Gender Role in Production, Demographic Transition and Economic Take-off*

Henry Tam

Assistant Professor of Economics, York University, TEL 2021, 4700 Keele Street, Toronto, ON, Canada M3J 1P3. Contact information: TEL 1-416-736-2100 x30112; FAX 1-416-736-5188; EMAIL thenry@yorku.ca

Abstract

This paper develops a growth model to explain the U-shaped female labor participation with economic development, the long-run historical inverted-U relationship between population growth and income, and the contemporary bipolarization of world income distribution in which some countries fail to take off with women playing a passive role in production. The model features structural transformation from a pre-modern regime, in which population growth increases and female participation decreases with mechanization complementary to physical human capital, to a modern regime, in which mental human capital plays a more important role and gender inequality narrows. Poor institution will however delay the timing of modernization. Accordingly, some LDCs fail to take off even nowadays and are still trapped at the pseudo steady state of the pre-modern regime, where economic growth is meager and population growth is high.

JEL classification: J13; J16; O11; O40

Keywords: fertility choice; female labor participation rate; gender gap; human capital; economic take-off

*I would like to thank Juan Cordoba, Paula Hernandez-Verme, Dennis Jansen, Sebnem Kalemli-Ozcan, Robert Tamura, conference participants at the Canadian Economic Theory Conference, as well as seminar participants at Brock University, Rice University, Ryerson University, Texas A&M University, the University of Houston and the University of Waterloo, for their helpful comments and suggestions.

1. Introduction

The present paper develops an economic model to understand the long-run relationship between demographic dynamics and economic development by emphasizing women's role in child-rearing and in production. From time past to present, child-rearing has predominantly been intensive in women's time. If one looks at the history of the US over the past 100 years, one observes a negative association between population growth and female labor participation: labor participation rate of women 25 to 44 years old rises from about 15% in 1890 to about 80% in 2000 (Goldin 1990, 2002), while population growth rate declines from around 2.5% in the late 1800s to around 1% in the late 1900s (Maddison 1995, 2001). In a longer-run perspective, the historical data of the nowadays developed countries demonstrates that population growth first increases and then later declines with economic development (Galor and Weil 2000). Figure 1 provides evidence for such demographic transitions for some representative OECD countries from 500AD-1990AD. Although longer time series data on female labor participation is scarce, Goldin (1994) suggests that female labor participation rate might have decreased during the early part of US history. Cross-country data in the post WWII period provides stronger evidence for a corresponding U-shaped relationship between female labor participation and economic development (Goldin, 1994, Mammen and Paxson, 2000). Figure 2 documents such U-shaped female labor participation with economic development. Figure 2 also illustrates an inverted-U relationship between population growth and per capita income in cross-country data.[1]

<<Insert Figures 1 and 2 about here>>

Theoretical works on fertility choice that link economic development with population dynamics, however, are mostly set up in a "genderless" framework. From the early models of Barro and Becker (1989), Becker et al. (1990) and Ehrlich and Lui (1991) to the more recent works of

[1] One might observe the inverted-U relationship between population growth and development is less apparent in the cross-country data. Moreover, the positions of the turning points are not the same in the U-shaped female participation function with economic development as in the inverted-U relationship between population growth and economic development in the cross-country data. However, we shall see later that such difference in the position of the turning point in fact follows directly from the present theory.

Galor and Weil (2000), Jones (2001), Lucas (2002) and Tamura (2002), parental altruism and the trade-off between the quantity and quality of children in "genderless" households constitute the key underlying mechanisms that are used to explain the demographic transition. One exception is Galor and Weil (1996), who model fertility choice by appealing to the increase in opportunity cost of having children due to the increase in the relative wages of women in modern economy. In their model, the substitution effect of rising wages dominates the income effect and leads population growth to fall with economic development. However, in their model, income growth is exogenous as in the Solow model. Moreover, they only model the decline in population growth with economic development in the more modern era, but not the longer-run inverted-U relationship between population growth and economic development.[2]

The present theory purports to endogenize fertility choice, gender education and labor participation choices, and structural transformation and growth in a framework of economic development to explain not only the inverted-U relationship between population growth and economic development, but also some fuller demographic dynamics including the U-shaped female labor participation with economic development and the evolution of gender inequality in education and earnings.

The model features two economic regimes: a pre-modern regime and a modern regime. Correspondingly, the model features two types of human capital: physical human capital and mental human capital. In the pre-modern regime, machines and physical human capital are complementary factors in the production function, whereas in the modern regime, machines and mental human capital are complementary factors of production. One could think about the former as mechanization in agriculture, mining and other heavy industries, in which bodily strength plays an important role in machine-usage, while the latter is the adoption of electric motors, computerization and the like in manufacturing and services which requires mental capability to work the machines.

[2] Lagerloff (2003) brings gender issue into a model of such longer-run inverted-U relationship between population growth and economic development. However, in his model, there is no intrinsic difference between the sexes. Ex post gender differences and hence the sexual division of labor and the dynamics of gender inequality are therefore exogenously determined by some outside factor.

One important difference between the two types of human capital is their differential endowments between the sexes. It is assumed that whereas only male is endowed with physical human capital and female is not, there is no difference in the initial endowment of male and female in mental human capital. The implication of such assumptions is that in the pre-modern regime, as machines complementary to physical strength accumulate, income rises and the relative wages of women fall. As a result, both the income and substitution effects lead households to have more children. Population growth increases with economic development and female labor participation declines. On the other hand, the modern regime uses mental human capital and machines. As mental human capital accumulates, the opportunity cost of women in rearing children rises. If the substitution effect of rising relative wages of women overwhelms the income effect, the rate of population growth falls, while female education and labor participation increase.[3]

Another important difference between the two types of human capital is that while physical human capital has limited potential for improvement, the same is not true for mental human capital accumulation. For simplicity, physical human capital endowment is assumed invariant with time. On the contrary, mental human capital accumulates in a fashion similar to Lucas (1988), where intergenerational externality plays an important role. The implication of such difference is that the engine of growth in the pre-modern regime is mainly physical capital accumulation and, like the Solow model without exogenous productivity changes, diminishing marginal returns eventually set in and the pseudo steady state of such pre-modern regime exhibits zero economic growth. On the other hand, economic growth in the modern regime is spurred by the accumulation of mental human

[3] The mechanisms underlying the population growth dynamics are therefore similar to but also substantively different from Galor and Weil (1996). First, both the income and substitution effects work in the same direction in the pre-modern regime in the present model, whereas in Galor and Weil (1996) the income effect and substitution effect work against each other. Second, the effect of the increase in relative wages of women in the modern regime is similar to Galor and Weil (1996). However, in Galor and Weil (1996), that is the only effect. In the present model, because productivity change is not exogenous but is dependent on gender education choice, there is an additional (resource) income effect of increased education (for both male and female) that induces lower population growth in the modern sector. Moreover, in the present model, relative wages of women rise in the modern regime mainly due to increase in relative skills of women, whereas in Galor and Weil (1996), relative wages of women rise due to increase in the skill premium in an exogenous growth model without education choice. We shall elaborate on these differences in later sections.

capital and physical capital and, like most endogenous growth models following Lucas (1988) and Romer (1990), long-run economic growth is procured.

Lastly, the model features structural transformation from a Solow-type model to a Lucas-type endogenous growth regime.[4] Structural transformation takes place because of "directed mechanization".[5] In the present model, machines are "skill"-specific. Investment in machines complementary to physical human capital will take place if and only if the marginal product of capital in the pre-modern regime is higher than that in the otherwise modern regime. At the pseudo steady state of the pre-modern regime, physical capital accumulates at the same rate as population increases and the marginal product of capital in the pre-modern regime is constant. On the other hand, because mental human capital accumulates with intergenerational externality, there is an additional scale effect. When population increases past a threshold, the marginal product of capital in the modern regime will be higher than that in the pre-modern regime. At this point, the economy switches to a modern regime, where mental human capital plays an important role, and the economy takes off to an eventual balanced growth path.[6]

The present model is capable of generating the long time series pattern of development in the nowadays developed countries. During the pre-modern age, female labor participation is relatively high, but women remain "unskilled" all the while. Female labor participation declines and the rate of population growth increases as the economy grows by physical capital accumulation. Eventually, diminishing marginal returns set in and the pre-modern age reaches a pseudo steady state that features meager economic growth, high population growth and low female labor

[4] The current structural transformation model is therefore closely related to Hansen and Prescott's (2002) seminal work "Malthus to Solow".

