elastic is its supply curve.
- The long-run elasticity of substitution between factors of production is relatively low, limiting the speed and capability of the oil- and commodities-producing firms to adjust their levels of production.
- The presence of strong real adjustment costs in the production of oil and commodities reduce the elasticity of substitution between factors of production in the short and medium run. Adjustment costs are greater in the oil sector than in the commodities sector. The effect of these adjustment costs on the price of oil is even more pronounced because oil-producing firms know in advance that it is only a temporary increase in oil demand.

As a result, the one per cent increase of world GDP induces a rise in the real price of oil that peaks at 14 per cent and an increase in the real price of commodities of 3 per cent. The amplitude of these links between the world output gap and the real prices of oil and commodities is consistent with the estimation results of Lalonde et al. (2004). Despite the increase in the real price of oil, the level of production of oil in Canada and CX increases by less than 0.1 per cent. The increase in commodities production is much larger (between

0.8 and 1.0 per cent depending on the region). Finally, in general, the real price of gasoline follows closely the real price of oil, increasing by 10 per cent.

5.2.2 A permanent shock to the productivity of the tradable and nontradable sectors of the commodity-importing regions (US, AS and RC)

Another mechanism for a permanent rise in the real prices of oil and commodities is a permanent increase in the productivity of the tradable and nontradable sectors of the three commodity-importing regions US, AS and RC (mainly the European Union and Japan). Figure 17 focuses on the effects on the two commodity-exporting regions (CX and CA) for this shock.

In all the commodity-importing regions, the shock is calibrated to create a permanent increase of 1.5 per cent of GDP. These regions increase their demand for all factors of production, including oil and commodities. Over the first two years, because of the real adjustment costs, there is a lack of response in the level of oil production, but a 9 per cent increase in the price of oil. Unlike a temporary demand shock, a permanent demand shock induces a gradual increase in the level of production, especially in Canada and CX, which limits the rise in the real price of oil and eventually contributes to a gradual reduction to a higher steady-state level. The shock elicits a similar, but more muted short-run response in the commodities sector, because of the weaker real adjustment costs (relative to the oil sector). The real price of commodities increases by only 2.9 per cent.

In the commodity-exporting regions, the rise in the real prices of oil and commodities imply a positive terms-of-trade shock, an appreciation of their currency and a reduction of the price of imported capital goods which increase potential output. The increase in the level of production is concentrated in the oil and commodities sectors. Furthermore, the positive wealth effect leads to higher consumption.

5.2.3 A permanent reduction in the supply of crude oil in the commodity exporter (CX)

The shock is an exogenous reduction of six percentage points of the oil production in the main oil exporting region (CX), but not in Canada (see Figures 18 and 19). In other words, the near-vertical supply curve for oil in CX is shifting to the left. During the first years after the shock, the inelasticity of the demand for oil creates a 20 per cent increase in the price of oil. Outside CX, oil-producing firms respond to the increase of the internationally-

determined price of oil by increasing their level of production. In other words, given their marginal cost curves, some fields become profitable at the new price and can be exploited more fully by increasing capital and labour inputs. The response of the production of oil is particularly important for Canada because it is the only oil exporter other than CX. The gradual and slow response of oil production outside CX combined with some substitution of the production processes away from oil and consumption away from gasoline (i.e. switching from sport-utility vehicles to small cars) contributes, in the long run, to a reduction in the real price of oil relative to its peak.

In the oil-importing regions like the United States, the long-lasting increase in the real price of oil implies a deterioration in the terms of trade, a depreciation of the real effective exchange rate and a sizable negative wealth effect. Therefore, the shock reduces consumption, increases the relative price of imported investment goods and reduces the general level of production. The increase in oil prices also raises the value of oil imports and leads to a deterioration in the U.S. current-account-to-GDP ratio by 0.20 percentage points. U.S. GDP falls by 0.23 percentage points. Given that we see the same sort of reaction in the European Union, Japan, and emerging Asia, world GDP falls by 0.25 percentage points (net of the gains in the oil-exporting regions).

In Canada, the shock leads to a positive terms-of-trade increase and an appreciation of its real effective exchange rate. This appreciation generates a positive wealth effect and reduces the relative price of imported goods, thereby raising consumption and imports. On the other hand, exports are affected negatively by the economic slowdown in the United States and in the other oil-importing regions. On net, Canadian GDP only increases slightly. While increase of GDP partly reflects the increase in oil production, it is mainly driven by the drop of the relative price of imported investment goods. This is in line with previous quantitative research for Canada (for example, Stuber (2001)).

As previously mentioned, the endogenous reaction of the world economy is very important for understanding the impact of the shock on Canadian GDP. The fall in exports cancels out a large share of the increase of consumption coming from the positive wealth effect. To illustrate that point, we can consider a similar shock in the commodities sector (Figure 20). This shock is a permanent reduction in the supply of commodities in the commodity exporting region (CX). Because the United States is a net exporter of commodities (mainly agricultural goods), in this scenario we see a slight increase in U.S. aggregate demand instead of a reduction which was the case with a negative shock to the world supply of oil. Thus, Canadian exports do not fall and, consequently, GDP increase substantially more than in

the case of the oil supply shock.

5.3 The impact of emerging Asia on the prices of imports, oil and commodities

Figures 21 and 22 present the impact of rapid growth in emerging Asia on the prices of oil and commodities worldwide, and the price of imported goods in the other regions. We will focus on the effect on the U.S. economy (as a major importer of oil and commodities) and on Canada (as both a major net exporter of oil and commodities). We generate a strong profile for growth in emerging Asia with a permanent increase of 5 per cent of productivity in both the tradable and nontradable sectors. The shock creates a depreciation of emerging Asia's real effective exchange rate.

This productivity shock affects the other regions mainly through the following channels:

- In emerging Asia, the shock increases the demand for factors of production, including oil, investment goods and commodities. Because of the important real adjustment costs in the production of oil and commodities, these prices increase substantially on impact. In the medium term, the production of oil and commodities adjust upwards and generate a gradual reduction of their relative prices relative to their peaks.
- The shock reduces the relative price of consumption and investment goods in emerging Asia, which in turn decreases the relative prices of imported goods in the other regions. The permanent drop of the price of imported investment goods worldwide generates a permanent increase of potential GDP in the other regions. In addition, the reduction of the price of imported consumption goods generates a positive wealth effect leading to higher consumption worldwide.
- The commodity-exporting regions export more oil and commodities to AS. This helps generate a positive wealth effect in CA and CX, allowing them to import more consumption goods from US and RC.

As seen in the previous section, the impact of the oil and commodities prices channel depends on whether a region is a net importer or a net exporter of these goods. For instance, in RC, this channel implies a negative terms-of-trade shock, a depreciation of the exchange rate and a negative wealth effect which reduces consumption. In CA and CX, increases in the real prices of oil and commodities cause opposite effects. In contrast, for all the regions

the reduction of the relative price of imports (consumption and investment goods) generates a positive wealth effect, an increase of consumption and a rise in potential output.

For oil-importing regions such as the United States, there are two wealth channels that have opposite effects on consumption and GDP. Initially the positive wealth effect associated with the reduction of imported goods slightly dominates the negative wealth effect induced by the increase of the price of oil. As the price of oil gradually falls over time, the fall of imported consumption and investment goods prices dominate and both consumption and GDP increase permanently.

In Canada, the two wealth effects are positive for consumption and GDP. Therefore, the increase in consumption and GDP is greater and occurs more rapidly than in the United States. The permanent rise in Canadian production is explained by the fall of investment goods prices (i.e. increase of potential GDP) and some increase of oil production.

As seen in Section 5.1, because the shock affects both the tradable and the nontradable sectors, it creates a depreciation of the real effective exchange rate in emerging Asia. In all the other regions, the real exchange rate is appreciating. As expected, the appreciation is larger in the oil-exporting regions (CA and CX) than in the oil-importing regions (US and RC). Therefore, the Canadian dollar appreciates relative to the U.S. dollar.

It is interesting to note that, outside emerging Asia, in spite of the reduction of the relative price of imported goods, the negative impact of the shock on the year-on-year core inflation rate is very small. Recall that the monetary authority is forward looking, and there is no uncertainty regarding the source, the persistence and the effect of the shock.

5.4 Shocks related to global imbalances

Now we turn to some issues related to the recent emergence of global imbalances. We start by presenting two risks that could generate a disorderly adjustment of global imbalances: a loss of investor confidence in U.S. dollar assets and a rise in protectionism. We also analyze to what extent a fiscal consolidation in the United States could help to eliminate global imbalances.

5.4.1 A loss of appetite for U.S. assets induced by an increase in the country risk premium

One risk that could generate a disorderly adjustment of global imbalances is a widespread loss of investor confidence in U.S. dollar assets because of mounting U.S. foreign liabilities and an expectation of significant currency losses (see Figures 23 and 24). If international investors suddenly perceive holding U.S. dollar-denominated assets as riskier than they originally thought, they would reduce the amount of U.S. dollar assets that they hold (all else being equal) and demand a higher rate of return on their remaining holdings of U.S. dollar assets. All of this would of course be accompanied by a sharp depreciation in the U.S. real effective exchange rate.⁴³

We implement this scenario through a long-run decline in the level of U.S. foreign indebtedness and an increase in the carrying cost to the U.S. of issuing debt to the international market. We can analyze the shock from either the domestic savings minus domestic investment perspective or alternatively through the net trade position and interest payments to abroad.

Consider the savings and investment perspective first. On net, non-U.S. residents sell a portion of their U.S. dollar-denominated assets, so U.S. citizens have to repurchase those liabilities, leading to a permanent rise in the U.S. private savings rate (more in the short-run than in the long-run). Coupled with the higher risk premium foreigners have also imposed on U.S. dollar-denominated assets, there is upward pressure on real interest rates and thus weaker U.S. domestic demand.

Similarly from a net trade perspective, there must be a surplus in the short run to achieve a long run increase in the U.S. net foreign asset position (a decrease in U.S. liabilities). This is achieved through a depreciation of the U.S. dollar, which, everything being equal, leads to a rise in inflation from both a stronger trade balance and higher imported goods prices. In turn, this leads to the monetary authority increasing the nominal interest rate.

The view in the rest of the world is a result of their reduction in their holdings of U.S. dollar-denominated assets. In the short run, there is a sharp real appreciation of their

⁴³Within the BoC-GEM, and most global DSGE models in general there are several weaknesses that are not addressed in this type of shock. Since markets are incomplete, it is assumed U.S. foreign assets are denominated in U.S. dollars, as are U.S. foreign liabilities. In reality, most U.S. foreign assets are denominated in foreign currencies, so a strong depreciation of the U.S. dollar by itself should reduce the U.S. net foreign asset position (assets increase in value offsetting the effects from liabilities being shed by foreigners), and diminish some of the adjustment effects discussed below (see Gourinchas and Rey (2007)).

currencies relative to the U.S. dollar and in the long run after the adjustment process is complete, a corresponding rise in interest income from U.S. dollar-denominated assets. Real interest rates fall and domestic demand rises over the near term as agents reduce their long-run holdings of U.S. dollar-denominated assets. On the other hand, trade surpluses shrink as the currencies strengthen. Everything else being equal, the effect of the stronger currency leads to disinflationary pressures and the monetary authorities have to lower their interest rates.

The net impact of these offsetting effects depends on the exposure of each region to the United States through trade (a negative income effect) and the size of its (net) holdings of U.S. dollar-denominated assets (a positive wealth effect). The larger the impact of the wealth effect relative to the income effect the more likely that a particular region sees an expansion of economic activity over the near term. This is certainly the case for emerging Asia, the commodity exporters and Japan (part of RC) because they hold a higher proportion of the stock of U.S. foreign liabilities.

Canada, on the other hand, holds relatively less U.S. assets (on net) but is much more dependent on the United States for its trade. As a result, the slowdown in U.S. domestic demand coupled with a stronger Canadian dollar more than offset the stimulative effect from rising interest income.

It is important to note that the GEM assumes that all assets are denominated in U.S. dollars. Therefore, it does not account for revaluation effects for assets that denominated in other currencies such as the Japanese yen or the euro. In the case of the United States, since most liabilities are in dollars, but most assets are held in foreign currencies, U.S. indebtedness should fall by more than is stated by the BoC-GEM in the face of a nominal (or real) depreciation. In the presence of the proper revaluation effects, the positive wealth effect that foreigners experience with this shock would be smaller.

5.4.2 An increase in trade protectionism worldwide

Another possibility that could arise in the current environment of global imbalances is increased protectionism, since there are some arguments in the United States that one possible solution could be raising tariffs against Emerging Asia, as they use export promotion policies to finance their holdings of U.S. assets (Faruqee et al. (2007b)). Trade literature suggests that increases in tariffs by one region against another will benefit the region imposing the tariff, but harm that region which is its victim - the so-called "beggar-thy-neighbour policy".

As past experience shows (particularly the Great Depression) a beggar-thy-neighbour policy eventually escalates into a worldwide tariff war, and theory suggests everyone loses under such an outcome. We can demonstrate that outcome in the BoC-GEM in Figures 25 and 26. However, we also use the BoC-GEM to demonstrate another wrinkle. Since Canada and the United States are separate regions in the world, we look at two cases. In the first case, all trading relationships break down worldwide, and there is a generalized increase in tariffs against tradable goods of ten percentage points.⁴⁴ In the second case, we assume that the North American Free Trade Agreement (NAFTA - or at least the Canada-United States portion of the agreement) survives unscathed, and the ten percentage point hike of tariffs by Canada and the United States are only against the other three regions.

As we expect, we see in the first case, that all regions' real GDP falls, as do their level of imports. In the second case, we begin to see a difference for Canada and the United States. Since both countries still maintain trade with one another without any tariffs, GDP falls by less in both regions. Moreover, while overall trade in Canada and the United States falls, trade between the two regions actually increases as the negative effects on consumption are less from import prices (since they rise less in CA and US when NAFTA continues to be in force). The real imports of CA from US are stronger if NAFTA is not broken in a worldwide tariff war, as are the real imports of US from CA (at least in the short to medium run). For Canada, the GDP loss is much smaller (0.9 per cent versus 3.5 per cent relative to control), since 80% of Canadian trade is with the United States, as the amount of its GDP affected by tariffs is much less if NAFTA remains in effect (about 7 per cent of its GDP instead of around 37 per cent of its GDP for either imports or exports).⁴⁵ GDP in the United States falls less as well if NAFTA is maintained but to a lesser extent than Canada (1.0 per cent with NAFTA versus 1.1 per cent). In this case the change is much less - first, imports only account for 13.7 per cent of GDP, and second, only about 30 to 40 per cent of its trade is with Canada. This pattern also holds for consumption in both countries. In the case where NAFTA is maintained, the U.S. real effective exchange rate depreciates less (0.7 versus 0.8 per cent in the long run) and the Canadian real effective exchange rate appreciates substantially less (1.4 versus 2.3 per cent in the long run). This is almost exclusively a result of the much smaller movement in the bilateral Canadian-U.S. dollar exchange rate.

For both global tariff war scenarios, we see similar effects across the other three regions

⁴⁴Note that we do not impose any tariffs on raw materials, but that the results would not be substantively different in the long run. We also assume all the new government revenues generated will go into government spending, not domestic tax cuts.

⁴⁵See Section 3.3 on the calibration of international linkages and Table 6.

CX, AS and RC. GDP falls substantially as consumption drops from higher import prices. Their net trade positions are not greatly changed, but this masks a huge fall in their real export and import volumes (around 6.5 per cent after the tariff war begins in CX, and around 10 per cent after the tariff war begins in both AS and RC). However notice that their exports (and hence their levels of GDP) are almost identical whether or not NAFTA is maintained between Canada and the United States during a global tariff war. Maintaining NAFTA does not divert trade - it seems to create it between Canada and the United States, and both nations are definitely better off.⁴⁶

So this shock demonstrates that a tariff war would be highly destructive, and any maintenance of freer trade amongst any of the regions would leave the regions involved better off, as is the case when NAFTA remains in force.

5.4.3 A fiscal consolidation in the United States

In this section we explore the impact of a fiscal consolidation in the United States on the current account imbalances (see Figures 27 and 28). We consider a permanent and credible reduction in the U.S. government-debt-to-GDP ratio by 20 percentage points, which translates into a reduction of the U.S. deficit-to-GDP ratio by about four percentage points. The fiscal consolidation considered here is financed through higher taxes on labour income. The link between the government debt and the desired net foreign asset position in the BoC-GEM means that the net foreign asset position improves by 10 percentage points of GDP, which is an improvement in the U.S. current-account-to-GDP ratio of about 0.7 percentage points in the short run and 0.5 percentage points at the steady state. This leads to a depreciation of the real effective exchange rate of roughly 1.1 per cent at its peak seven to eight years after the beginning of the consolidation. In the short run, we see a large shift in the real net export position of the United States in order to finance the shift in the net foreign asset position. After five years, export volumes are up by 2.2 per cent relative to control, while import volumes are down about 3.2 per cent.