[5] Acemoglu (2002) puts forth a similar idea of "directed technological change", in which skill-biased technological change in recent decades happens endogenously when the supply of skills creates a large enough market for investment in skill-biased type of technology to be profitable. Here, when intergenerational externality is large enough, investment in machines complementary to mental human capital becomes profitable and mechanization will be directed towards such ends.

[6] The switch-of-regime mechanism, which depends on scale effect, is therefore similar to Goodfriend and McDermott (1995), who are the first to model industrialization and modernization in an endogenous structural transformation framework. However, in their model, population dynamics is exogenous.

participation. As population continues to grow, however, intergenerational externality eventually becomes large enough and the economy switches to a modern regime. Although initially women, because of their role in rearing children, have lower schooling and labor participation than men in the modern era, as the economy grows through mental human capital accumulation, female education and labor participation increase, while the rate of population growth falls. Because the fall in population growth rate releases time for women disproportionately, the rate of increase in female mental human capital accumulation will be faster than that for male. Gender inequality in earnings therefore narrows. It is important to notice that although female labor participation is high both during the early part of the pre-modern age and the later part of the modern era, the nature of their participation is very different. During the pre-modern age, female labor is unskilled and their relative wages fall relative to the male counterpart. During the modern era, female participation is accompanied by rising skills and education of women, and their relative wages increase relative to that of men.

The present model could also be used to shed light on the cross-sectional difference in the patterns of development among the world. In the model, productivity parameters determine the timing of modernization. Following the empirical studies of Hall and Jones (1999) and Acemoglu et al. (2001), differences in institutional arrangement that determine the country-specific productivity parameters could therefore potentially explain the enormous income disparity across countries. Accordingly, Western Europe and North America industrialize and modernize first because of their efficient institutions. As a result, Western Europe and North America experience persistent economic growth, declining population growth, rising female labor participation, reduction in gender inequality and improvement in education for both male and female for more than a hundred years. This explains the Great Divergence between the developed countries and the LDCs since the Industrial Revolution.

More recently, some newly industrialized countries (NICs) succeed to take off in the last fifty or so years because of their better or improved institutions. On the contrary, other LDCs fail to modernize even nowadays because of their poor or unimproved institutions. These LDCs are still trapped in the pseudo steady state of the pre-modern regime, where population growth is high, female labor participation is largely unskilled and economic growth is meager. This accounts for the more recent observation that the world income distribution has become bipolar (Quah 1996) with the NICs joining the league of advanced nations and the rest of LDCs lagging behind. Moreover, in accordance with the present model, the pattern of development, where economic take-off is accompanied by the demographic transition and the rise of skills and labor participation of women, is shared by the early modernizers as well as the NICs. As Young (1995) notes, rising female labor participation and education are key determinants of the extraordinary growth in East Asia.

Figure 3 illustrates the relationship between economic take-off and the demographic transition for a number of representative countries. For the early modernizers, demographic transitions took place in the mid- to late 1800s and their economies took off since. For the NICs, economic take-off and demographic transition took place in the mid 1900s. For other LDCs, demographic transition has not taken place yet and their economies stagnate.

<<Insert Figure 3 about here>>

The layout of the paper is as follows. Section 2 sets up the model specification. In Section 3, we provide the analysis for the pre-modern regime. Section 4 describes the dynamics in the modern era. Section 5 analyzes the transition from the pre-modern to modern regime. The time series and cross-country implications of the present model will be discussed in Section 6. Section 7 concludes.

2. Model Specification

We consider a closed economy with overlapping generations of couples. Time is discrete. Each generation t consists of a continuum of individuals with population measure N_t, with half the population as male and half female. Marriage is monogamous and there are no singles in any generation. Economic activities of each generation take place in two periods. In the first period, each couple decides on the allocation of time to invest in education, to rear children and to work.

There is no consumption when young and couples save all their earnings. They also decide on what types of physical capital to invest in. In the second period, they retire and consume their savings.

To focus on the effect of relative wages on fertility choice, we abstract from altruistic and old-age support motive. Utility of each couple depends only on the couple's consumption when old and the number of children they have, but not on the consumption of their children or the utility of their children. Thus, each couple maximizes the per couple utility u_t given by

$$u_t = \gamma \ln n_t + (1-\gamma)\ln c_t,$$

where n_t is the number of survived children (i.e. the net fertility rate or equivalently the gross population growth rate), c_t is consumption of the couple when old and $0 < \gamma < \frac{1}{2}$.[7] The net fertility rate is $n_t = b_t - d_t(y_t)$, where b_t is the the gross fertility (birth) rate and d_t is the infant mortality (death) rate, the latter depending exogenously on per capita income y_t. Even though b_t is the actual choice variable, since the mortality rate is exogenously determined in a deterministic fashion, the gross fertility choice b_t will determine the net fertility rate n_t. Thus, without loss of generality, we treat it as if the couple is making a choice on net fertility rate n_t.[8] The time constraint of an individual i is given by

$$1 = \sum_j \tau_{jt}^i = \begin{cases} \tau_{ct}^i + \tau_{pt}^i & \text{in the pre-modern regime} \\ \tau_{ct}^i + \tau_{mt}^i + \tau_{\text{ht}}^{\text{i}} & \text{in the modern regime} \end{cases}$$

where the superscript i=m or f refers to the male and female in each couple and the subscript j refers to the type of economic activities one engages in at time t. Accordingly, τ_{ct}^i is i's time spent in

---

7 Following Galor and Weil (1996), utility is additive in the log. This simplifies the analysis by making fertility, education and work hours choice independent of the interest rate. In fact, such choices will be independent of the interest rate insofar as utility is additive separable and is log in consumption. The log in number of children is not necessary. The restriction of γ to be less than 1/2 is made to ensure that women will not always spend all the time rearing children.

8 The distinction between gross fertility rate (b_t) and net fertility rate (n_t) is empirically important though. As Kalemli-Ozcan (2002) notes, the decline in gross fertility rate needs not be an indication of a true demographic transition. If the mortality rate declines faster than the decline in gross fertility rate, the net fertility rate and hence population growth will increase.

children-rearing, τ^i_{pt} is i's time spent working in the pre-modern regime, τ^i_{mt} is i's time spent working in the modern regime, and τ^i_{ht} is i's time spent in education in the modern regime. Child-rearing requires a fixed time cost z per survived child. Total time cost of child-rearing per couple is therefore zn_t.

There are two types of human capital: physical human capital and mental human capital, with gender differences in human capital endowments. We assume that only male is endowed with physical human capital and such endowment is fixed. Thus, we have

$$h^m_{pt} = h_p \text{ and } h^f_{pt} = 0 \ \forall t$$

where h^m_{pt} and h^f_{pt} are the physical human capital endowments per male and female respectively.

On the other hand, both male and female are endowed with the same initial mental human capital, $h^m_{m0} = h^f_{m0} = h_{m0}$. Unlike physical human capital, mental human capital accumulates according to

$$\hbar^i_{mt} = D\left(\frac{\mathcal{H}_{m,t-1}}{R} + h_{m0}\right)^{\beta} \tau^i_{ht},$$

where $\hbar^i_{mt}$ is the improvement in mental human capital for i due to education and training and $\mathcal{H}_{m,t-1}$ is the aggregate human capital of the previous generation given by

$$\mathcal{H}_{m,t-1} = \frac{N_{t-1}}{2}\left(h^m_{m,t-1} + h^f_{m,t-1}\right),$$

where

$$h^i_{m,t-1} = \hbar^i_{m,t-1} + h_{m0}$$

is mental human capital (initial plus improvement) of individual i of generation $t-1$. $D>0$ is a productivity parameter of this human capital accumulation sector, whereas $R>0$ is a conversion factor. For example, if R measures the area of an economy, then it will be the density of human capital of the previous generation that affects the productivity of human capital accumulation.

According to (2), even though initial mental human capital endowment is the same for male and female, an individual's ultimate level of mental human capital depends on how much time he/she spends in education. Mental human capital accumulation exhibits intergenerational externality, where aggregate human capital of the previous generation affects the productivity of the mental human capital accumulation. Unlike Lucas (1988), $\beta \neq 1$ but is a constant $\in$ (0,1). This creates non-scale economic growth along the balanced growth path as in Jones (1995).