The U.S. fiscal consolidation imposes an important cost for the U.S. economy as the increase of the labour income tax rate reduces labour effort and consequently consumption and output. In fact, U.S. real GDP falls on average by about one percent relative to control over the first eight to ten years. These factors lead to a fall in the world real U.S. dollar price of oil, despite the offsetting effect of the depreciation of the U.S. dollar. Real commodity

⁴⁶This would be true even if we used a welfare measure based on consumption and leisure (and stated in terms of consumption), as in Faruqee (2007b).

prices rise in U.S. dollar terms, but are actually falling in value in other currencies. The different behavior of the raw materials prices reflects the degree of real adjustment costs in demand (commodities demand adjusts more rapidly) and the fact that the United States has a smaller portion of world commodities consumption, relative to its share of world oil consumption.

Despite the reduced U.S. demand the world is for the most part better off. Canada and the commodity exporter gain less than the other regions, because of an increase in domestic currency earnings from commodities (the real U.S. dollar commodity price increases coupled with notable currency appreciations in CA and CX) and oil (where currency appreciations in CA and CX overwhelm the small short-run fall in the real U.S. dollar oil price).

Overall, although a fiscal consolidation in the United States helps to reduce the size of the U.S. current account imbalance, an attempt to reduce the U.S. federal debt by around 50 per cent relative to its current level only has a relatively small impact of 0.7 per cent of GDP, compared to the actual current account deficit of around six per cent of GDP, with notable short-run costs for the United States.

6. Conclusion

The BoC-GEM is a useful and necessary complement to the other models used at the Bank of Canada (ToTEM and MUSE). In particular, for issues that need a global and/or a multisectoral perspective, the BoC-GEM can be used to generate risk scenarios around the base-case projection generated by Staff using ToTEM and MUSE. Also, we would like to use the BoC-GEM annually or semi-annually to validate the consistency of the world projection underlying the Staff's base case projection scenario. In more specific terms, the projections for Canada, the United States, Europe, Japan and the rest of the world are constructed using a mix of different models and consensus forecasts from outside the Bank of Canada. Up until the introduction of the BoC-GEM, the Staff of the Bank of Canada has not had a formal tool to evaluate the consistency of the different projections done with different models. In a context where the issues require an increasingly integrated global perspective, the BoC-GEM, with its fully articulated and consistent world framework, will be a useful tool to verify the consistency of the Staff's global outlook. For instance, the BoC-GEM could be useful to make sure that the trade flows implied by the staff's projection are consistent with the forecasts for domestic demand and bilateral exchange rates of the different regions, while accounting for a global stock-flow perspective. This validation exercise is very ambitious because it implies

the (at least partial) use of the model in a real environment (i.e. with actual time series data). Currently, all our work with the BoC-GEM has been around a steady state constructed with calibrated, but artificial data, or a generated state of disequilibrium as the base case that approximates some salient features of the global and national economies.

The BoC-GEM is also a powerful and flexible tool for research, particular into issues directly relevant to the baseline global projection used at the Bank of Canada. The number of sectors of production and regions can be easily reduced or adjusted to better fit the needs of a particular research project. Recently, the Bank of Canada's Staff have used smaller versions of the model to analyze issues like the cause and the effects of the recent increase in the world price of oil and risks regarding global imbalances originating with the large current account deficits of the United States in recent years. In the near future, the International Department of the Bank of Canada plans to use the model to address the issues such as the effect of emerging Asia on traded goods prices, the relative merits of inflation and price-level-path targeting in a global framework, the relative merits of targeting headline versus core CPI inflation, exchange rate pass-through to inflation and the implications of differing forms of trade protectionism in different regions of the world.

During the process of the above research agenda, we will be able to further improve the model along various dimensions. This includes fully operationalizing the distribution sector (introduced in Laxton and Pesenti (2003)) which will allow us to break the long-run law of one price and thus to be consistent with the data, and refining the calibration by adding more differentiation between regions. Eventually, it may be worthwhile to estimate a scaled-down Canada / United States version of the model using the Bayesian approach, building on Juillard et al. (2006).

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Table 2: Core Parameters of GEM

Parameter	CA	US	CX	AS	RC
Rate of time preference $(1/\beta^4 - 1) * 100$	1.9	1.9	1.9	1.9	1.9
Depreciation rate δ	0.02	0.02	0.02	0.02	0.02
Intertemporal elasticity of substitution $1/\sigma$	0.70	0.70	0.70	0.70	0.70
Habit persistence in consumption b_c	0.85	0.85	0.85	0.85	0.85
Frisch elasticity of labour $1/\zeta$	0.20	0.20	0.20	0.20	0.20
Share of liquidity-constrained consumers SLC	0.20	0.20	0.20	0.50	0.20
Habit persistence in labour b_ℓ	0.75	0.75	0.75	0.75	0.75

CA = CAnada, CX = Commodity-eXporter, US =United States,
AS = emerging ASia and RC = Remaining Countries (incl. Japan and EU)

Table 3: Distribution of Oil Reserves Around the World

Parameter	CA	US	CX	AS	RC
Calibration	14	6	75	4	2
Data	11	4	81	3	1

Table 4: Parameterization of the Production Functions

Parameter	CA	US	CX	AS	RC
Tradable Intermediate Goods					
Substitution between factors of production ξ_T	0.70	0.70	0.70	0.70	0.70
Bias towards capital α_{KT}	0.67	0.71	0.72	0.82	0.73
Bias towards oil α_{OT}	0.06	0.04	0.07	0.03	0.04
Bias towards commodities α_{ST}	0.15	0.08	0.16	0.07	0.10
Nontradable Intermediate Goods					
Substitution between factors of production ξ_N	0.70	0.70	0.70	0.70	0.70
Bias towards capital α_{KN}	0.68	0.59	0.65	0.73	0.68
Bias towards oil α_{ON}	0.04	0.03	0.06	0.03	0.02
Bias towards commodities α_{SN}	0.11	0.05	0.12	0.05	0.06
Oil					
Substitution between factors of production ξ_O	0.60	0.60	0.60	0.60	0.60
Bias towards capital α_{KO}	0.68	0.53	0.12	0.44	0.59
Bias towards crude oil reserves (<i>OIL</i>) α_{OILO}	0.32	0.46	0.87	0.53	0.40
Gasoline					
Substitution between factors of production ξ_{GAS}	0.70	0.70	0.70	0.70	0.70
Bias towards capital α_{KGAS}	0.45	0.47	0.42	0.43	0.46
Bias towards oil α_{OGAS}	0.54	0.51	0.57	0.52	0.53
Commodities					
Substitution between factors of production ξ_S	0.60	0.60	0.60	0.60	0.60
Bias towards capital α_{KS}	0.50	0.38	0.23	0.37	0.36
Bias towards natural resources (<i>LAND</i>) α_{LANDS}	0.49	0.60	0.77	0.61	0.63

Table 5: Parameterization of the Final Demand Functions

Parameter	CA	US	CX	AS	RC
Final Consumption Goods					
Substitution between domestic and imported goods μ_A	1.5	1.5	1.5	1.5	1.5
Bias towards domestic goods ν_A	0.36	0.80	0.50	0.34	0.92
Substitution between domestic tradables and nontradables ε_A	0.5	0.5	0.5	0.5	0.5
Bias towards tradable goods γ_A	0.65	0.66	0.67	0.47	0.66
Substitution between gasoline and the rest ε_{GAS}	0.3	0.3	0.3	0.3	0.3
Bias towards gasoline γ_A	0.13	0.15	0.11	0.06	0.16
Final Investment Goods					
Substitution between domestic and imported goods μ_E	1.5	1.5	1.5	1.5	1.5
Bias towards domestic goods ν_E	0.17	0.69	0.13	0.23	0.80
Substitution between domestic tradables and nontradables ε_E	0.5	0.5	0.5	0.5	0.5
Bias towards tradable goods γ_E	0.81	0.76	0.81	0.84	0.76
Demand for Oil in Production					
Substitution between domestic and imported oil $\mu_{OT}, \mu_{ON}, \mu_{OGAS}$	10.0	10.0	10.0	10.0	10.0
Bias towards domestic oil for producing tradables ν_{OT}	0.07	0.00	0.00	0.83	0.96
Bias towards domestic oil for producing nontradables ν_{ON}	0.04	0.00	0.00	0.56	0.90
Bias towards domestic oil for producing gasoline ν_{OGAS}	0.22	0.00	0.00	0.97	0.99
Demand for Commodities in Production					
Substitution between domestic and imported commodities μ_{ST}, μ_{SN}	5.0	5.0	5.0	5.0	5.0
Bias towards domestic commodities for producing tradables ν_{ST}	0.41	0.79	0.01	0.01	0.78
Bias towards domestic commodities for producing nontradables ν_{SN}	0.50	0.70	0.02	0.00	0.65

Table 6: Steady-State National Accounts - Expenditure Side

Ratio of GDP	CA	US	CX	AS	RC
Total Consumption $A + P_N G_N$	80.8	81.4	83.4	69.0	81.1
Private C	57.3	66.3	66.3	54.7	60.5
Public $G_C + P_N G_N$	23.5	15.2	17.1	14.3	20.7
Total Investment $P_E E$	19.1	18.0	16.9	31.4	19.1
Private $P_E I$	16.6	16.0	14.9	29.9	16.6
Public $P_E G_I$	2.5	2.0	2.0	1.5	2.5
Trade balance $TBAL$	0.1	0.5	-0.2	-0.4	-0.2
Imports IM	37.0	13.7	24.0	26.5	9.0
Consumption Goods $P_{MA} MA$	19.9	7.0	10.7	8.1	3.2
Investment Goods $P_{ME} ME$	11.6	4.0	8.4	12.5	3.2
Oil $P_{OT} MOT + P_{ON} MON + P_{OGAS} MOGAS$	2.2	1.7	1.7	2.8	1.2
Commodities $P_{MS}(MST + MSN)$	3.3	1.0	3.1	3.0	1.4
Oil Demand $P_{OT} OT + P_{ON} ON + P_{OGAS} OGAS$	3.7	3.5	3.9	4.8	2.7
Gasoline Demand $P_{GAS}(1 + \tau_{GAS})GAS$	3.1	3.3	2.7	3.0	3.0
Commodities Demand $P_{ST} ST + P_{SN} SN$	4.6	2.6	4.5	5.2	2.9
Government Debt B	25.0	50.0	15.0	24.0	67.0
Net Foreign Assets B^*	-7.5	-50.0	21.4	35.0	20.0
Share of World GDP (percent)	2.4	30.1	9.3	10.6	47.7

Table 7: Steady-State National Accounts - Production Side

Ratio of GDP	CA	US	CX	AS	RC
Tradables	43.3	40.1	35.3	50.9	41.1
Nontradables	53.5	56.0	62.3	46.5	56.1
Oil	7.3	2.0	11.9	2.8	2.2
Commodities	8.0	3.3	6.6	4.1	2.1
Factor Incomes (% share of oil)					
Capital	27.6	19.6	10.8	20.5	25.2
Labour	14.0	11.2	10.1	24.5	11.5
Crude Oil	58.4	69.2	79.1	55.0	63.3
Factor Incomes (% share of commodities)					
Capital	25.1	21.9	19.0	23.3	22.2
Labour	20.8	23.1	21.8	22.0	25.0
Land	54.0	55.1	59.2	54.7	52.9

Columns will not sum to 100, as the measures include both final goods (included in GDP) and intermediate goods (not included in GDP).

Table 8: Price and Wage Markups

Parameter	CA	US	CX	AS	RC
Tradables					
Markup $\theta_T/(\theta_T - 1)$	1.20	1.15	1.18	1.14	1.20
θ_T	6.00	7.67	7.67	8.00	5.89
Nontradables					
Markup $\theta_N/(\theta_N - 1)$	1.42	1.28	1.23	1.25	1.38
θ_N	3.38	4.58	4.58	5.00	3.63
Oil					
Markup $\theta_O/(\theta_O - 1)$	1.01	1.01	476	1.01	1.01
θ_O	100	100	1.21	100	100
Gasoline					
Markup $\theta_{GAS}/(\theta_{GAS} - 1)$	1.16	1.17	1.17	1.17	1.16
Markup and Taxes	1.50	1.34	1.34	1.34	1.50
θ_{GAS}	7.15	7.00	7.00	7.00	7.15
Wages					
Markup $\psi/(\psi - 1)$	1.20	1.16	1.30	1.16	1.30
ψ	6.00	7.30	4.30	7.30	4.30

Table 9: Per Cent Share of Gasoline in Consumption and GDP

Gasoline as a % share of	CA	US	CX	AS	RC
Consumption	5.4	4.9	4.1	4.9	4.1
GDP	3.1	3.3	2.7	3.0	3.1

Table 10: Real Adjustment Costs and Rigidities

Parameter	CA	US	CX	AS	RC
Real Adjustment Costs					
Capital accumulation ϕ_{I1}	1.00	1.00	1.00	1.00	1.00
Investment changes ϕ_{I2}	100	100	100	100	160
Imports of consumption goods ϕ_{MA}	0.95	0.95	0.95	0.95	0.95
Imports of investment goods ϕ_{ME}	0.95	0.95	0.95	0.95	0.95
Real Adjustment Costs in the Oil and Gasoline Sectors					
Capital for producing oil ϕ_{KO}	400	300	200	300	300
Capital for producing gasoline ϕ_{KGAS}	500	500	500	500	500
Labour for producing oil ϕ_{LO}	400	300	200	300	300
Labour for producing gasoline ϕ_{LGAS}	500	500	500	500	500
Demand for oil in production. $\phi_{OT}, \phi_{ON}, \phi_{OGAS}$	300	300	300	300	300
Real Adjustment Costs in the Commodities Sector					
Capital for producing commodities ϕ_{KS}	200	200	200	200	200
Labour for producing commodities ϕ_{LS}	200	200	200	200	200
Demand for commodities in production. ϕ_{ST}, ϕ_{SN}	200	200	200	200	200

Table 11: Nominal Rigidities

Parameter	CA	US	CX	AS	RC
Weight on past versus steady-state inflation	0.5	0.5	0.5	0.5	0.5
Wages for liquidity-constrained consumers ϕ_{WLC}	900	800	800	800	1050
Wages for forward-looking consumers ϕ_{WFL}	900	800	800	800	1050
Price of domestically-produced tradables ϕ_{PQ}	700	700	700	700	900
Price of nontradables ϕ_{PN}	700	700	700	700	900
Price of imported intermediate goods ϕ_{PM}	4000	4000	4000	4000	4000

Table 12: Monetary Policy Reaction Function Parameters

Parameter	IFB Rule	Fixed Exchange Rate Regime
Lagged interest rate at t-1 ω_i	0.95	1.00
Year-on-year core inflation gap at t+3 ω_1	0.90	0.00
Contemporaneous output gap ω_2	0.00	0.00
Change in the nominal exch. rate at t ω_3	0.00	1000000 (proxy for ∞)