There are two regimes of production. The production function in each regime is given by

$$Y_{pt} = \left[\left(A_p L_{pt} \right)^{\sigma} + \left(K_{pt}^{\alpha} \left(B_p H_{pt} \right)^{1-\alpha} \right)^{\sigma} \right]^{\frac{1}{\sigma}},$$

$$Y_{mt} = \left[\left(A_m L_{mt} \right)^{\sigma} + \left(K_{mt}^{\alpha} \left(B_m H_{mt} \right)^{1-\alpha} \right)^{\sigma} \right]^{\frac{1}{\sigma}},$$

where Y_{pt} is production in the pre-modern regime and Y_{mt} is production in the modern regime. Parameters are restricted to be $0<\sigma<1$, $0<\alpha<1$, and A_p, B_p, A_m and B_m are positive productivity parameters. The production function in the each regime r is a special case of a CES production function $Y_{rt} = \left[\left(A_r L_{rt} \right)^{\sigma} + \left(\alpha K_{rt}^{\rho} + (1-\alpha)\left(B_r H_{rt} \right)^{\rho} \right)^{\frac{\sigma}{\rho}} \right]^{\frac{1}{\sigma}}$, where $1 \geq \sigma > 0 > \rho = 0$. The production functions therefore exhibit capital-"skill" complementarity because $\sigma > \rho$. Machines are "skill"-specific and are complementary to the "skills" in that regime, where the relevant "skills" are physical human capital in the pre-modern regime and mental human capital in the modern regime.

Production in the pre-modern regime depends on L_{pt} (the total hours of raw labor used), H_{pt} (the total hours of physical human capital used), and K_{pt} (the amount of skill-specific machines used). Because only male is endowed with physical human capital, we have

$$L_{pt} = \frac{N_t}{2}\left(\tau_{pt}^{m} + \tau_{pt}^{f} \right),$$

$$H_{pt} = \frac{N_t}{2} h_p \tau_{pt}^m,$$

Similarly, H_{mt} is total hours of mental human capital used in modern regime and K_{mt} is the amount of skill-specific machines used in the modern regime. L_{mt} is the total hours of "unskilled" labor used in the modern regime. "Unskilled" labor in the modern regime consists of time usage of raw labor as well as bodily strength. This drives a wedge between the unskilled wages for female and male and produces the sexual division of labor in the modern regime. This assumption accords with Goldin's (2002) observation of a rather constant wedge between female and male wages in "brawn" jobs in modern economy. Thus, we have

$$L_{mt} = \frac{N_t}{2}\left(E_m h_p \tau_{mt}^m + \tau_{mt}^m + \tau_{mt}^f\right),$$

$$H_{mt} = \frac{N_t}{2}\left(h_{mt}^m \tau_{mt}^m + h_{mt}^f \tau_{mt}^f\right),$$

where E_m is a positive productivity parameter.

3. Analysis of the pre-modern regime

3.1. Couple decision in pre-modern regime

In this section, we shall first analyze the dynamics in the pre-modern regime. The existence of the pre-modern regime and when it will switch to the modern regime will be discussed in Section 5. This section assumes its existence and its continuation throughout. There is no education choice in the pre-modern regime. Moreover, only male is endowed with bodily strength. There is therefore a fixed "skill" differential between male and female in the pre-modern regime. If female spends a marginal hour in production, she will only earn $w_{pt}^u = MPL_{pt}$, where w_{pt}^u is the hourly "unskilled" wages. On the contrary, if male spends a marginal unit of time in production, he will earn $w_{pt}^u + h_p w_{pt}^s = MPL_{pt} + h_p MPH_{pt}$, where w_{pt}^s is the hourly "skill" premium per unit of physical human capital. As a result, when $zn_t \le 1$, only women will take care of children. The budget constraint per couple therefore reads

$$w^u_{pt} zn_t + i_{pt} = 2w^u_{pt} + h_p w^s_{pt} \text{ if } zn_t \le 1,$$

$$\left(zn_t - 1\right)\left(w^u_{pt} + h_p w^s_{pt}\right) + i_{pt} = w^u_{pt} + h_p w^s_{pt} \text{ if } zn_t > 1,$$

and

$$c_{t+1} = i_{pt}\left(1 + r_{p,t+1}\right),$$

where i_{pt} is per couple investment in machines specific to the pre-modern regime.

Suppose $zn_t \le 1$. The maximization of per couple utility in (1) subject to (9a) and (10) gives the first order condition

$$zn_t \le \gamma\left(2 + \frac{h_p w^s_{pt}}{w^u_{pt}}\right)$$

with "=" if $\gamma\left(2 + \frac{h_p w^s_{pt}}{w^u_{pt}}\right) \le 1$ and "<" if $\gamma\left(2 + \frac{h_p w^s_{pt}}{w^u_{pt}}\right) > 1$. Note that the assumption $\gamma < \frac{1}{2}$ is necessary for interesting dynamics in female labor participation choice, otherwise women will always spend all their time rearing children.

On the other hand, if $zn_t > 1$, maximizing (1) subject to (9b) and (10) gives the first order condition that

$$\frac{\gamma}{n_t} - \frac{\left(1-\gamma\right)z}{2 - zn_t} \le 0 \Rightarrow zn_t \le 2\gamma < 1 \text{ since } \gamma < \frac{1}{2}$$

which contradicts supposition of $zn_t > 1$. Therefore, the assumption $\gamma < \frac{1}{2}$, which allows for interesting dynamics in female labor participation, also precludes men from spending any time rearing children.

Provided that $\gamma < \frac{1}{2}$, we have net fertility in the pre-modern regime given by

$$zn_t = \min\left\{\gamma\left(2 + \frac{h_p w_{pt}^s}{w_{pt}^u}\right), 1\right\}.$$

Equation (11) shows that time spent in rearing children rises with wage inequality between skilled and unskilled. This is so because child-rearing is intensive in women's time and in the pre-modern regime women are unskilled while men are skilled and their skill difference is fixed. Thus, increasing skill premium relative to unskilled wages have both an income effect (because wages on average increase and children is a normal good) and a substitution effect (because relative wages of women reduce) that induce higher net fertility. Population growth increases with $\frac{h_p w_{pt}^s}{w_{pt}^u}$ until $\frac{h_p w_{pt}^s}{w_{pt}^u} \geq \frac{1}{\gamma} - 2$ at which point population growth rate will remain at its high level $\overline{n} = \frac{1}{z}$. Note that female labor participation is $\tau_{pt}^f = 1 - zn_t$. As net fertility increases with $\frac{h_p w_{pt}^s}{w_{pt}^u}$, female labor participation decreases. On the other hand, male labor participation is invariant with time $\left(\tau_{pt}^m = 1 \ \forall t\right)$.

With the fertility choice in (11), we could write down the investment decision per couple as

$$i_{pt} = \begin{cases} (1-\gamma)\left(2w_{pt}^u + h_p w_{pt}^s\right) \text{ if } \gamma\left(2 + \frac{h_p w_{pt}^s}{w_{pt}^u}\right) \leq 1 \\ w_{pt}^u + h_p w_{pt}^s \text{ if } \gamma\left(2 + \frac{h_p w_{pt}^s}{w_{pt}^u}\right) > 1 \end{cases}.$$

Note that because of additive separable log utility, all the choices above are independent of the interest rate.