Figure 1: Structure of the BoC-GEM - Overview of the Production Sectors

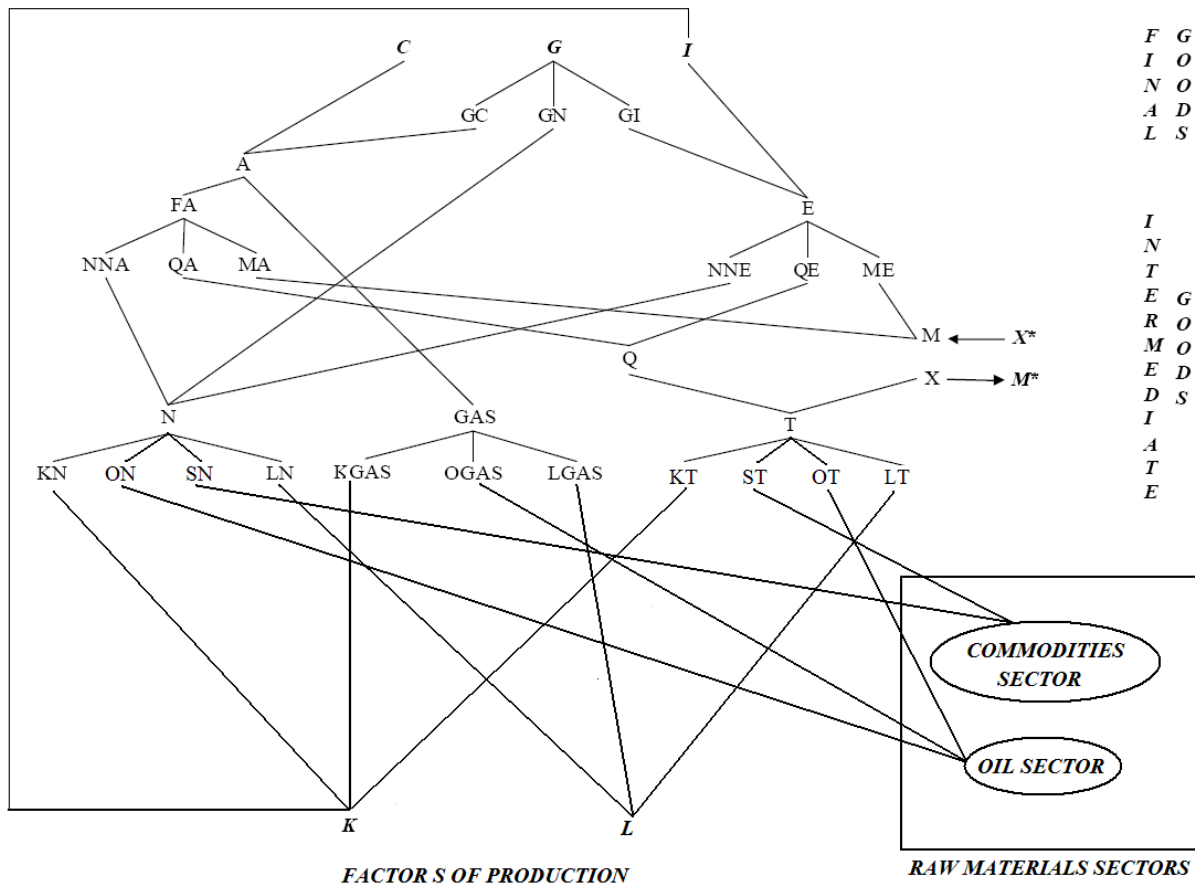


Figure 2: Structure of the BoC-GEM - The Oil Sector

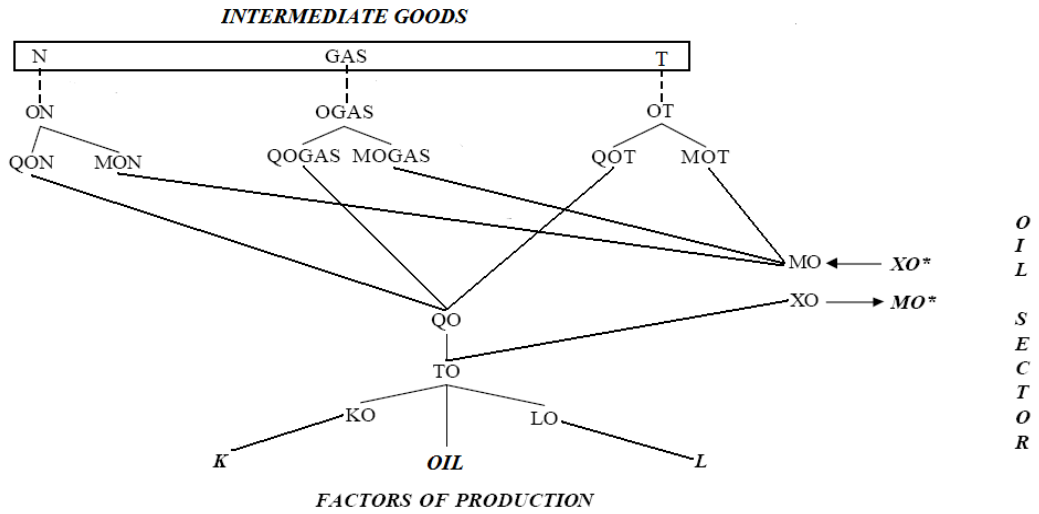


Figure 3: Structure of the BoC-GEM - The Commodities Sector

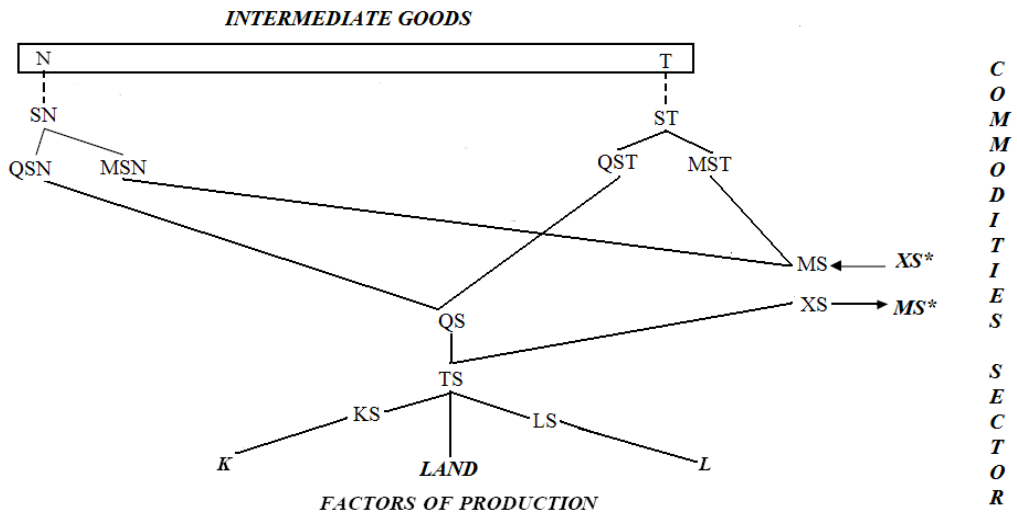


Figure 4: Global Bilateral Trade Flows - All Goods (% of World GDP)

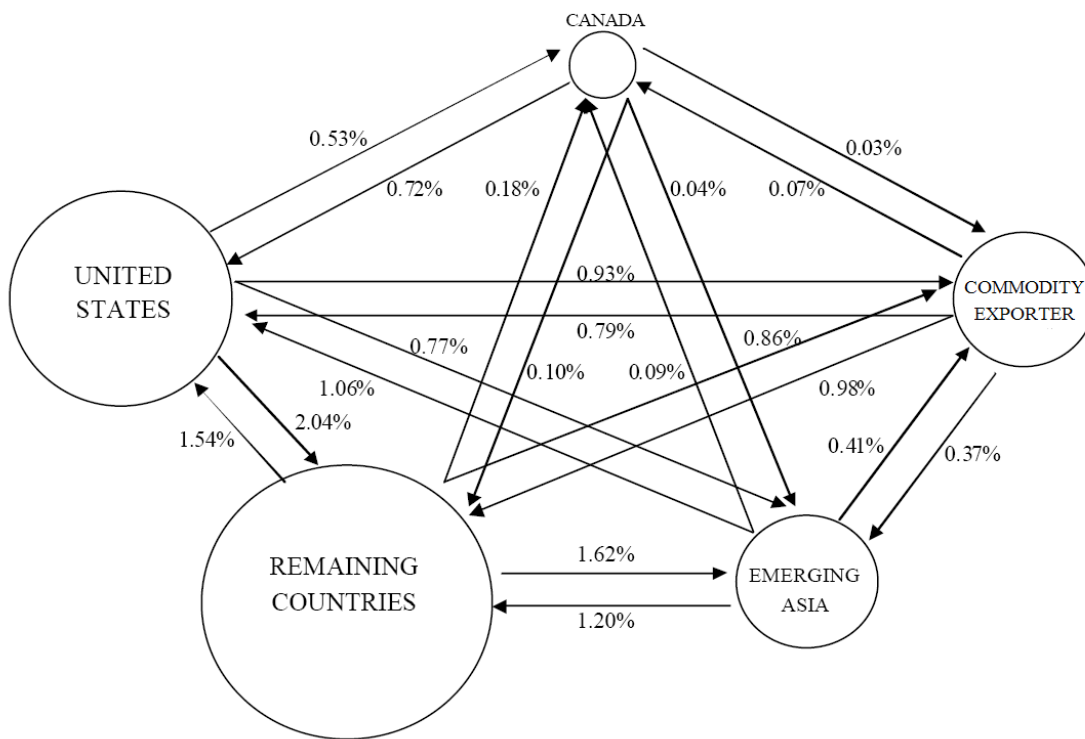


Figure 5: Global Bilateral Trade Flows - The Oil Sector (% of World GDP)

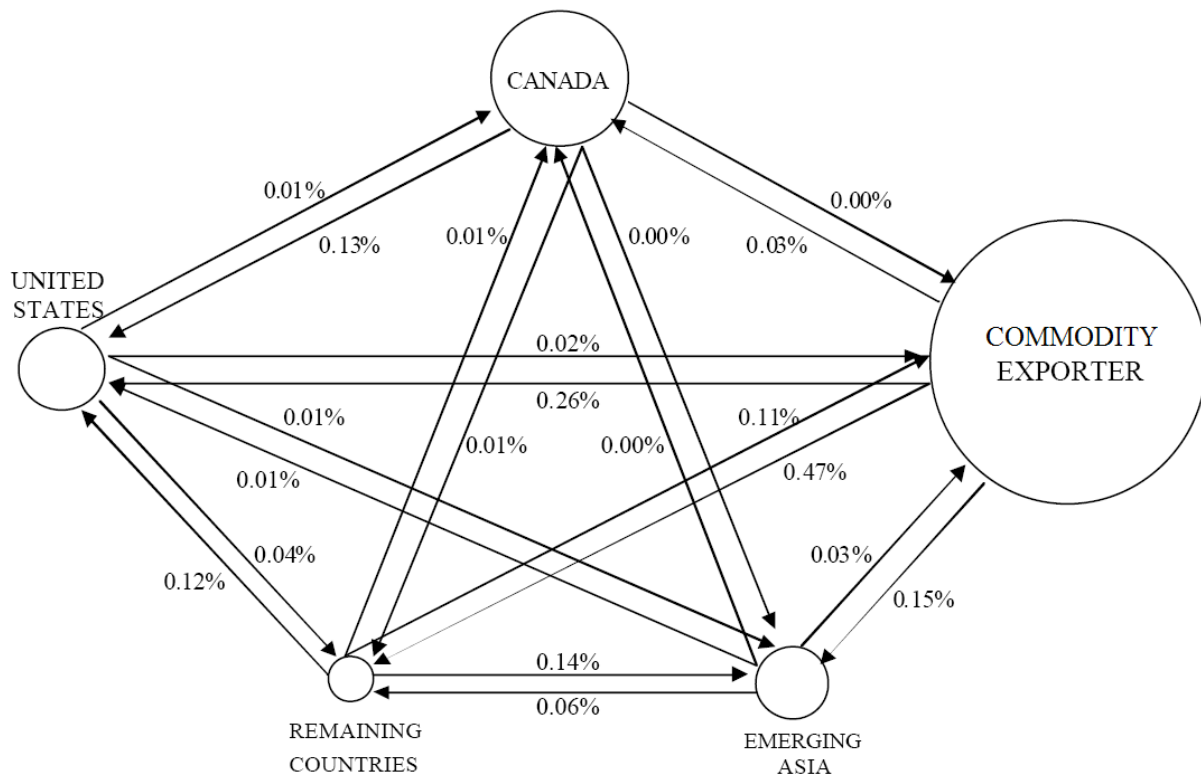


Figure 6: Global Bilateral Trade Flows - The Commodities Sector (% of World GDP)

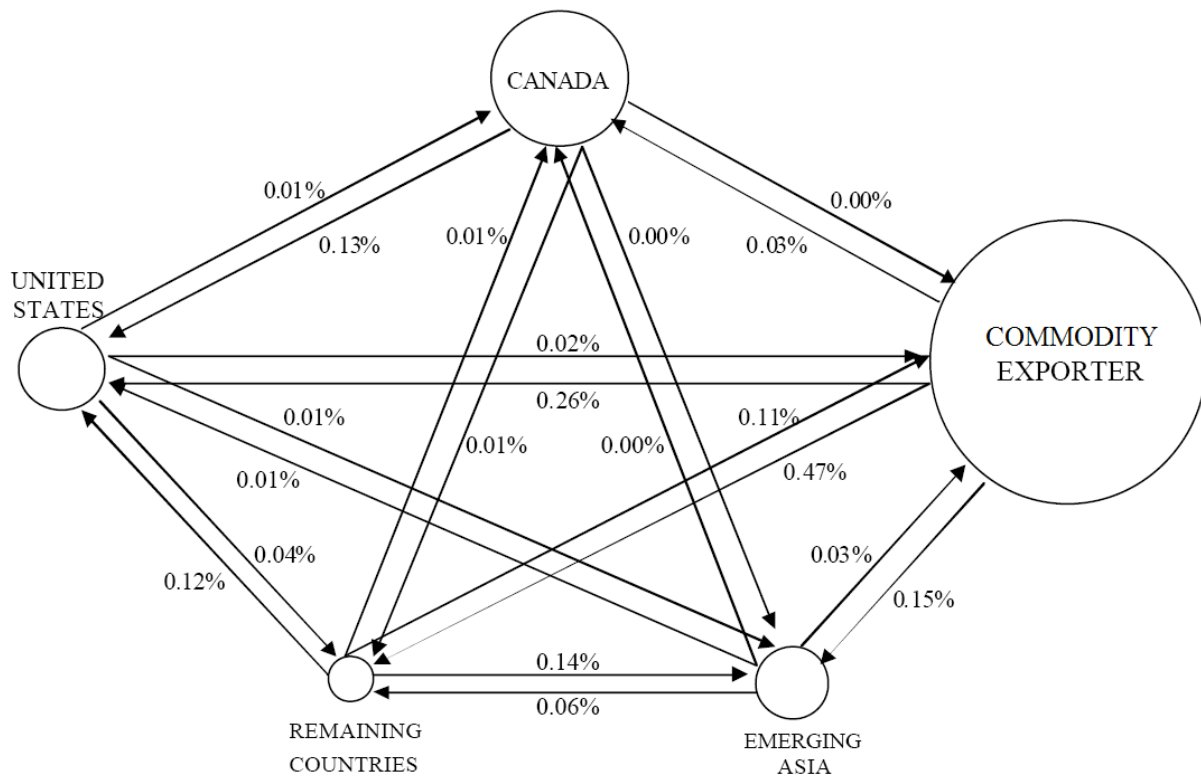


Figure 7: A Stylized Representation of the Crude Oil Reserves Market

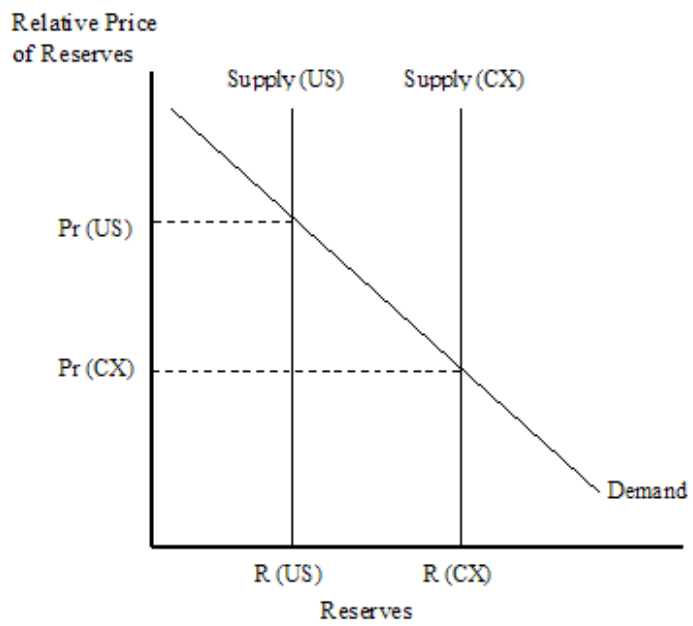


Figure 8: A Temporary Increase in Consumption in Canada

(Deviation from control, in percent)