3.2. General equilibrium and macro-dynamics in the pre-modern regime

To find the general equilibrium of this economy, we first need to find the respective market wages. From the production function in (4a), we obtain

$$w_{pt}^{u} = \left(\frac{A_p L_{pt}}{Y_{pt}} \right)^{\sigma-1} A_p ,$$

$$w_{pt}^{s} = \left(\frac{K_{pt}^{\alpha} \left(B_p H_{pt} \right)^{1-\alpha}}{Y_{pt}} \right)^{\sigma-1} \left(1-\alpha\right) B_p \left(\frac{K_{pt}}{B_p H_{pt}} \right)^{\alpha} .$$

The skilled-unskilled wage ratio $\frac{h_p w_{pt}^{s}}{w_{pt}^{u}}$ is therefore given by

$$\frac{h_p w_{pt}^{s}}{w_{pt}^{u}} = \left(1-\alpha\right) b \left(2 - z n_t\right)^{1-\sigma} k_{pt}^{\sigma\alpha} ,$$

where $b = A_p^{-\sigma} 2^{\sigma\alpha} \left(B_p h_p \right)^{\sigma(1-\alpha)}$ and $k_{pt} \equiv \frac{K_{pt}}{N_t}$ is the physical capital per young person at time *t*. Because of capital-skill complementarity, as k_{pt} increases, the skilled-unskilled wage ratio increases. With the expression of $\frac{h_p w_{pt}^{s}}{w_{pt}^{u}}$, the fertility decision in (11) becomes an implicit equation in n_t:

$$\left(\frac{z n_t}{\gamma} - 2 \right) \left(2 - z n_t\right)^{\sigma-1} = \left(1-\alpha\right) A_p^{-\sigma} 2^{\sigma\alpha} \left(B_p h_p \right)^{\sigma(1-\alpha)} k_{pt}^{\sigma\alpha} .$$

Using the implicit function theorem, we could show that the net fertility rate n_t increases as k_{pt} accumulates.[9] We could therefore write

[9] This conclusion holds for any general CES production function that exhibits capital-skill complementarity, i.e., $\forall \sigma > \rho$. It can be shown that $\frac{h_p w_{pt}^{s}}{w_{pt}^{u}}$ is increasing with k_{pt} and decreasing in n_t for such CES production function, which is what is required for n_t to increase with k_{pt}.

$$zn_t = \begin{cases} \varphi\left(k_{pt}\right) \text{ if } k_{pt} \le \overline{k}_p \\ 1 \text{ if } k_{pt} > \overline{k}_p \end{cases}$$

where $\varphi'\left(k_{pt}\right) > 0$ and $\overline{k}_p$ is given by $\overline{k}_p = \left(\dfrac{\left(\frac{1}{\gamma} - 2\right) A_p^{\sigma}}{2^{\sigma\alpha}\left(1-\alpha\right)\left(B_p h_p\right)^{\sigma(1-\alpha)}} \right)^{\frac{1}{\sigma\alpha}}$.

Physical capital accumulation is given by $K_{p,t+1} = N_t i_{pt}$. Since $N_{t+1} = n_t N_t$, we have

$$k_{p,t+1} = \frac{K_{p,t+1}}{N_{t+1}} = \frac{i_{pt}}{n_t}$$
$$= \begin{cases} z\dfrac{1-\gamma}{\gamma} w_{pt}^{u} = \phi\left(k_{pt}\right) \text{ if } k_{pt} \le \overline{k}_p \\ z\left(w_{pt}^{u} + h_p w_{pt}^{s}\right) = \psi\left(k_{pt}\right) \text{ if } k_{pt} > \overline{k}_p \end{cases}$$

where $\phi\left(k_{pt}\right) = \dfrac{1-\gamma}{\gamma} zA_p \left(1 + \dfrac{bk_{pt}^{\sigma\alpha}}{\left(2-\varphi\left(k_{pt}\right)\right)}\right)^{\frac{1-\sigma}{\sigma}}$ and $\psi\left(k_{pt}\right) = zA_p\left(1+bk_{pt}^{\sigma\alpha}\right)\left(1+\left(1-\alpha\right)bk_{pt}^{\sigma\alpha}\right)^{\frac{1-\sigma}{\sigma}}$.

It could be shown that $\phi'(.) > 0$, $\psi'(.) > 0$, $\psi''(.) < 0$, $\phi\left(\overline{k}_p\right) = \psi\left(\overline{k}_p\right)$, and $\phi\left(k_{pt}\right)$ has at most one fixed point.

<<Insert Figure 4 about here>>

The graphs in Figure 4 show the dynamic time paths for k_{pt}, zn_t and τ_{pt}^{f} in the pre-modern regime. There are two subcases. In subcase 1, $\overline{k}_p < k_p^*$, where k_p^* solves the implicit equation $\psi\left(k_p^*\right) = k_p^*$. In subcase 2, $\overline{k}_p > k_p^*$. Subcase 2 happens iff

$$\frac{B_p h_p}{A_p} < \left(\frac{\gamma \left(\frac{1}{\gamma} - 2 \right)^{\frac{1}{\sigma\alpha}}}{2z(1-\alpha)^{\frac{1}{\sigma\alpha}}(1-\gamma)\left(1 + \frac{\frac{1}{\gamma} - 2}{1-\alpha} \right)^{\frac{1-\sigma}{\sigma}}} \right)^{\frac{\alpha}{1-\alpha}} .$$

Suppose in the initial period (period 0), each young couple is endowed with $k_{p0} < \overline{k}_p$. Then, $zn_0 < 1$. In subcase 1, zn_t increases smoothly to 1 as k_{pt} accumulates toward $\overline{k}_p$. After $\overline{k}_p$ is reached, $zn_t = 1$ all the time thereafter during the whole pre-modern era, while k_{pt} keeps on accumulating. In fact, after $\overline{k}_p$ is reached, the model is essentially a Solow model with constant (gross) population growth $\overline{n} = \frac{1}{z}$. Without exogenous productivity changes, therefore, diminishing marginal returns set in and k_{pt} will be stuck at a pseudo steady state level of k_p^*. In subcase 2, k_{pt} accumulates toward its pseudo steady state k_p^{**}, where k_p^{**} solves the implicit equation $\phi\left(k_p^{**}\right)=k_p^{**}$. All the while, zn_t increases toward its pseudo steady state value $\varphi\left(k_p^{**}\right)$.

3.3. Model implications of the pre-modern regime

The conclusion of this section is that in the pre-modern regime, population growth increases and female labor participation decreases during the transition to the pseudo steady state of the pre-modern economy. This result is due to the gender difference in physical human capital endowment and capital-skill complementarity in the production function. The engine of economic growth in the pre-modern regime is predominantly physical capital accumulation. The model of such pre-modern economy resembles the Solow model without exogenous productivity changes, but with endogenous population growth that increases with time. Like the Solow model, therefore, economic growth eventually tapers off as diminishing returns set in.

The model of the pre-modern regime is compatible with the time-series evidence of Western Europe and North America before the Industrial Revolution, where economic growth is meager, while population growth increases with per capita income. Moreover, the theory offered in this section could also shed light on the downward-sloping part of the U-shaped female labor participation in cross-country data in the post WWII period. Suppose the LDCs nowadays are still in the pseudo steady states of the pre-modern regime. Now, suppose the LDCs are different in terms of the parameters B_p, which reflects the degree of property rights protection. For low enough B_p, the inequality in (13) will be satisfied. It can be shown that, given (13), the lower B_p, the lower the pseudo steady state value of k_p^{**}. With lower value of B_p, therefore, both the pseudo steady state population growth $\varphi\left(k_p^{**}\right)$ and per capita income are lower. Correspondingly, the pseudo steady state value of female labor participation is higher. Thus, for the subset of countries with sufficiently low income, female labor participation is negatively correlated with income. We shall discuss in more details the time series and cross-country implications of the model in Section 6.

4. Analysis of the Modern Regime

4.1. Couple decision in the modern regime

In this section, we shall analyze the economics in the modern regime. As will be shown in Section 5, when population increases past a threshold, the economy will switch from the pre-modern regime to the modern regime of production. In the modern regime, even though initial mental human capital endowment is the same for male and female, because there is a wedge in the unskilled wages between male and female due to male endowment in physical human capital, again it will always be that only women take care of children first, if $zn_t \le 1$. The budget constraint per couple now reads

$$i_{mt} = \left(1-\tau_{ht}^{m}\right)\left(w_{mt}^{mu} + h_{mt}^{m} w_{mt}^{s}\right) + \left(1-\tau_{ht}^{f} - zn_t\right)\left(w_{mt}^{fu} + h_{mt}^{f} w_{mt}^{s}\right) \text{ if } zn_t \le 1,$$

$$\left(zn_t - 1\right)\left(w_{mt}^{mu} + h_{mt}^{m} w_{mt}^{s}\right) + i_{mt} = \left(1-\tau_{ht}^{m}\right)\left(w_{mt}^{mu} + h_{mt}^{m} w_{mt}^{s}\right) \text{ if } zn_t > 1,$$

and

$$c_{t+1} = i_{mt}\left(1 + r_{m,t+1}\right),$$

where τ_{ht}^{m} and τ_{ht}^{f} are the time spent on education per male and female respectively, h_{mt}^{m} and h_{mt}^{f} are the mental human capital per male and female given by (2) and (3b), w_{mt}^{s} is the skill premium (the hourly wages per unit mental human capital) in the modern regime, w_{mt}^{mu} and w_{mt}^{fu} are the unskilled male and female wages respectively, and i_{mt} is per couple investment in machines specific to the modern regime.

4.1.1. The first order conditions

As in the pre-modern regime, the assumption $\gamma < \frac{1}{2}$ implies that $zn_t > 1$ never arises in the modern regime. The maximization of per couple utility in (1) with respect to n_t subject to (14a), (14b) and (15) gives the fertility choice

$$zn_t = \min\left\{\gamma\left(\left(1 - \tau_{ht}^{m}\right)\frac{h_{mt}^{m} w_{mt}^{s} + w_{mt}^{mu}}{h_{mt}^{f} w_{mt}^{s} + w_{mt}^{fu}} + \left(1 - \tau_{ht}^{f}\right)\right), 1\right\}.$$

Like the fertility decision (11) in the pre-modern regime, the fertility choice (16) in the modern regime depends on gender wage inequality. The larger the male-to-female wage ratio, the higher the demand for children because of strong substitution effect. Unlike the pre-modern regime, however, the skill differential between female and male is not fixed, but is dependent on gender education choice. Thus, we need to understand how the gender education choices are made to fully understand fertility decision of the representative couple.

Gender education choice affects the fertility decision in two ways: first, it determines the relative skill levels of female and male and therefore the gender wage gap $\frac{h_{mt}^{m} w_{mt}^{s} + w_{mt}^{mu}}{h_{mt}^{f} w_{mt}^{s} + w_{mt}^{fu}}$; second, it determines the time available for working and child-rearing and therefore has an additional (resource) income effect on fertility decision. Suppose both female and male education increases, but female education increases faster than male education, the partial equilibrium effect (that is, taken the market unskilled wages and skill premium as given) will be that the net fertility falls. First, the gender wage gap $\frac{h_{mt}^{m} w_{mt}^{s} + w_{mt}^{mu}}{h_{mt}^{f} w_{mt}^{s} + w_{mt}^{fu}}$ reduces, hence inducing a substitution effect (stronger than the opposing income effect) toward having fewer children. Second, the increase in schooling in general reduces resource for working and rearing children, i.e. $\left(1-\tau_{ht}^{m}\right)$ and $\left(1-\tau_{ht}^{f}\right)$ decrease, which has an additional (resource) income effect toward having fewer children.

Given $zn_t \leq 1$, gender education choice is determined by the maximization of (1) with respect to τ_{ht}^{m} and τ_{ht}^{f} subject to (14a) and (15). For notational convenience, we shall define the following:

$$\theta\left(\mathcal{H}_{m,t-1}\right) \equiv D\left(\frac{\mathcal{H}_{m,t-1}}{R} + h_{m0}\right)^{\beta} \text{ and } x_t \equiv \frac{h_{m0}}{\theta};$$

$$q_t^{m} \equiv x_t + \frac{w_{mt}^{mu}}{\theta w_{mt}^{s}} \text{ and } q_t^{f} \equiv x_t + \frac{w_{mt}^{fu}}{\theta w_{mt}^{s}}.$$

All these variables depend on the aggregate mental human capital of the previous generation $\mathcal{H}_{m,t-1}$. The first order conditions in gender education choice give:

$$\tau_{ht}^{m} = \max\left\{\frac{1}{2}\left(1-q_t^{m}\right), 0\right\},$$

$$\tau_{ht}^{f} = \max\left\{\frac{1}{2}\left(1-zn_t-q_t^{f}\right), 0\right\}.$$

Given net fertility and gender education choices, the ultimate levels of mental human capital per male and female are:

$$h_{mt}^{m} = \frac{\theta}{2}\left(1+2x_t-q_t^{m}\right),$$

$$h_{mt}^{f} = \frac{\theta}{2}\left(1-zn_t+2x_t-q_t^{f}\right).$$

Accordingly, the male-to-female wage ratio is given by

$$\frac{h_{mt}^{m}w_{mt}^{s}+w_{mt}^{mu}}{h_{mt}^{f}w_{mt}^{s}+w_{mt}^{fu}}=\frac{1+q_t^m}{1-zn_t+q_t^f}$$

and gender labor participation choice is given by

$$\tau_{mt}^{m}=1-\tau_{ht}^{m}=\min\left\{\frac{1}{2}\left(1+q_t^m\right),1\right\},$$

$$\tau_{mt}^{f}=1-zn_t-\tau_{ht}^{f}=\min\left\{\frac{1}{2}\left(1-zn_t+q_t^f\right),1-zn_t\right\}.$$

4.1.2. The representative couple's fertility choice

Given the first order conditions of net fertility decision (16), the female education choice (18b) and the male-female wage ratio (20), we could solve the simultaneous equations to get an explicit solution for female time allocation toward child-rearing. Assuming interior solution,[10] we have

$$zn_t=\frac{1-\sqrt{1-\gamma\left(2-\gamma\right)\left(1+\left(\frac{1+q_t^m}{1+q_t^f}\right)^2\right)}}{2-\gamma}\left(1+q_t^f\right).^{11}$$

[10] Interior solutions require $q_t^m\leq 1$ and $zn_t\leq 1-q_t^f$. In fact, the former condition has to hold in order for the pre-modern regime to exist (see footnote 15). This means that the male education and labor participation choices will always be interior solutions in the modern regime. On the other hand, the latter condition holds only for large enough $\mathcal{H}_{m,t-1}$. Therefore, if the switch of regime takes place at large enough $\mathcal{H}_{m,t-1}$, the interior solutions hold throughout the modern regime for the female choices as well. But if the switch of regime takes place at small $\mathcal{H}_{m,t-1}$, female education will be at the corner solution of zero for a while. For simplicity, we shall assume that $\mathcal{H}_{m,t-1}$ is large enough at the switch of regime.

[11] In order for interior solution to exist, $\gamma<1-\frac{\sqrt{2}}{2}$. Otherwise, women will always spend all their time rearing children. We shall assume such restriction on γ to allow for interesting dynamics in female education and labor participation choices. Note that this restriction nests the restriction of $\gamma<\frac{1}{2}$ in the pre-modern regime.

Net fertility therefore depends positively on $\left(1+q_t^f\right)$ and on $\dfrac{1+q_t^m}{1+q_t^f}$. We first consider the partial equilibrium effect of $\mathcal{H}_{m,t-1}$ on the representative couple's decision. That is, we take the market wages, w_{mt}^{mu}, w_{mt}^{fu} and w_{mt}^{s}, as given. As mental human capital accumulates, the intergenerational externality effect on the productivity of the education sector becomes larger. According to (17a) and (17b), θ increases with $\mathcal{H}_{m,t-1}$ and therefore q_t^m and q_t^f decrease with $\mathcal{H}_{m,t-1}$. Thus, the $\left(1+q_t^f\right)$ in (22) decreases with $\mathcal{H}_{m,t-1}$. Furthermore, the ratio $\dfrac{1+q_t^m}{1+q_t^f}$ decreases with $\mathcal{H}_{m,t-1}$. From the definitions in (17b), the ratio $\dfrac{1+q_t^m}{1+q_t^f}$ reflects the wage ratio between male and female for the case where male education equals to female education. When $\mathcal{H}_{m,t-1}$ is small, such ratio largely reflects the differential unskilled wages between the sexes. When $\mathcal{H}_{m,t-1}$ becomes large, the skilled wages (skills times skill premium) dominates the unskilled wages. The ratio $\dfrac{1+q_t^m}{1+q_t^f}$ therefore decreases to 1 as $\mathcal{H}_{m,t-1}\rightarrow\infty$. As a result, with mental human capital accumulation, net fertility falls.