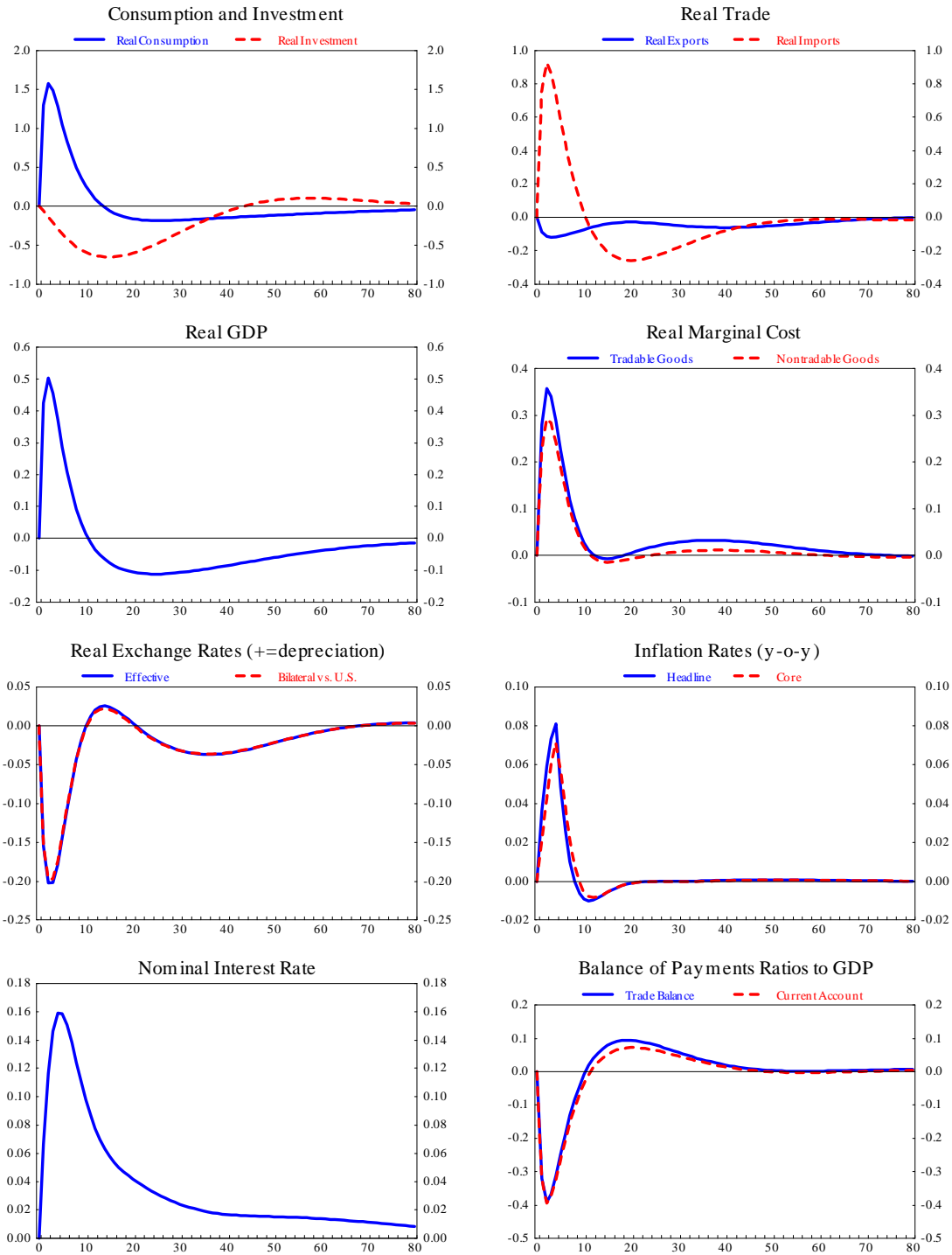


Figure 9: A Temporary 100 bp Increase in the Canadian Interest Rate

(Deviation from control, in percent)

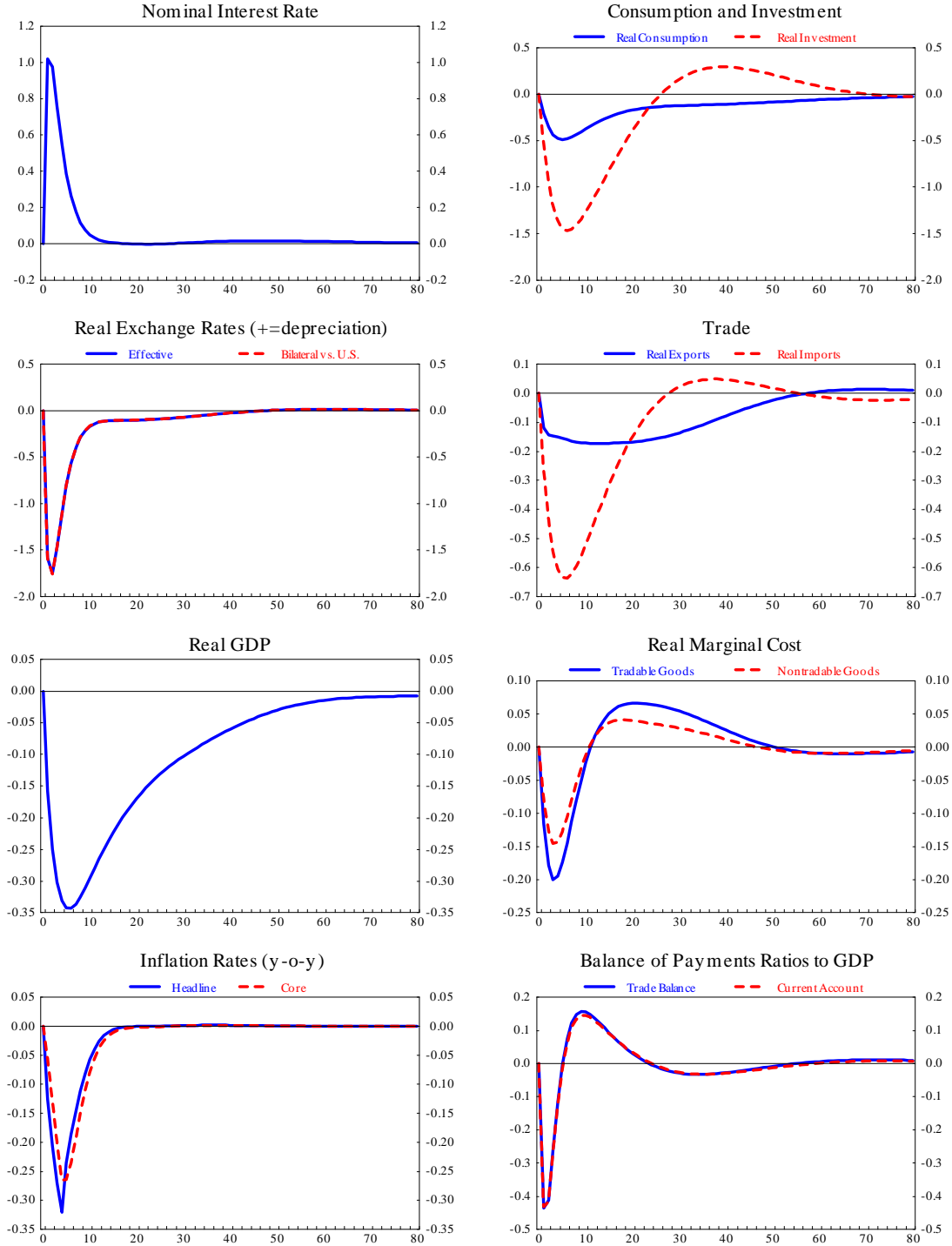


Figure 10: A Temporary Increase in Consumption in the United States - U.S. Effects

(Deviation from control, in percent)

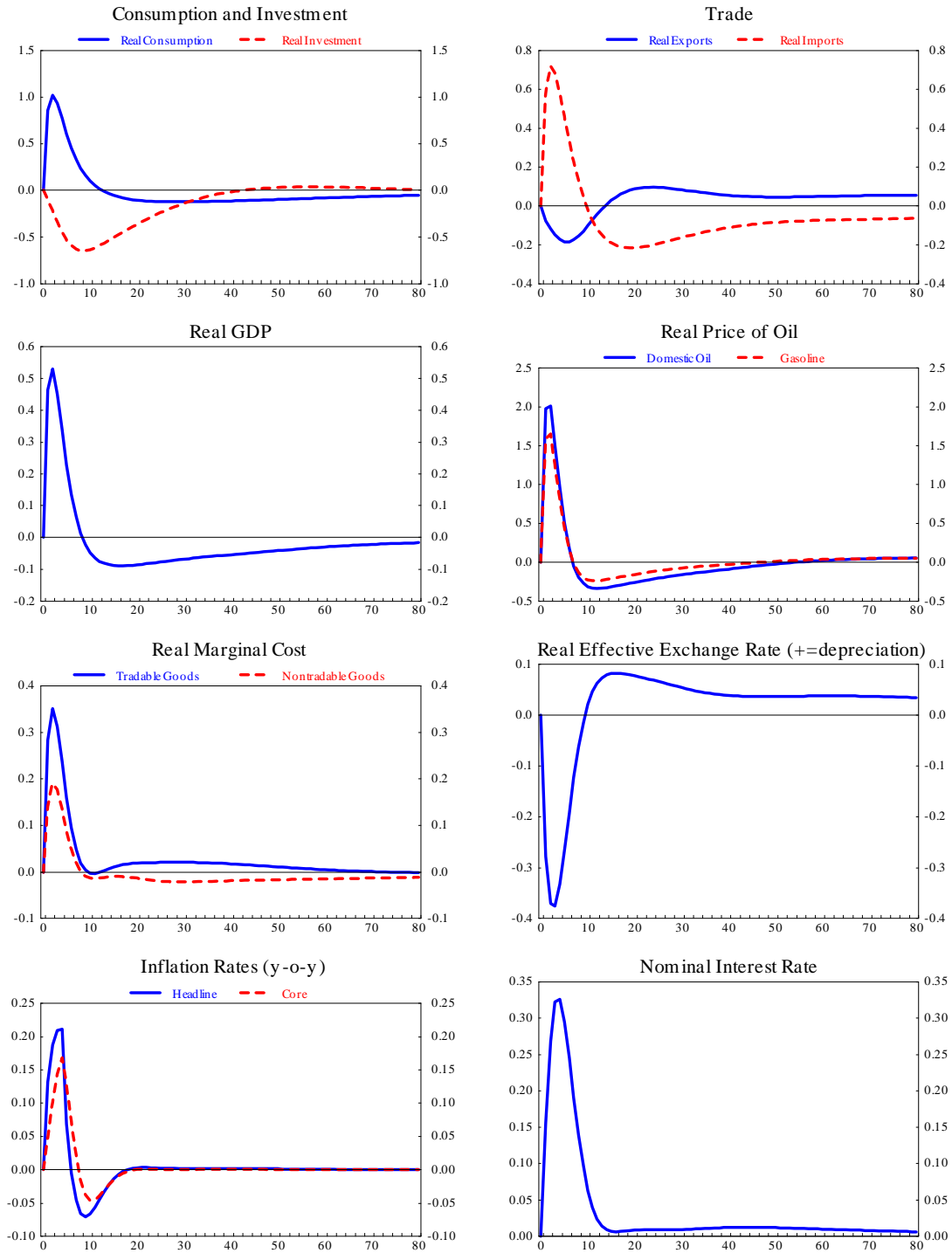


Figure 11: A Temporary Increase in Consumption in the United States - Effects on Canada

(Deviation from control, in percent)

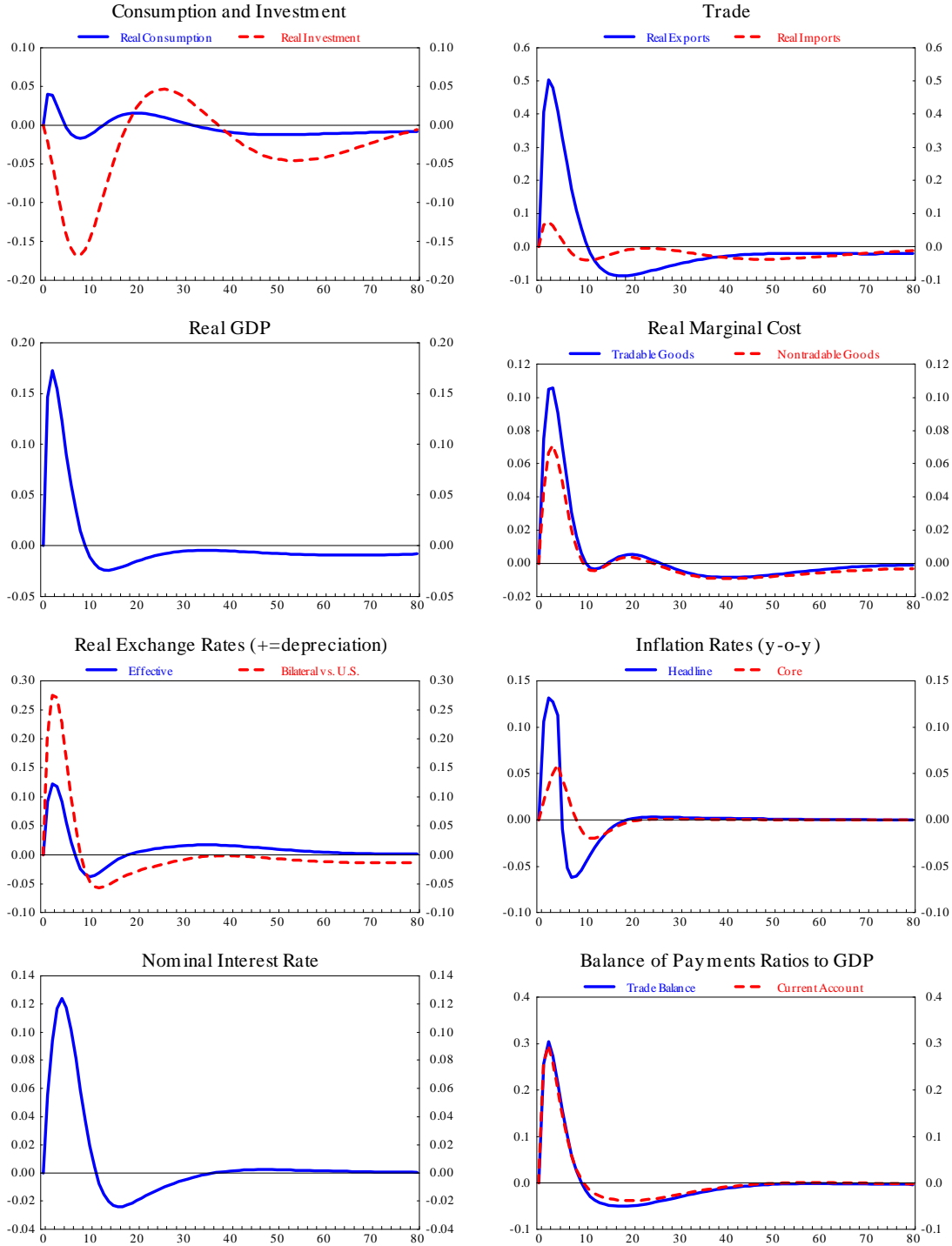


Figure 12: A Temporary 100 bp Increase of the Interest Rate in the United States - U.S. Effects

(Deviation from control, in percent)

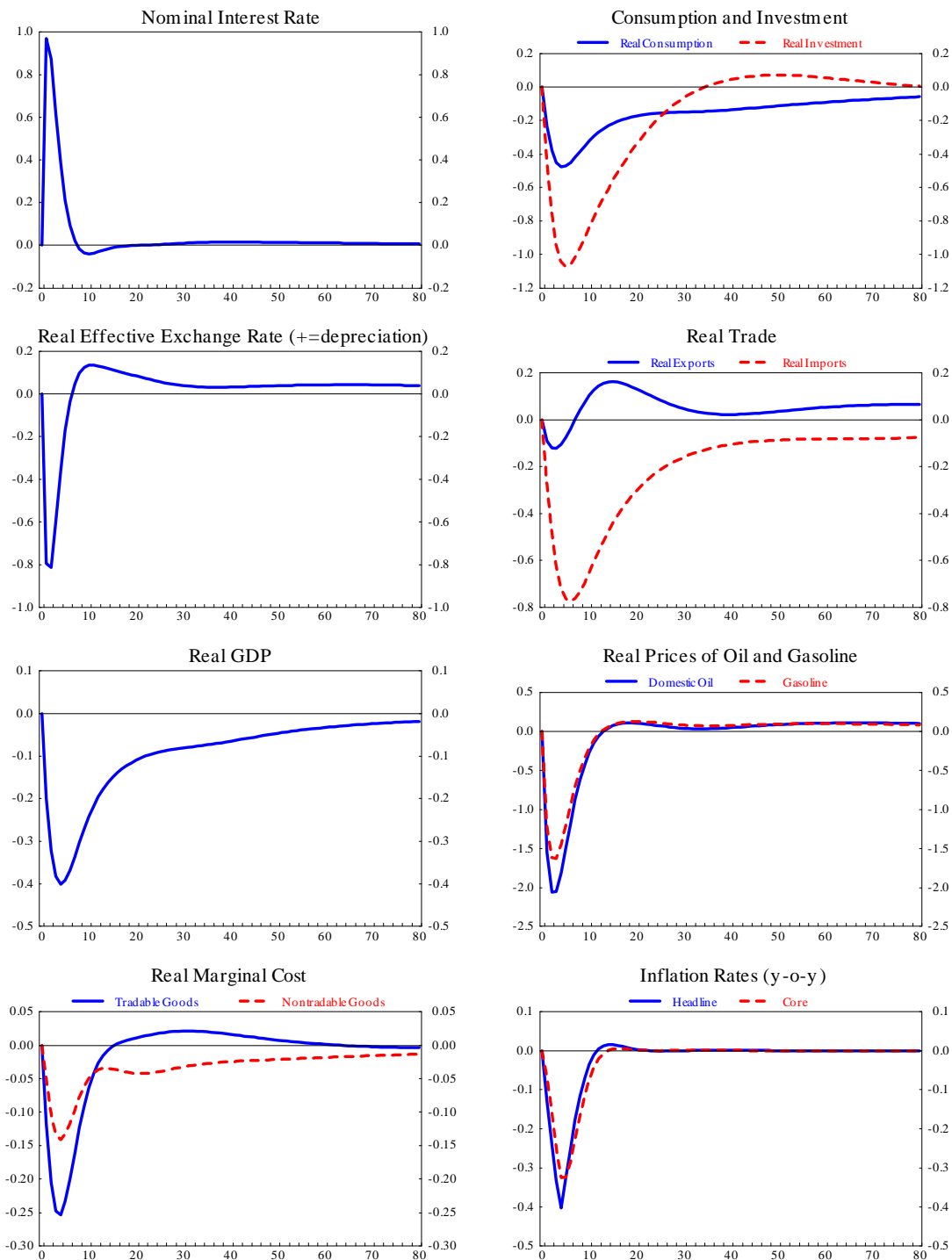


Figure 13: A Temporary 100 bp Increase in the Interest Rate in the United States - Effects on Canada

(Deviation from control, in percent)

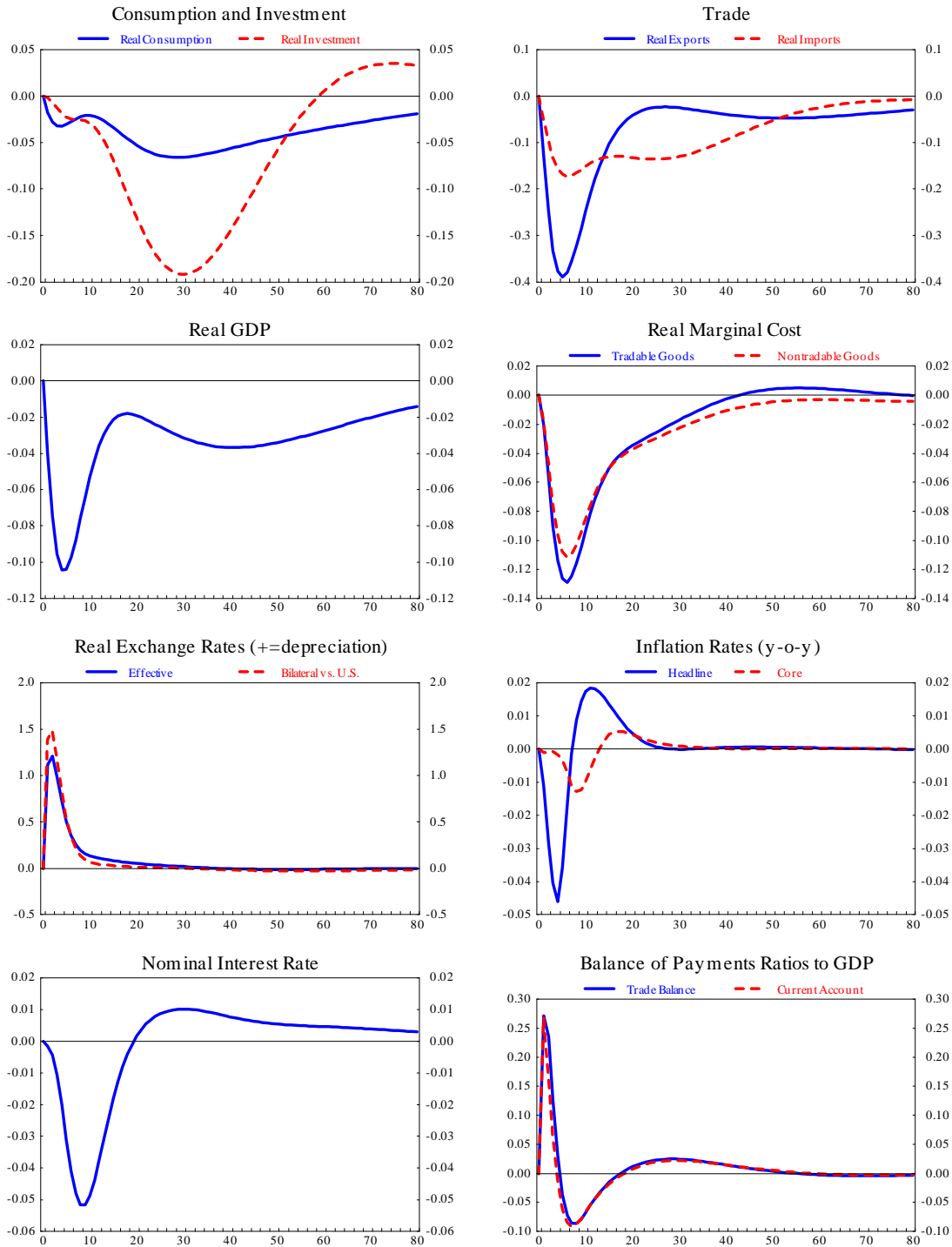


Figure 14: U.S. Productivity and the Balassa-Samuelson Effect - U.S. Effects

Permanent Increase of Productivity in All the Sectors (Solid)
 Permanent Increase of Productivity in the Traded Sectors (Dashed)
 (Deviation from control, in percent)

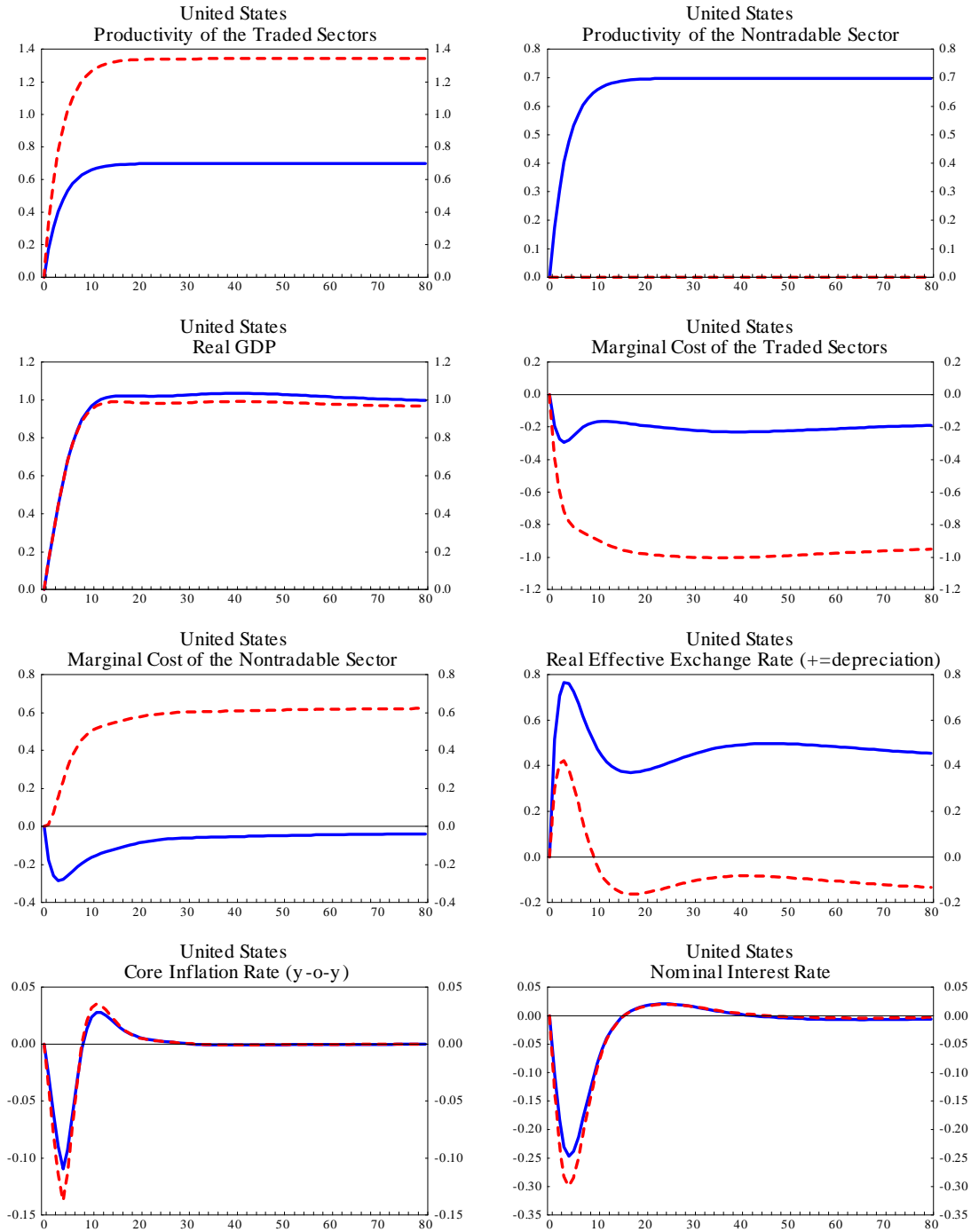


Figure 15: U.S. Productivity and the Balassa-Samuelson Effect - Effects on Canada

Permanent Increase of Productivity in All the Sectors (Solid)
 Permanent Increase of Productivity in the Traded Sectors (Dashed)
 (Deviation from control, in percent)

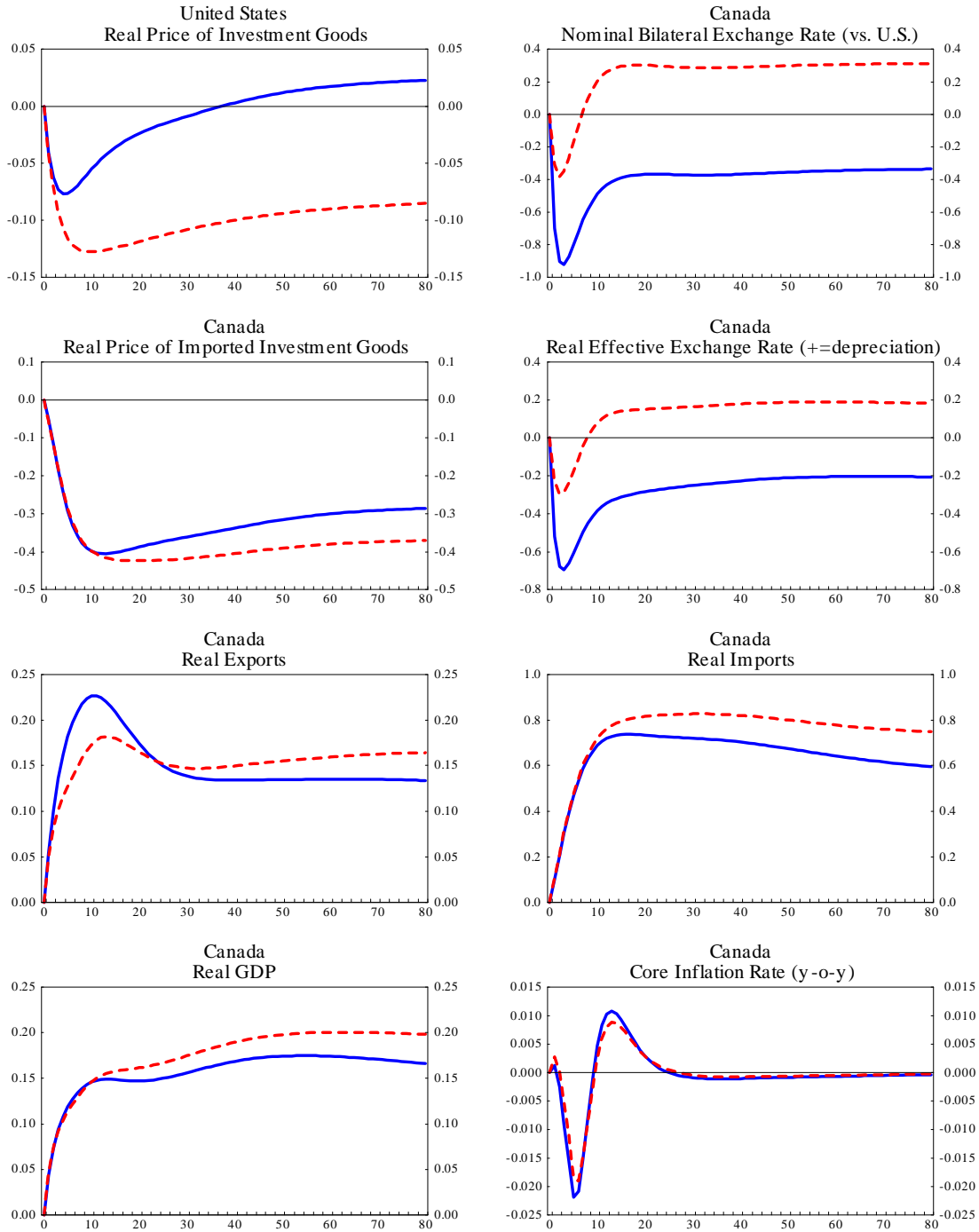


Figure 16: A Temporary World Consumption Shock and its Effects on Oil and Commodities Prices

(Deviation from control, in percent)

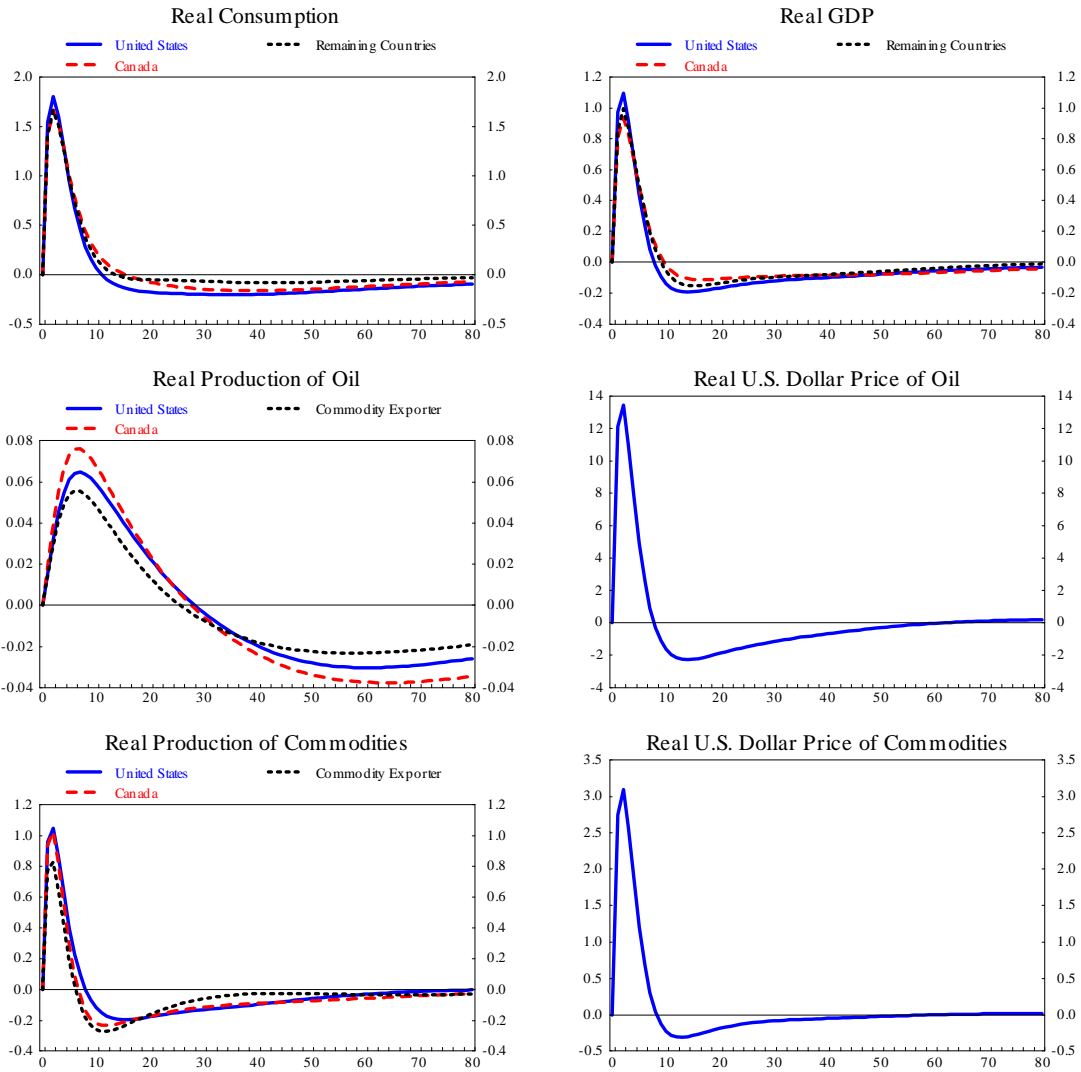


Figure 17: A Permanent Increase in the Productivity of the Commodity-Importing Regions

(Deviation from control, in percent)