To understand more intuitively as to why zn_t decreases with $\mathcal{H}_{m,t-1}$, we shall look at the first order conditions (16), (18a) and (18b). When $\mathcal{H}_{m,t-1}$ becomes large, productivity of the education sector increases. Therefore, for given zn_t, there is a substitution effect for both male and female to get more educated. That is, time allocation shifts from working toward getting educated, for given time spent in child-rearing. Such increase in time in education has a "resource" effect on reducing the net fertility as reflected in the RHS of (16). Moreover, for given zn_t, larger $\mathcal{H}_{m,t-1}$ diminishes the significance of the differential unskilled wages between the sexes in their wage ratio. This lowers the gender wage gap and contributes to a substitution effect on reducing the net fertility as reflected in the RHS of (16). As net fertility decreases, it releases time disproportionately for women (to get educated and to work). This contributes to reducing the gender gap in education and hence the gender wage gap, thus reinforcing the fall in net fertility.

4.1.3. Gender education and labor participation choices

We now turn to how the gender education and labor participation choices are determined. For men, such decisions are independent of fertility decisions, because they never partake in rearing children. Their decisions depend only on the summary variable q_t^m, which depends negatively on θ and hence $\mathscr{H}_{m,t-1}$. Assuming interior solutions, the male education and labor participation choices are given by

$$\tau_{ht}^m = \frac{1}{2}\left(1-q_t^m\right)$$

$$\tau_{mt}^m = \frac{1}{2}\left(1+q_t^m\right)$$

As aggregate mental human capital accumulates, q_t^m decreases. Therefore, male education increases while male labor participation decreases. This happens because for large $\mathscr{H}_{m,t-1}$, the education sector becomes very productive. There is therefore a substitution effect for male to allocate larger fraction of their lifetime getting educated rather than working at early age.

For women, education and labor participation choices are affected by fertility choice, as well as directly affected by the summary variable q_t^f. Assuming interior solutions, such choices are given by

$$\tau_{ht}^f = \frac{1}{2}\left(1-zn_t-q_t^f\right)$$

$$\tau_{mt}^f = \frac{1}{2}\left(1-zn_t+q_t^f\right)$$

As $\mathcal{H}_{m,t-1}$ increases, female education increases with $\mathcal{H}_{m,t-1}$. First, as in the case of male education choice, the direct substitution effect of high productivity in education sector on q_t^f induces women to allocate more time toward getting educated, for given zn_t. Second, the increase in $\mathcal{H}_{m,t-1}$ lowers time allocated toward child-rearing, thus releasing time for women to get educated. Both the substitution effect and the resource effect tend to increase female schooling.

In terms of female labor participation, there are again two effects of increasing $\mathcal{H}_{m,t-1}$: substitution effect and resource effect. As in the case of male labor participation choice, the direct substitution effect of high productivity in education sector on q_t^f induces women to allocate less time toward working, for given zn_t. However, the increase in $\mathcal{H}_{m,t-1}$ lowers time allocated toward child-rearing, thus releasing more time for women to work as well. If the resource effect of lowering zn_t overwhelms the substitution effect, women's labor participation increase with $\mathcal{H}_{m,t-1}$.[12]

4.2. General equilibrium and macro-dynamics in the modern regime

The unskilled wages for male and female and the skill premium in the modern regime are given by

$$w_{mt}^{mu} = \left(\frac{A_m L_{mt}}{Y_{mt}}\right)^{\sigma-1} A_m \left(1 + E_m h_p\right),$$

[12] The explicit solution for female labor participation is given by $\tau_{mt}^f = \frac{1}{2}\left(1+q_t^f\right)\left[\frac{1-\sqrt{1-\gamma\left(2-\gamma\right)\left(1+\left(\frac{1+q_t^m}{1+q_t^f}\right)^2\right)}}{2-\gamma}\right]$. The term $\left(1+q_t^f\right)$ decreases with $\mathcal{H}_{m,t-1}$ while the term in [...] increases with $\mathcal{H}_{m,t-1}$ because $\frac{1+q_t^m}{1+q_t^f}$ decreases with $\mathcal{H}_{m,t-1}$. Thus, insofar as the term in [...] increases faster than the term $\left(1+q_t^f\right)$, female labor participation increases with $\mathcal{H}_{m,t-1}$.

$$w_{mt}^{fu} = \left(\frac{A_m L_{mt}}{Y_{mt}} \right)^{\sigma-1} A_m,$$

$$w_{mt}^{s} = \left(\frac{K_{mt}^{\alpha} \left(B_m H_{mt} \right)^{1-\alpha}}{Y_{mt}} \right)^{\sigma-1} \left(1-\alpha\right) B_m \left(\frac{K_{mt}}{B_m H_{mt}} \right)^{\alpha}.$$

In the modern regime, however, the ratios $\frac{w_{mt}^{mu}}{w_{mt}^{s}}$ and $\frac{w_{mt}^{fu}}{w_{mt}^{s}}$ need not fall with physical capital accumulation. This is so because even though the production function exhibits complementarity between machines and mental human capital, aggregate mental human capital also accumulates. The transitional behavior of the skill premium depends on the initial imbalance between physical capital and mental human capital. However, unless the initial imbalance effect is opposing and too large, the ratios $\frac{w_{mt}^{mu}}{\theta w_{mt}^{s}}$ and $\frac{w_{mt}^{fu}}{\theta w_{mt}^{s}}$ will decrease with $\mathcal{H}_{m,t-1}$. This contrasts with Galor and Weil (1996), which is a model of exogenous growth with fixed skill differential. In their case, capital-skill complementarity drives up the wage per mental human capital relative to wage per physical labor, which reduces the gender wage gap to produce the fall in net fertility. Their relative wage mechanism therefore depends on the rise in (mental) skill premium. In contrast, in the present model, skill level is a choice variable, and skilled wages rise relative to unskilled wages mainly because skill level rises, while the skill premium could increase or decrease.[13] Similar to both models, however, the rise of female relative wages with economic development contributes to the decline in net fertility.

[13] In fact, if we model the modern regime in a slightly different way, we could get rid of the skill premium effect altogether. Instead of assuming unskilled wages for male are higher than that for female because of bodily strength, we could assume that the initial mental human capital endowment is higher for male than for female, i.e., $h_{m0}^{m} > h_{m0}^{f}$ (this follows Zhang et al 1998). Furthermore, instead of a CES production, we could simply have a Cobb-Douglas production function with only physical capital and mental human capital as inputs. In this case, the male-to-female wage ratio depends only on the ratio of the skill levels but not on the skill premium. This alternative formulation gives us essentially the same qualitative results.

Therefore, insofar as the general equilibrium imbalance effect is not opposing or too large, q_t^m and q_t^f decrease with $\mathcal{H}_{m,t-1}$. Moreover, the ratio $\frac{1+q_t^m}{1+q_t^f}$ declines with $\mathcal{H}_{m,t-1}$ regardless of the general equilibrium effect, because $\frac{w_{mt}^{mu}}{\theta w_{mt}^s}$ and $\frac{w_{mt}^{fu}}{\theta w_{mt}^s}$ differ only by a proportionate constant $\left(1+E_m h_p\right)$. The conclusions of how net fertility choice, gender education and labor participation choices change with $\mathcal{H}_{m,t-1}$ from the previous subsection therefore carry over to the general equilibrium analysis of the present model.

Because the decline in time allocated toward child-rearing releases time for women disproportionately, the gender wage gap will narrow with $\mathcal{H}_{m,t-1}$. From (20) and (22), the male-to-female wage ratio is given by

$$\frac{h_{mt}^m w_{mt}^s + w_{mt}^{mu}}{h_{mt}^f w_{mt}^s + w_{mt}^{fu}} = \frac{\left(\frac{1+q_t^m}{1+q_t^f}\right)}{1-\frac{1-\sqrt{1-\gamma\left(2-\gamma\right)\left(1+\left(\frac{1+q_t^m}{1+q_t^f}\right)^2\right)}}{2-\gamma}}$$

which is an increasing function with $\frac{1+q_t^m}{1+q_t^f}$. Therefore, as $\mathcal{H}_{m,t-1}$ increases, $\frac{1+q_t^m}{1+q_t^f}$ falls and therefore the gender wage gap will narrow.

In conclusion, our model could generate the implications that, as $\mathcal{H}_{m,t-1}$ increases, population growth falls, female education and labor participation increases, male education increases but male labor participation falls, and gender wage gap narrow. Figure 5 illustrates such plausible dynamic paths for population growth, female and male years of schooling, female and male labor participation and gender wage gap in the modern regime.