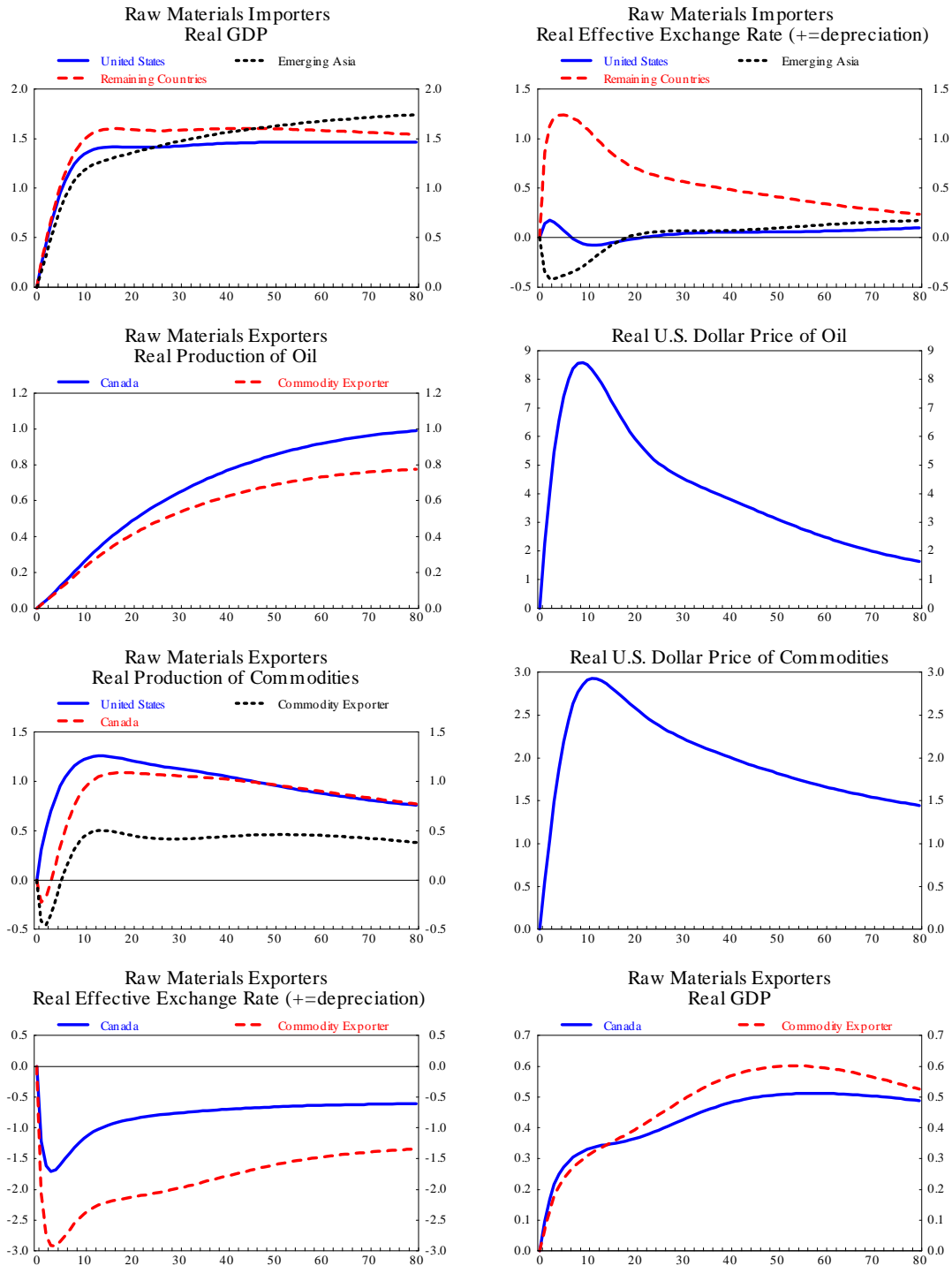


Figure 18: A Permanent Decrease in the Oil Production of the Commodity Exporter - Part I

(Deviation from control, in percent)

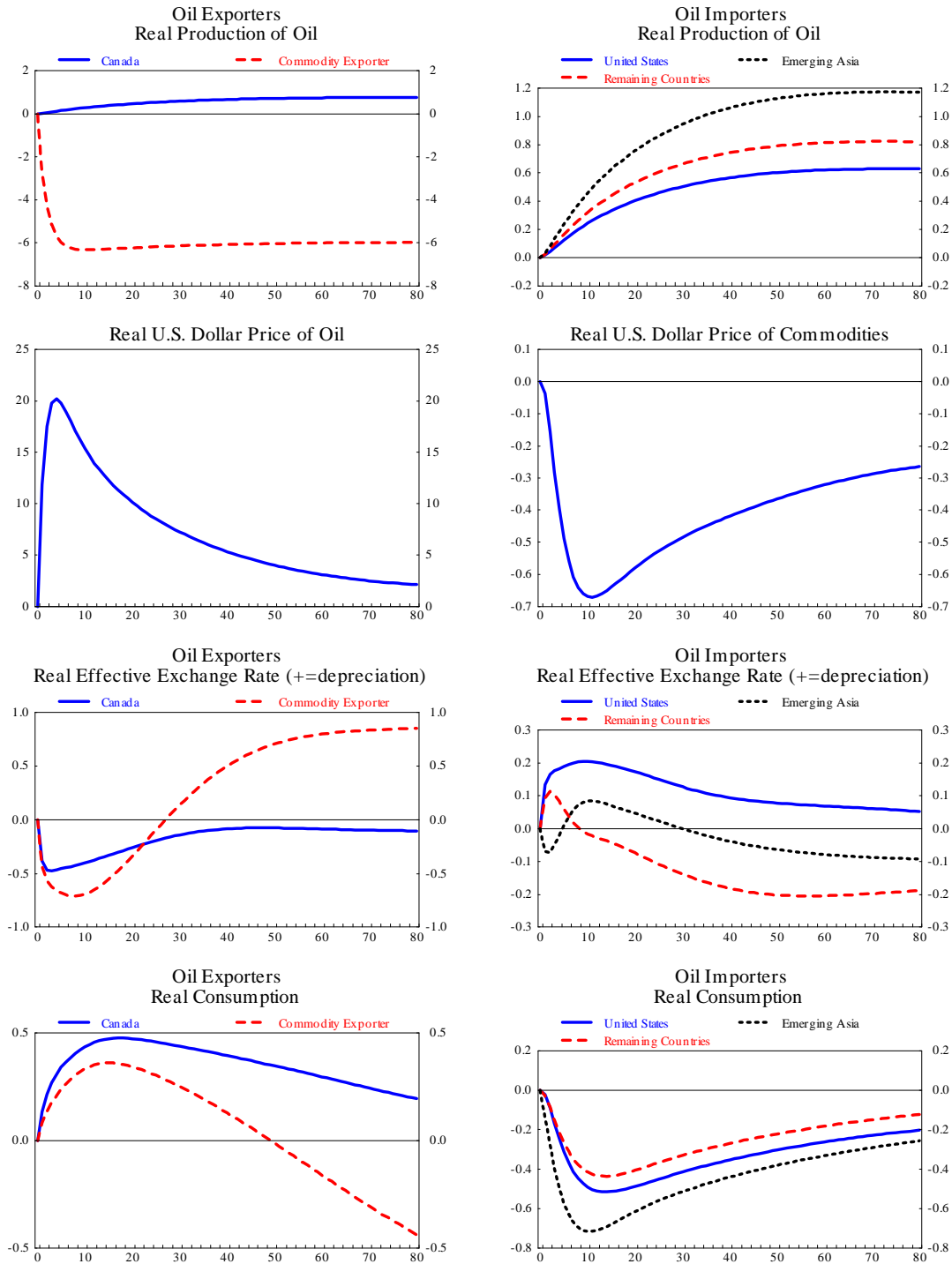


Figure 19: A Permanent Decrease in the Oil Production of the Commodity Exporter - Part II

(Deviation from control, in percent)

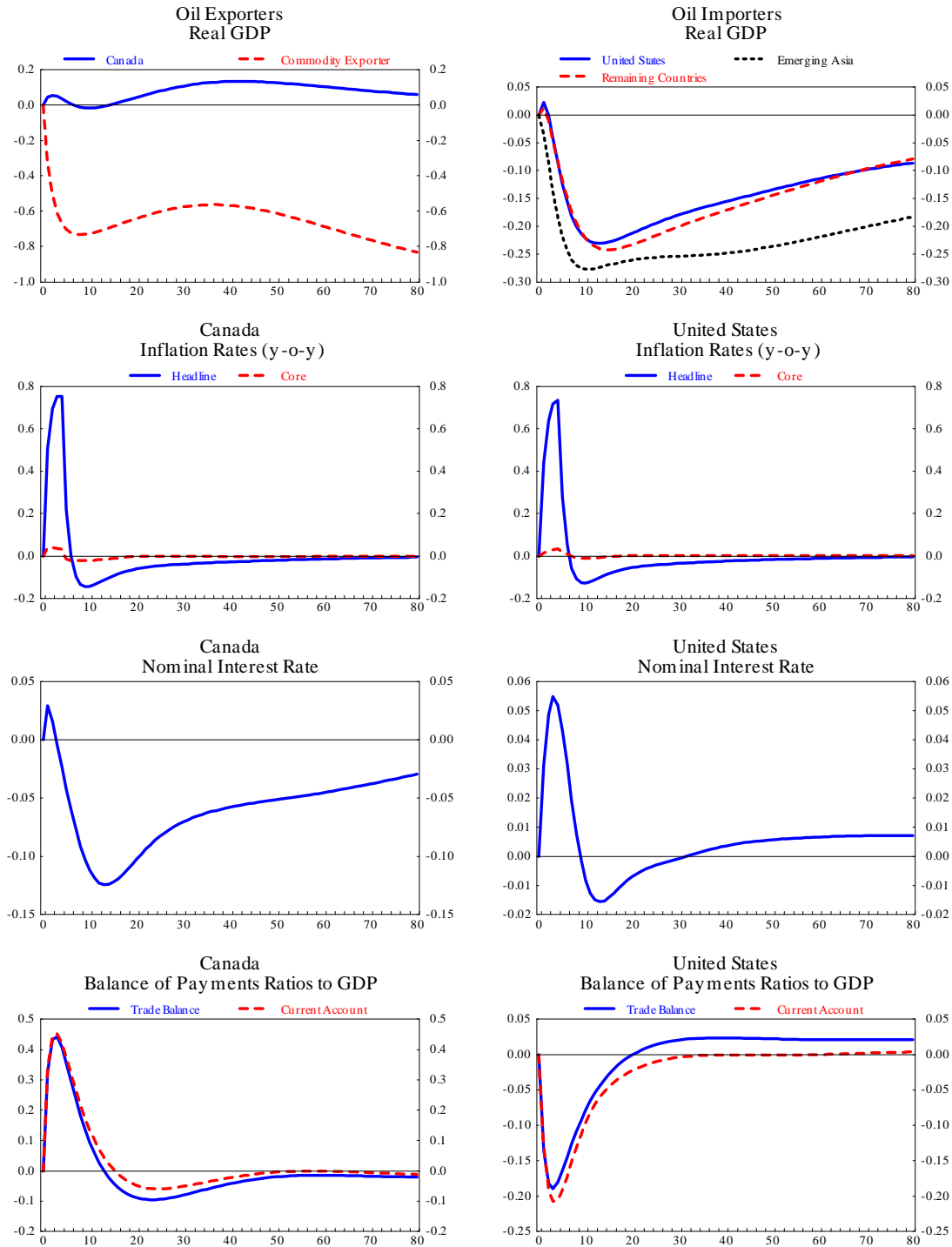


Figure 20: Permanent Decrease in the Commodities Production of the Commodity Exporter

Reduction of Oil Production in the Commodity Exporter (Solid)
 Reduction of Commodity Production in the Commodity Exporter (Dashed)
 (Deviation from control, in percent)

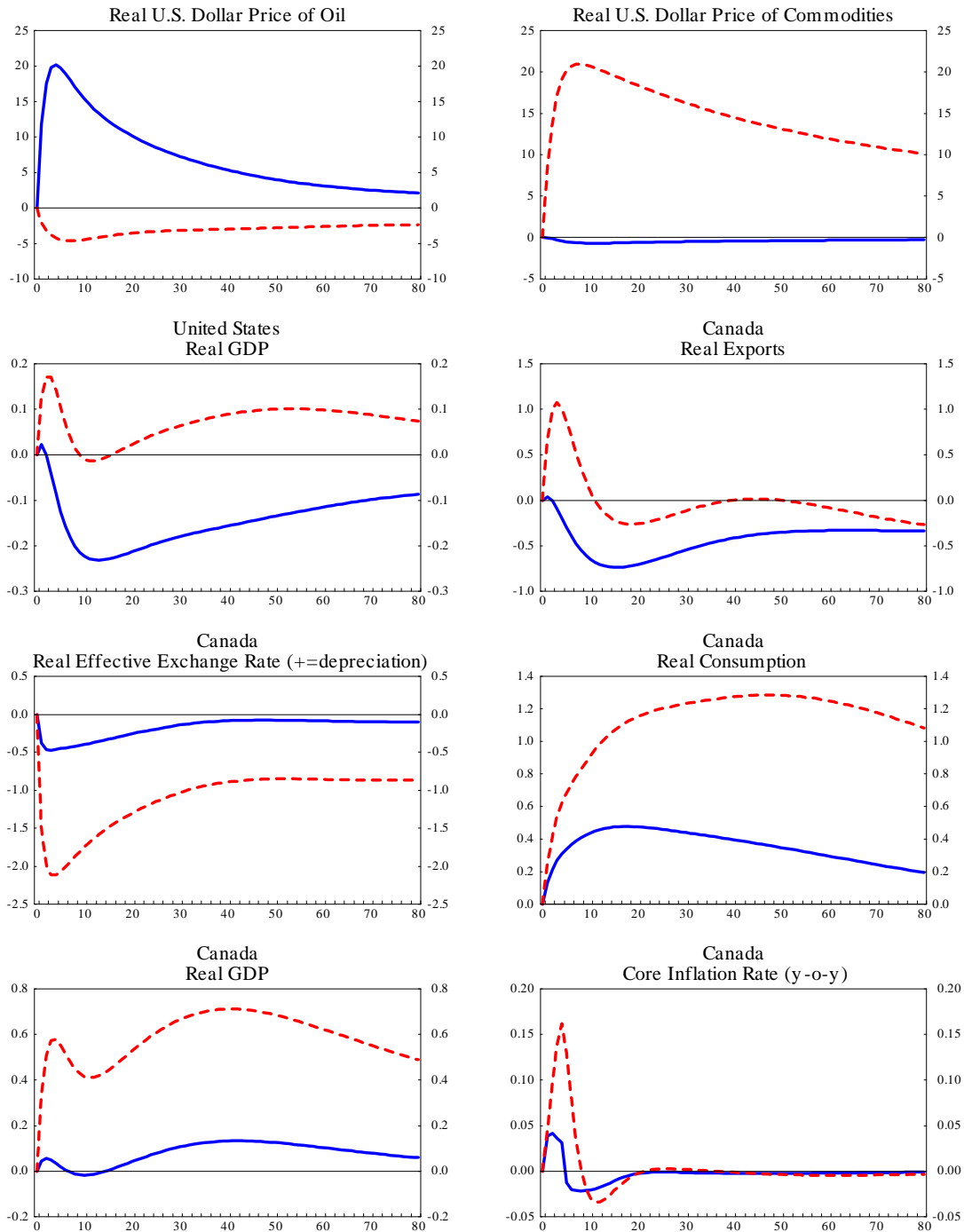


Figure 21: Permanent Increase in Productivity in Emerging Asia - Part I

(Deviation from control, in percent)