<<Insert Figure 5 about here>>

4.2.1. Mental human capital accumulation in the modern regime

To complete the model, we turn to the dynamics of mental human capital accumulation. From the aggregate human capital function in (3), we have $\mathcal{H}_{mt} = \frac{N_t}{2}\left(h_{mt}^m + h_{mt}^f\right)$. Substituting (19a) and (19b) into the aggregate human capital function, we obtain

$$\mathcal{H}_{mt} = \frac{N_t}{4}\left[4h_{m0} + \theta\left(1 - zn_t - q_t^f\right) + \theta\left(1 - q_t^m\right)\right]$$

Suppose the economy switches to the modern sector at time t_0. The expression on the RHS of (23) implies that $\mathcal{H}_{mt_0} = N_{t_0}h_{m0} > N_{t_0-1}h_{m0} = \mathcal{H}_{m,t_0-1}$. The expression in the bracket of (23) is increasing with $\mathcal{H}_{m,t-1}$, since zn_t, q_t^f and q_t^m are decreasing with $\mathcal{H}_{m,t-1}$ and θ is increasing with $\mathcal{H}_{m,t-1}$.[14] Now, θ is concave, but the term $1 - zn_t - q_t^f$ and $1 - q_t^m$ might convexify the aggregate human capital function in (23). Thus, for given N_t, the human capital accumulation function could have multiple fixed-points as in Becker et al. (1990). However, in the present model, there is intergenerational externality as in Lucas (1988). As N_t increases through time, the human capital function continuously shifts up and rotate counterclockwise. Thus, these "fixed-points" increase through time as well. Figure 6 illustrates the evolution of $\mathcal{H}_{mt}$ through time. As can be seen, $\mathcal{H}_{mt}$ increases monotonically throughout the modern era.

<<**Insert Figure 6 about here**>>

4.2.2. Balanced growth path in the modern regime

As $\mathcal{H}_{mt}$ increases indefinitely through time, $x_t \to 0$, $q_t^m \to 0$, $q_t^f \to 0$ and $\frac{1+q_t^m}{1+q_t^f} \to 1$.

Therefore, in the long-run, net fertility is given by

$$n^* = \frac{1}{z}\frac{1 - \sqrt{1 - 4\gamma + 2\gamma^2}}{2 - \gamma}$$

[14] Interior solutions also require $1 - zn_t - q_t^f > 0$ and $1 - q_t^m > 0$, as remarked in footnote 10.

Given the long-run fertility in (24), in the long run, aggregate human capital used in goods production is given by

$$H_{mt} \to \frac{N_t}{2R^{\beta}} \frac{D}{4}\left[\left(1 - zn^*\right)^2 + 1\right] \mathscr{H}_{m,t-1}^{\beta},$$

$$\mathscr{H}_{m,t-1} \to \frac{2\left(2 - zn^*\right)}{\left(1 - zn^*\right)^2 + 1} H_{m,t-1}.$$

Define $H_{mt}^a \equiv \frac{H_{mt}}{N_t}$ as the average human capital used in goods production at time *t*. The dynamic equation for H_{mt}^a in the long run is given by

$$H_{mt}^a = \xi\left(N_{t-1} H_{m,t-1}^a\right)^{\beta},$$

where $\xi \equiv \frac{D}{8R^{\beta}} 2^{\beta}\left(2 - zn^*\right)^{\beta}\left[\left(1 - zn^*\right)^2 + 1\right]^{1-\beta}$. This dynamic equation gives rise to non-scale growth in H_{mt}^a in an eventual balanced growth path (BGP). Let $g \equiv \frac{H_{mt}^a - H_{m,t-1}^a}{H_{m,t-1}^a}$ be the growth rate of H_{mt}^a. Also, let $\eta_t = \frac{\left(H_{mt}^a\right)^{1-\beta}}{N_t^{\beta}}$. Then, (25) can be rewritten as

$$\eta_t = \frac{\xi^{1-\beta}}{n^{*\beta}} \eta_{t-1}^{\beta}.$$

Since the RHS of the above equation is concave, η_t eventually reaches a steady state given by $\eta^* = \frac{\xi}{n^{*\frac{\beta}{1-\beta}}}$. Since η_t reaches a steady state constant in the long run, H_{mt}^a must be growing at

$$g^* = n^{*\frac{\beta}{1-\beta}} - 1$$

along the BGP, where n^* is given by (24). This is essentially the non-scale growth in Jones (1995) but with endogenous fertility.

We could also find out the dynamics of physical capital accumulation in the long run. Now, $K_{mt} \to w^s_{m,t-1} H_{m,t-1} \to (1-\alpha) K^{\alpha}_{m,t-1} \left(B_m H_{m,t-1} \right)^{1-\alpha}$. Therefore, in the long run, physical capital per young person is given by $k_{mt} \to \frac{(1-\alpha)}{n^*} k^{\alpha}_{m,t-1} \left(B_m H^a_{m,t-1} \right)^{1-\alpha}$. Define $\chi_t \equiv \frac{k_{mt}}{H^a_{mt}}$. We have

$$\chi_t = \frac{(1-\alpha) B_m^{1-\alpha}}{n^* \left(1+g^*\right)} \chi^{\alpha}_{t-1}.$$

Since the RHS of the above equation is concave, χ_t eventually reaches a steady state given by $\chi^* = \left[\frac{(1-\alpha)}{n^* \left(1+g^*\right)} \right]^{\frac{1}{1-\alpha}} B_m$. Therefore, along the BGP, k_{mt} will be growing at the same rate g^* as H^a_{mt}. Real GDP per capita is similarly given by $y_{mt} \equiv \frac{Y_{mt}}{N_t + N_{t-1}} \to \frac{k^{\alpha}_{m,t-1} \left(B_m H^a_{m,t-1} \right)^{1-\alpha}}{1+n^*}$. Therefore, along the BGP, y_{mt} is also growing at rate g^*.

5. Modernization

The above two sections delineate the full dynamics under each individual regime before and after modernization. The question remains: when does the switch from the pre-modern regime to the modern regime occur? We shall discuss the timing of economic take-off in this section. To derive explicit expression, we shall assume σ=1 in this section. The qualitative conclusion is unchanged for σ<1.

We first assume that in the initial period the economy is only endowed with K_{p0}. Given that, the initial period production must involve only Y_p. When a generation decides on what types of machines to invest in, they do so to maximize their consumption when old. The types of machinery investment determine the rent they earn at old age. Therefore, when young if they anticipate $MPK_p > MPK_m$ for any given *K* when old, investment will take the form i_p but not i_m.

Conversely, if $MPK_m > MPK_p$, investment will take the form i_m, but not i_p. Note, because of the additive separable log utility, the *amount* of investment does not depend the anticipated marginal product. The decisions of the *amount* of investment and the *type* of investment are therefore independent of each other.

The marginal product of capital in pre-modern regime at time t is

$$MPK_{pt} = \left(\frac{B_p h_p}{2k_{pt}}\right)^{1-\alpha}$$

whereas the marginal product of capital in the modern regime at time t is given by

$$MPK_{mt} = \left(\frac{B_m H^a_{mt}}{k_{mt}}\right)^{1-\alpha} = \left(\frac{B_m}{8k_{mt}}\right)^{1-\alpha}\left[\psi\left(\frac{N_{t-1}}{R}\right)\right]^{1-\alpha}$$

where $\psi\left(\frac{N_{t-1}}{R}\right) = \left[(1+x_t)^2 + (1-zn_t+x_t)^2 - (q^m_t - x_t)^2 - (q^f_t - x_t)^2\right]\theta\left(\mathcal{H}_{m,t-1}\right)$. Suppose that at *t-1* the economy is still in the pre-modern era, then $\mathcal{H}_{m,t-1} = N_{t-1}h_{m0}$. Because the amount of investment is the same regardless of the type of investment, when comparing the MPK's to decide on what type of machinery to invest in, households take $k_{pt} = k_{mt}$. Therefore, the economy will switch to the modern regime iff

$$\psi\left(\frac{N_{t-1}}{R}\right) > \frac{4B_p h_p}{B_m}.$$

Now, $\psi(\infty) \to \infty$. Moreover, $\psi'\left(\frac{N_{t-1}}{R}\right) > 0$ for large enough $\frac{N_{t-1}}{R}$. Therefore, there exists a threshold $\widehat{\left(\frac{N}{R}\right)}$ such that $MPK_p > MPK_m$ when $\frac{N_{t-1}}{R} < \widehat{\left(\frac{N}{R}\right)}$ and $MPK_p < MPK_m$ when $\frac{N_{t-1}}{R} > \widehat{\left(\frac{N}{R}\right)}$. Suppose the conversion factor *R* denotes area and therefore $\frac{N}{R}$ denotes population density. Then, before the population density reaches $\widehat{\left(\frac{N}{R}\right)}$, the economy remains in the pre-modern regime as $MPK_p > MPK_m$. Economic takes off once the scale of the economy reaches the threshold $\widehat{\left(\frac{N}{R}\right)}$ at which point the intergenerational externality on the productivity of education is strong enough to guarantee higher returns to building machines complementary to mental human capital in the modern regime.[15]