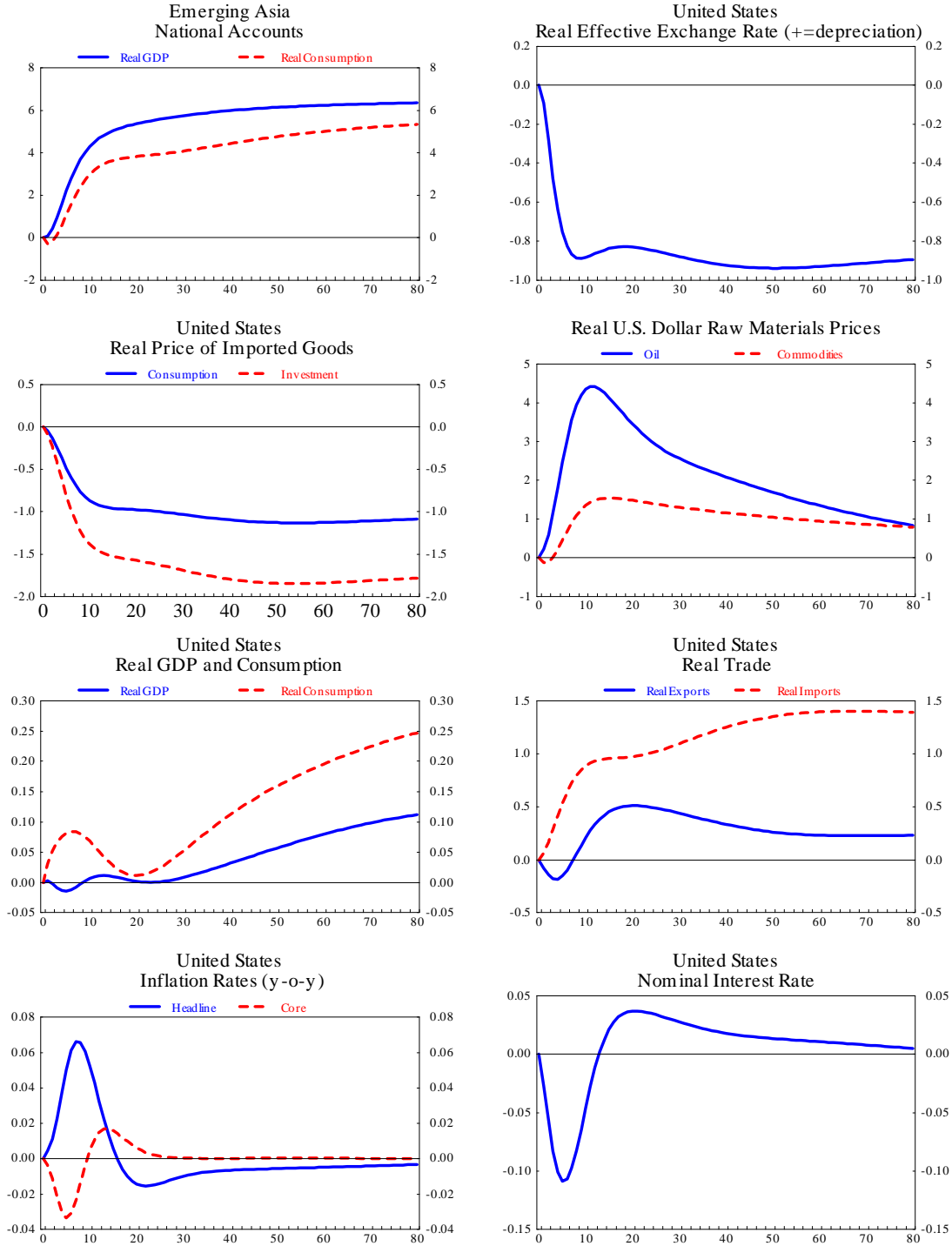


Figure 22: Permanent Increase in Productivity in Emerging Asia - Part II

(Deviation from control, in percent)

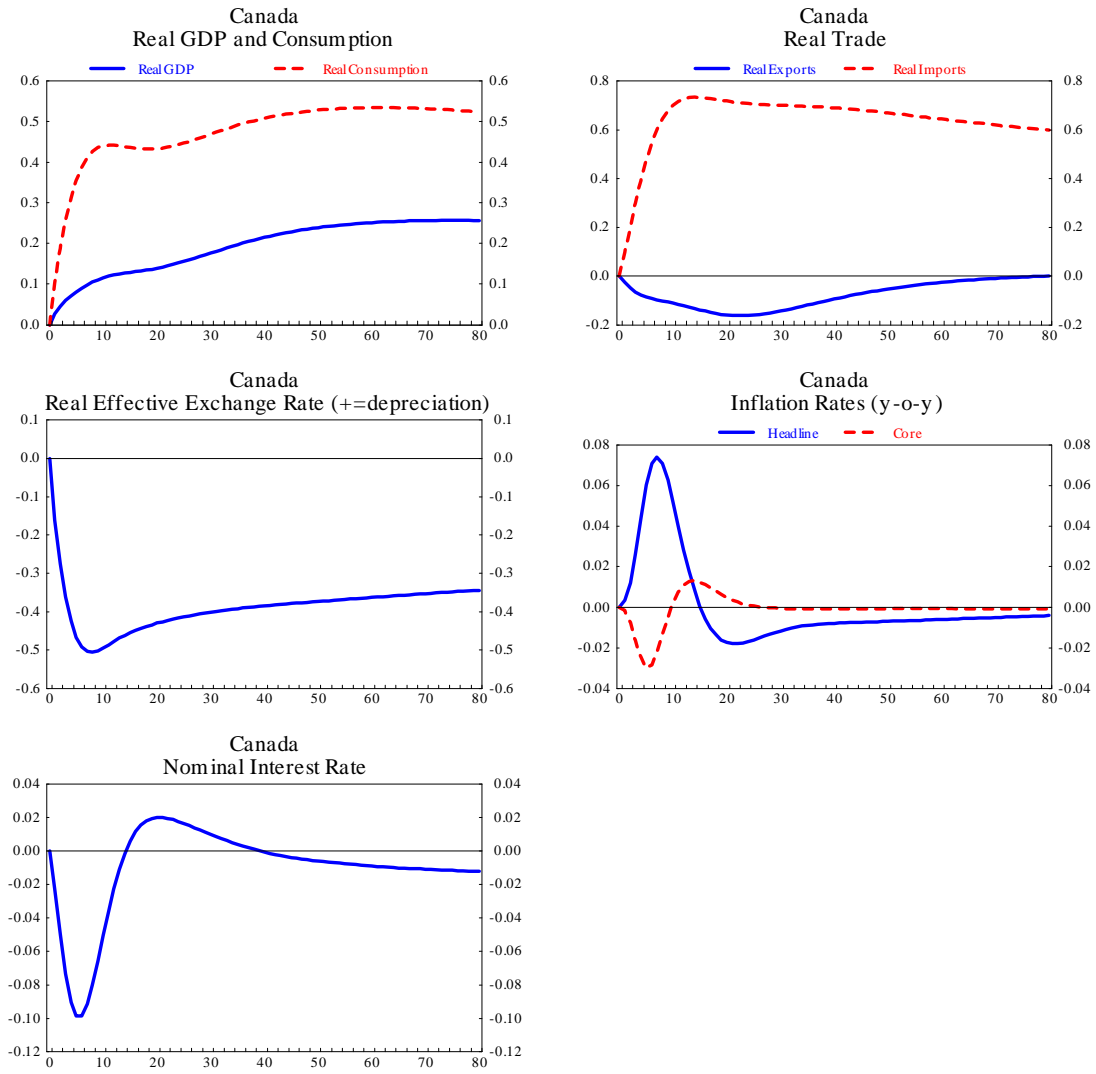


Figure 23: A Loss of Appetite for U.S. Assets Induced by a Higher U.S. Country Risk Premium - U.S. Effects

(Deviation from control, in percent)

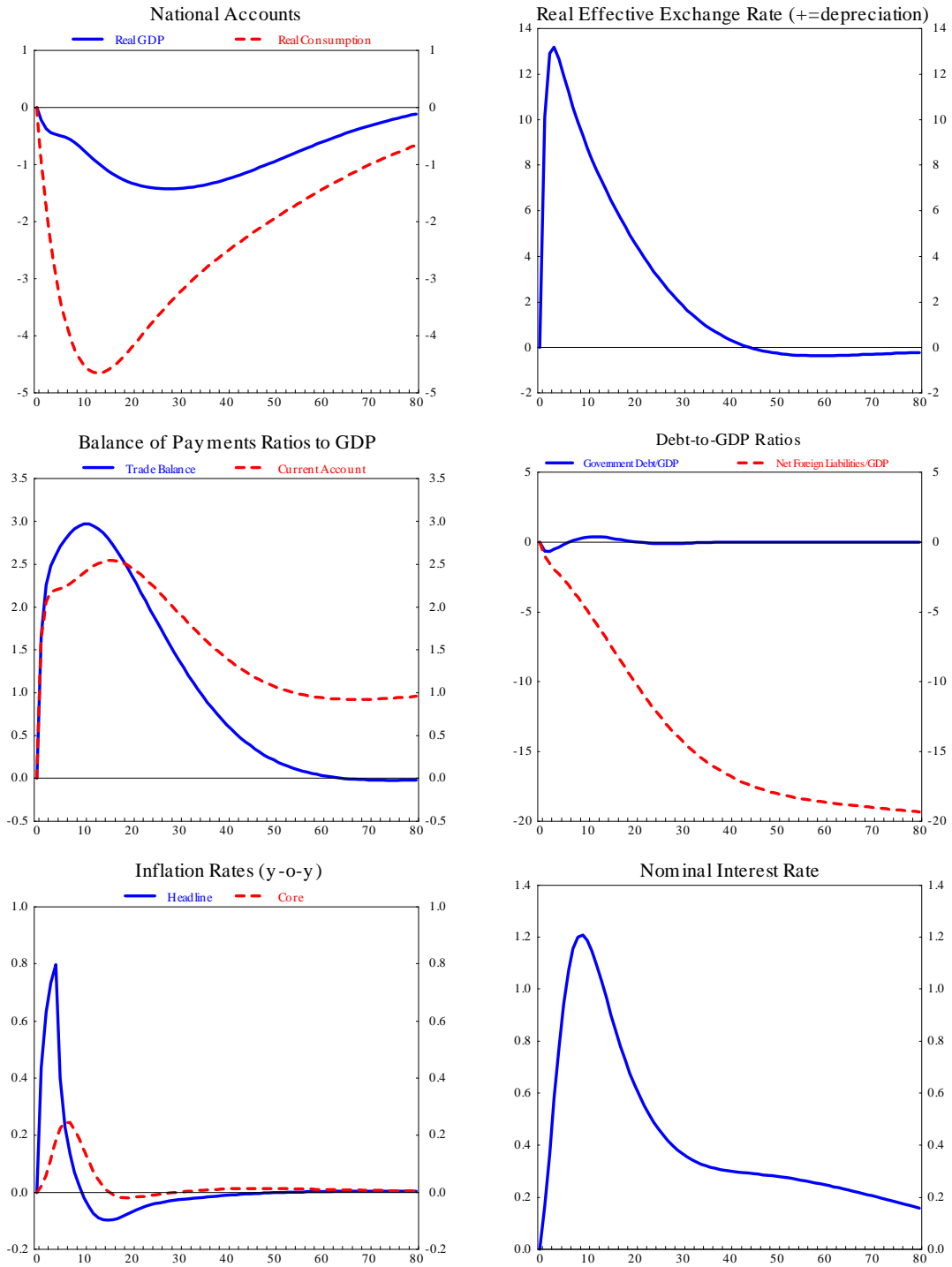


Figure 24: A Loss of Appetite for U.S. Assets Induced by a Higher U.S. Country Risk Premium - Effects in the Rest of the World

(Deviation from control, in percent)

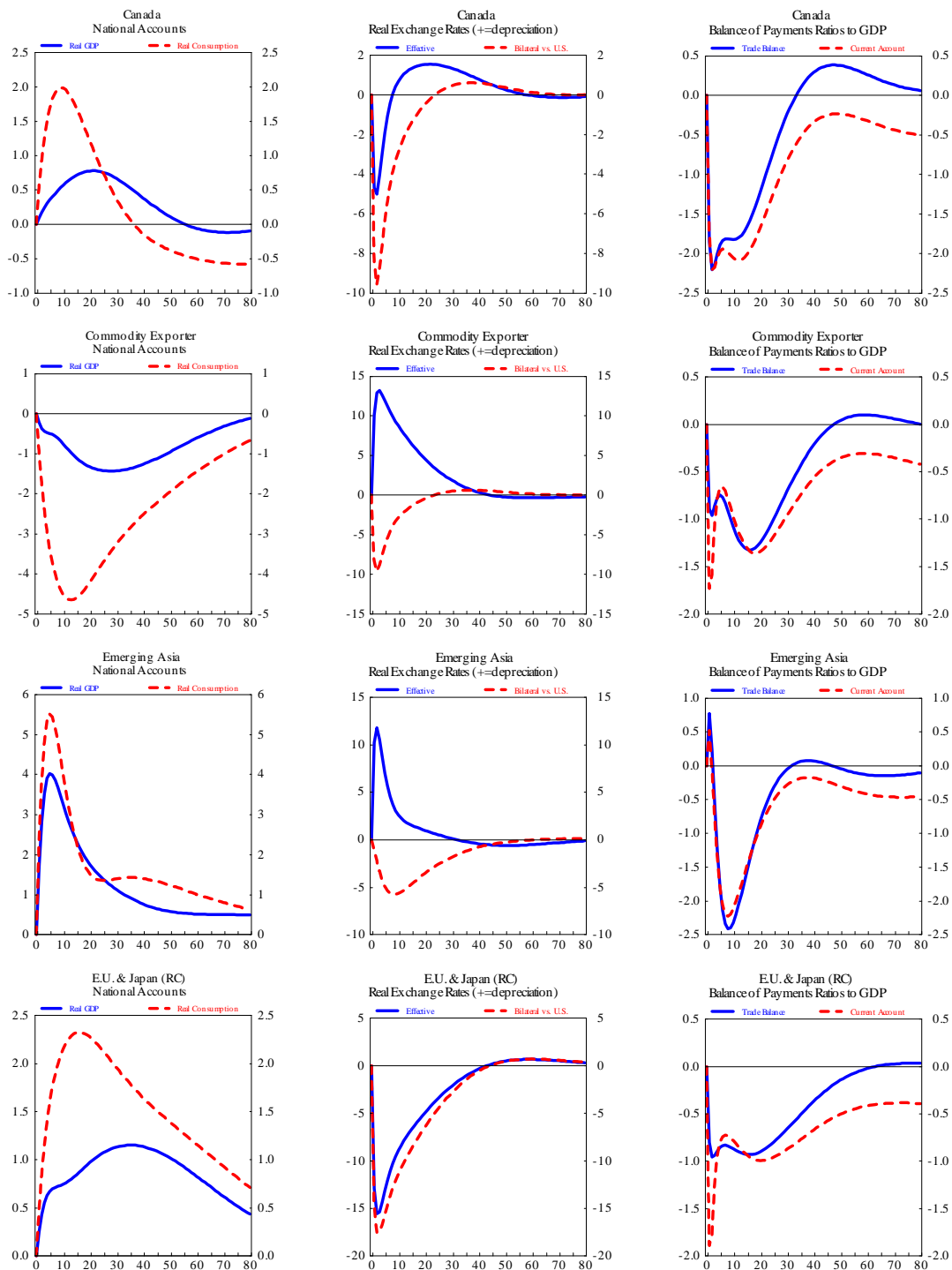


Figure 25: An Increase in Protectionism Worldwide - Part I

NAFTA collapses (Solid)
 NAFTA is maintained (Dashed)
 (Deviation from control, in percent)

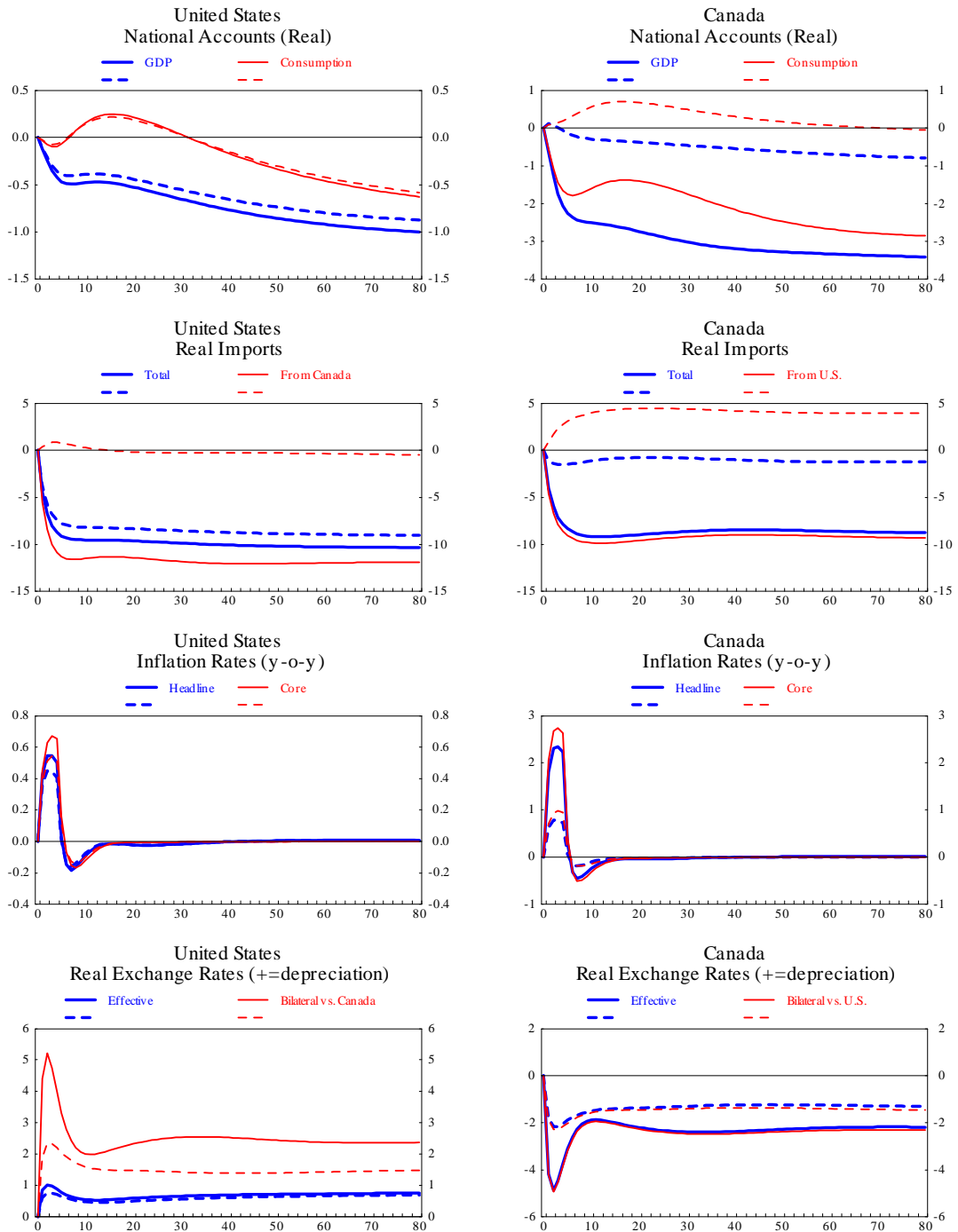


Figure 26: An Increase in Protectionism Worldwide - Part II

NAFTA collapses (Solid)
 NAFTA is maintained (Dashed)
 (Deviation from control, in percent)

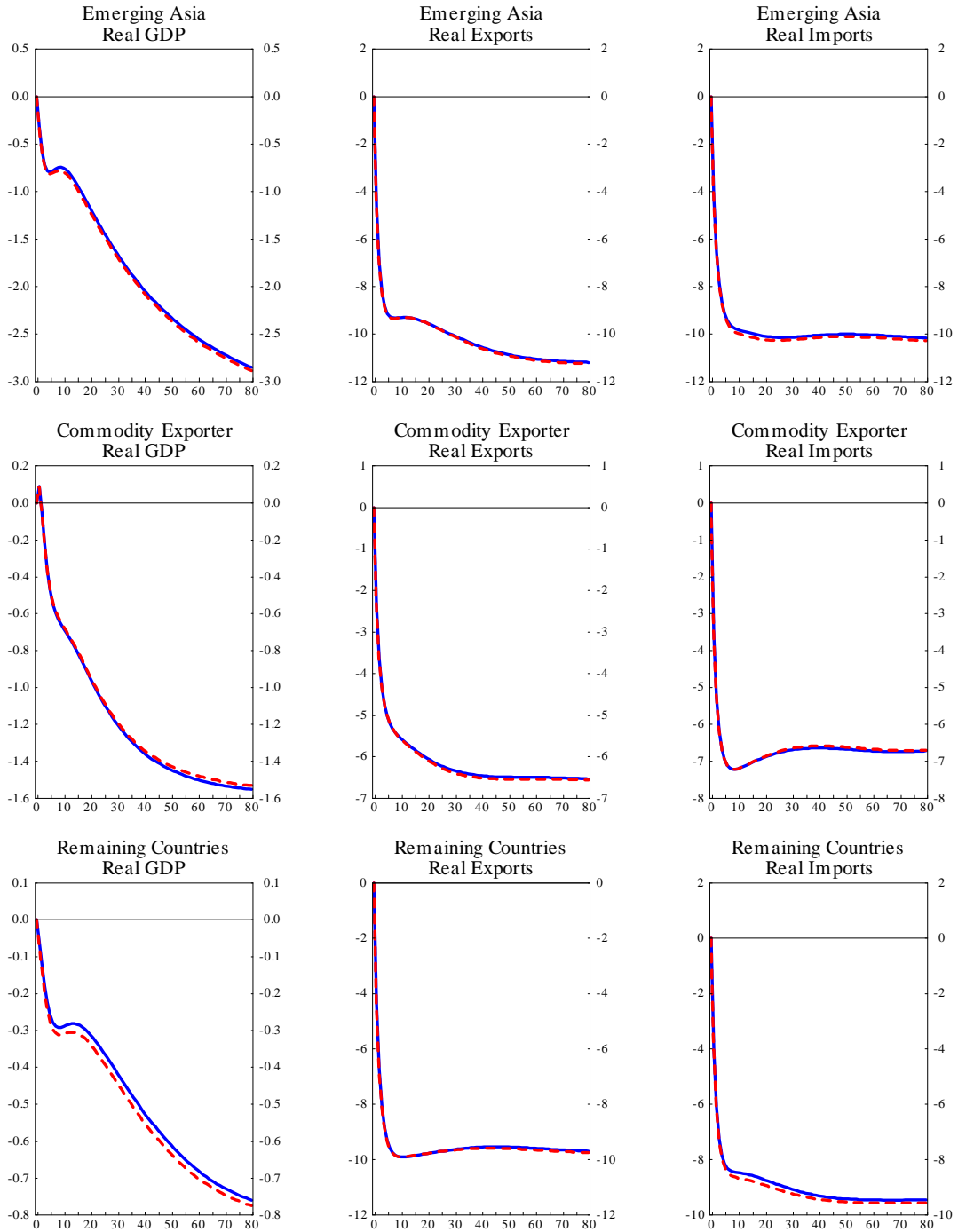


Figure 27: The Effects of a Fiscal Consolidation in the United States - U.S. Effects

(Deviation from control, in percent)

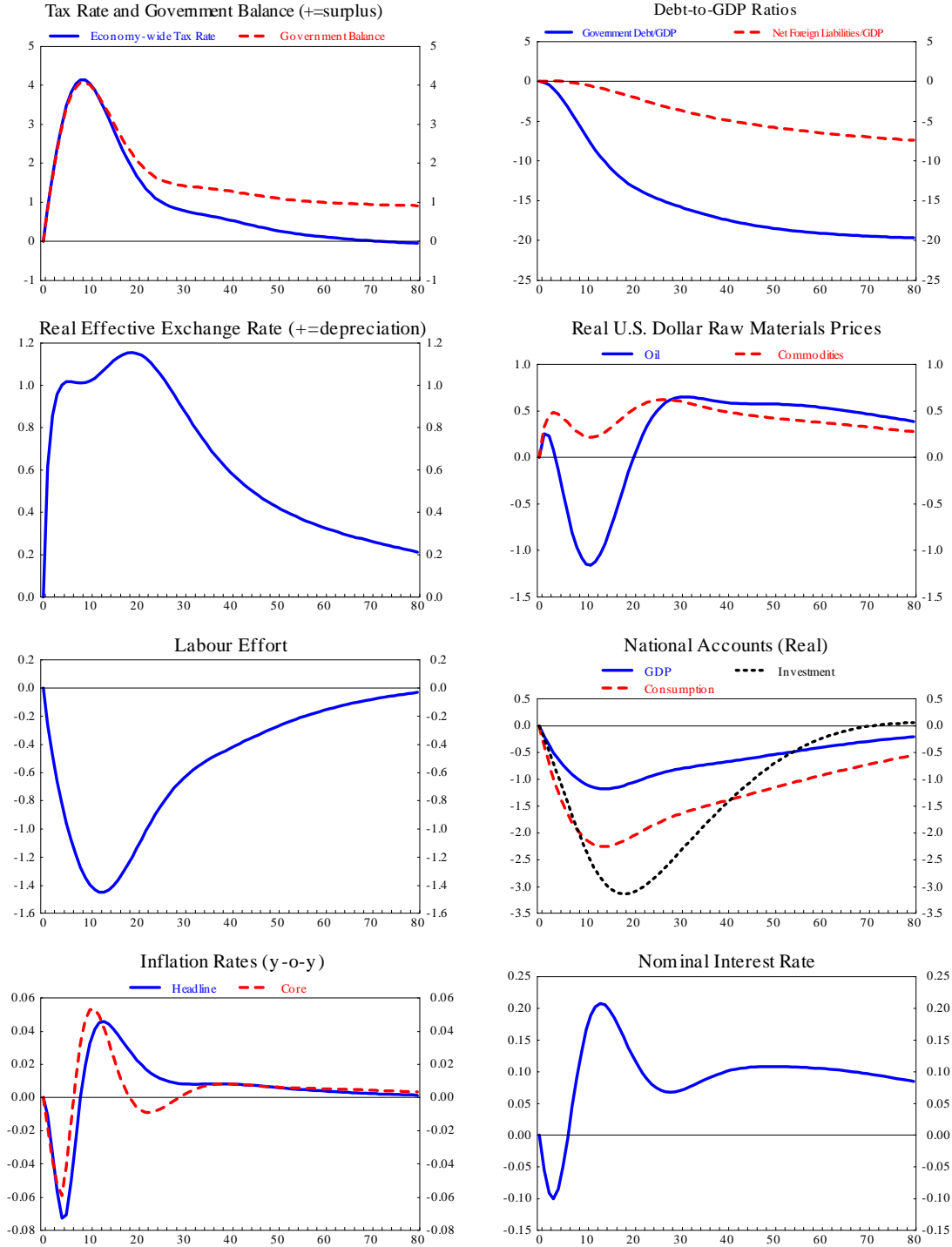
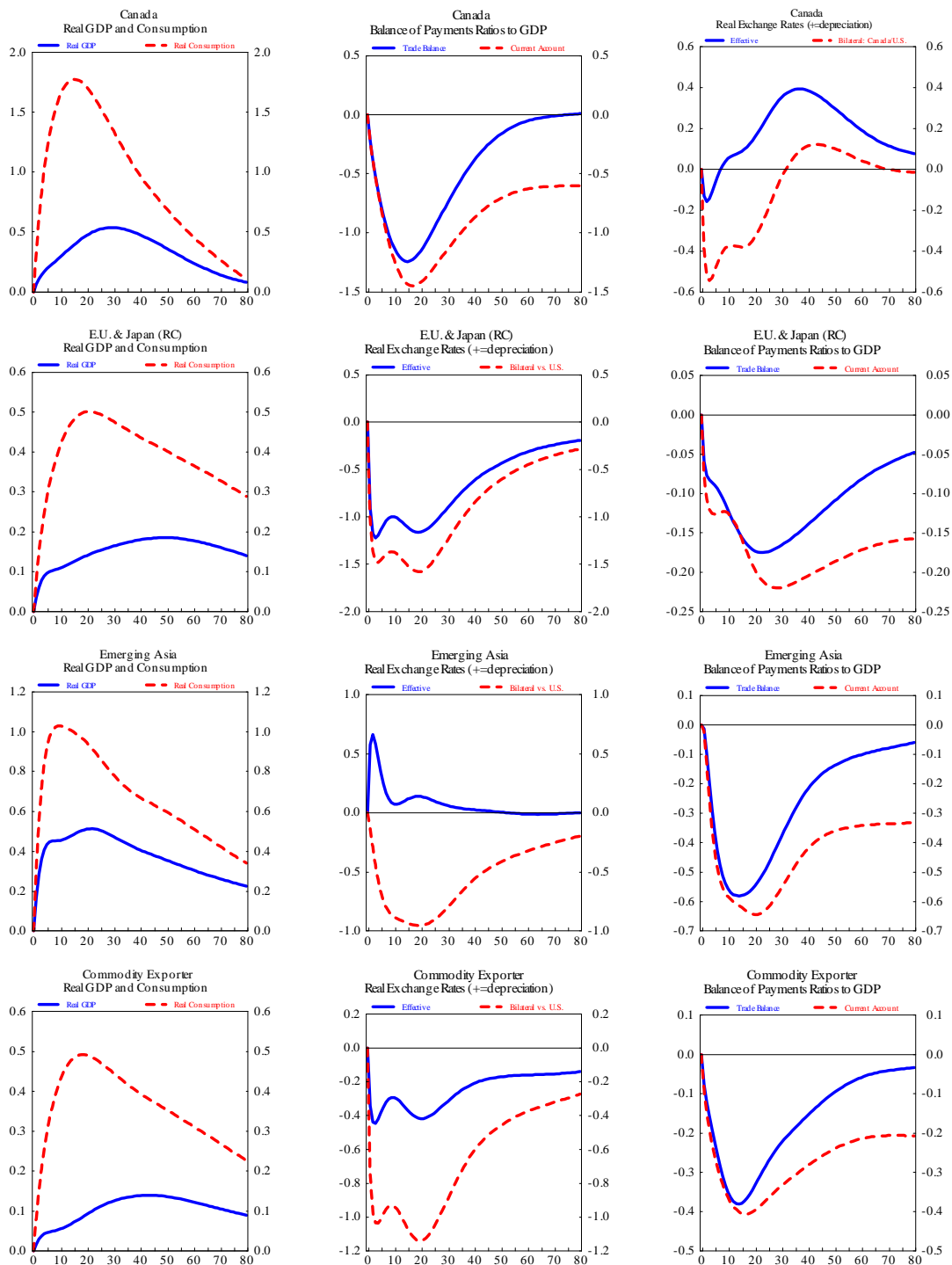


Figure 28: The Effects of a Fiscal Consolidation in the United States - Effects on the Rest of the World

(Deviation from control, in percent)



Appendix A: Composition of the regions in BoC-GEM

Five regions:

(i) CA – CAⁿada

(ii) US – United States

(iii) CX – Commodity eXporter = 22 countries

OPEC = Iran, Iraq, Kuwait, Libya, Nigeria, Qatar, Saudi Arabia, Venezuela (Note: data of good quality is not available for the United Arab Emirates)

Algeria; Argentina; Australia; Azerbaijan; Bahrain; Brazil; Chile; Indonesia; Mexico; New Zealand; Norway; Oman; Russia; South Africa

(iv) AS – emerging ASia (IMF definition, excluding Indonesia) = 8 countries

China; Hong Kong Special Administrative Region of China; India; Republic of Korea; Malaysia; Philippines; Singapore; Thailand

(v) RC – Remaining Countries

Includes all other countries in the world not yet named, but its properties in the model are focused mainly on members of the entire European Union (=25 countries) and Japan.

Appendix B: Volume, price and current dollar measures of the national accounts

To finish exploring the accounting structure of the BoC-GEM, we will consider once again the current account.

Expression (122) can be rewritten as:

$$CURBAL_t^H = \varepsilon_t^{H,US} \left(B_t^{*H} - \frac{B_{t-1}^{*H}}{\pi_{t-1,t}^{US} g_{t-1,t}} \right) = \frac{i_{t-1}^* \varepsilon_t^{H,US} B_{t-1}^{*H}}{\pi_{t-1,t}^{US} g_{t-1,t}} + TBAL_t^H \quad (B1)$$

The left hand side of (B1) is region H 's current account (in nominal terms, in consumption units). The first term on the far right hand side equation are net factor payments from the rest of the world to region H and $TBAL$ is the trade balance:

$$TBAL_t = EX_t - IM_t \quad (B2)$$

where total exports EX are defined by equation (124) and total imports IM are defined by equation (125). We define the model-based Gross Domestic Product (in consumption units) as before:

$$\begin{aligned} GDP_t &= A_t + p_{E,t} E_t + p_{N,t} G_{N,t} + EX_t - IM_t \\ &= p_{N,t} N_t + p_{T,t} T_t + (1 + \tau_{GAS,t}) p_{GAS,t} GAS_t + TBAL_{S,t} + TBAL_{O,t} \end{aligned} \quad (B3)$$

so that:

$$\begin{aligned} CURBAL_t^H &= TBAL_t^H + \frac{i_{t-1}^* \varepsilon_t^{H,US} B_{t-1}^{*H}}{\pi_{t-1,t}^{US} g_{t-1,t}} \\ &= GDP_t^H - (C_t^H + p_{E,t}^H I_t^H + G_t^H) + \frac{i_{t-1}^* \varepsilon_t^{H,US} B_{t-1}^{*H}}{\pi_{t-1,t}^{US} g_{t-1,t}} \end{aligned} \quad (B4)$$

While theoretically sound, this measure of output (and those measures for its associated components) bears little similarity to real GDP as measured by the system of national accounts in countries such as the United States and Canada. The problem is particularly severe for relatively open economies facing large swings in real exchange rates and relative prices. We would like to approximate as closely as possible We therefore construct ‘national

accounts' concepts for GDP and its components, whereby we create new definitions of GDP, consumption, investment, government spending, exports and imports (GDP , C , I , G , EX and IM respectively) along with their associated price indices. We construct them using the Tornqvist approximation to chain-weighted Fisher indices for the data.¹ Now for each region we have the following relationships between model concepts, current dollars and prices multiplied by volumes:

$$CPI_t GDP_t = GDP_t^{NOM} = p_{GDP,t}^{NAT} GDP_t^{NAT} \quad (B5)$$

$$CPI_t C_t = C_t^{NOM} = p_{C,t}^{NAT} C_t^{NAT} \quad (B6)$$

$$CPI_t p_{I,t} I_t = I_t^{NOM} = p_{I,t}^{NAT} I_t^{NAT} \quad (B7)$$

$$CPI_t p_{G,t} G_t = G_t^{NOM} = p_{G,t}^{NAT} G_t^{NAT} \quad (B8)$$

$$CPI_t EX_t = EX_t^{NOM} = p_{X,t}^{NAT} EX_t^{NAT} \quad (B9)$$

$$CPI_t IM_t = IM_t^{NOM} = p_{M,t}^{NAT} IM_t^{NAT} \quad (B10)$$

¹A straightforward and clear explanation of the use of chain-weighted Fisher indices by the U.S. system of national accounts can be found in Whelan (2000).