The timing of modernization depends on the value of the threshold $\widehat{\left(\frac{N}{R}\right)}$, among other things. The lower the threshold, other things the same, the earlier the timing of modernization. The threshold value depends in part on B_m as shown in (27). The larger B_m is, the lower the threshold value $\widehat{\left(\frac{N}{R}\right)}$. B_m is a productivity parameter that reflects property rights protection in the modern regime. Therefore, other things the same, countries with better institutions in procuring property rights protection in the modern sector will modernize first. Note also that for early modernizers, because the threshold population density is low, they might not have reached the pseudo steady state of the pre-modern regime before switching to the modern regime.

6. Empirical implications of the Model

6.1. Time series implications

The above three sections depict the whole development process from the pre-modern regime to the modern era. Figure 7 illustrates an example of the time paths of population growth, population density, female and male years of schooling, female and male labor participation rate,

[15] To guarantee that the economy will not switch to the modern regime in the first period, we shall assume that $\frac{B_p h_p}{B_m h_{m0}} > 2(1-\gamma)$, so that for small enough N_{t-1}, $MPK_p > MPK_m$. This assumption also ensures that at switch of regime $q_t^m < 1$. This implies that male education will not be zero (corner solution) at the switch of regime.

gender gap in earnings and log real GDP per capita from the pre-modern age through the modern era.[16]

<<Insert Figure 7 about here>>

As the graphs show, population growth first rises then falls with time. There is zero education in the pre-modern era. Assuming interior solutions at switch of regime, both male and female begin schooling as soon as the modern era begins, with initial male schooling larger than female schooling. Male devote all their time to production during the pre-modern era. But once the economy modernizes they spend their early young adulthood in school and participate fully in labor only in later part of their adulthood. Female spend time working as unskilled labor at the beginning of the pre-modern era, but gradually spend more time rearing children and their labor participation in the pre-modern production sector decreases. After the economy switches to the modern regime, however, their labor participation increases with time, but because they do spend part of the young adulthood rearing children, their labor participation never reaches the level of their male counterpart. Gender gap in earnings widens as time passes in the pre-modern era. Once women start working again in the modern era, however, gender gaps in education and earnings narrow as time passes, though the gaps never totally vanish. Output increases slowly during the pre-modern era before it reaches a pseudo steady state, but takes off once population density reaches a threshold to generate enough intergenerational externality in education production and eventually reaches a balanced growth path.

The present model is therefore capable of generating some time series implications that are compatible with the historical development patterns of Western Europe and North America. Population growth is inverted-U with economic development, female labor participation is increasing the modern era, while male labor participation slightly decreases, male and female education both increase with time in the modern era, while gender gap in education narrows, gender

[16] For illustration purpose, the example in Figure 7 assumes interior solutions at switch of regime and the switch of regime occurs before the pre-modern economy reaches its pseudo steady state.

gap in earnings narrow during the modern era, economic growth was meager before the Industrial Revolution, but the economy takes off ever since and the timing of take-off coincides with the demographic transition.

6.2. The timing of modernization and cross-country implications

In this section, we shall discuss in more details how the degree of property rights protection affects the timing of modernization and therefore cross-country pattern in economic development and performance. Consider the lack of property rights protection as an extortionary tax on capital income. The lower the degree of property rights protection, the lower the productivity parameters B_m and B_p in both the modern and pre-modern regimes. Moreover, the modern regime involves accumulation of mental human capital as well. Poor intellectual property rights protection will therefore likely further reduces B_m. In what follows, we shall therefore treat poorer institution to mean both B_m and B_p are lower and the ratio $\frac{B_m}{B_p}$ is also lower. We assume preference parameter γ, time cost per child z and other productivity parameters A_p, A_m, E_m and D are the same.

Institutions affect the timing of modernization in two ways. First, countries with poorer institutions have lower $\frac{B_m}{B_p}$. According to (27), the lower $\frac{B_m}{B_p}$, the higher the threshold value $\widehat{\left(\frac{N}{R}\right)}$ for modernization to take place. Therefore, poorer institutions delay the timing of modernization. Second, the pace of population increase in the pre-modern regime depends on the relative magnitude of B_p versus A_p. With poorer institution, B_p is small relative to A_p. This means that male wages are not much higher than female wages in the pre-modern sector. As a result, female participates more in work and population growth is lower. This implies that even with the same threshold $\widehat{\left(\frac{N}{R}\right)}$, a country with lower property rights protection will reach the threshold population density more slowly. Again, poorer institutions delay the timing of modernization. This has the following cross-country implications.

Consider five countries ranked in terms of their degree of property rights protection. Country 1 has the poorest institution and the country 5 has the best institution. To highlight the importance of institution, we assume further that all countries have the same initial conditions: they have the same initial population density, physical capital endowment and human capital endowment. Figure 8 illustrates a plausible scenario for the relative time paths for the five countries. The shaded area could be seen as the post WWII period.

<<Insert Figure 8 about here>>

Country 5 represents the early modernizers such as Western Europe and North America. Country 4 represents a later modernizer such as Japan. Country 3 represents the NICs such as East Asia. Countries 2 and 1 represent countries like sub-Saharan African or South Asian countries that have not yet taken off even today. For illustration, we also assume B_p is low enough in countries 1 and 2 that the inequality in (13) is satisfied. According to Section 2, this means that the pseudo steady state population growth rate for country 2 will be higher than that for country 1.

At the beginning, when all the countries are still in the pre-modern regime, because the ratio $\frac{B_p}{A_p}$ is higher for countries will better institutions, population growth rates are somewhat higher in these countries. More importantly, because the threshold population density at which modernization takes place is lower for countries with better institutions, as population increases, these countries reach their respective threshold population densities faster than those with poorer institutions. Demographic transitions take place in these countries one after another, with country 5 taking off first, followed by country 4 and then later by country 3.[17] After the demographic transitions take place, population growth rate falls in these countries. All the while, population

[17] In the way we draw the graphs, we assume that the countries that modernized have not reached their pseudo steady states in the pre-modern regime at the switch of regime. This happens if the threshold population density is low enough due to efficient institution.

growth rate of countries 1 and 2 continue to increase. As a result, the population growth rates of countries 1 and 2 eventually surpass those of the modernizers. If we look at the cross-country pattern of population growth during the post WWII period, then for the subset of countries that have modernized (countries 3, 4 and 5), their population growth rates are inversely related to their economic development. On the other hand, for the subset of countries that still have not modernized (countries 1 and 2), their population growth rates are positively related to their economic development. As illustrated in Figure 9, this generates an inverted-U relationship between population growth and economic development in the post WWII cross-country data.

<<Insert Figure 9 about here>>

The reverse happens to female time allocation toward labor participation. At the beginning, when all the countries are still in the pre-modern regime, because the ratio $\frac{B_p}{A_p}$ is higher for countries will better institutions, female labor participation is lower in these countries. But as countries 5, 4 and 3 modernize one after another, their female labor participation increase. All the while, the female labor participation continues to decrease in countries 1 and 2. As a result, if we look at the cross-country pattern of female labor participation during the post WWII period, then for the subset of countries that have modernized (countries 3, 4 and 5), their female labor participation rates are positively related to their economic development. On the other hand, for the subset of countries that still have not modernized (countries 1 and 2), their female labor participation rates are inversely related to their economic development. This generates a U-shaped female labor participation function with economic development in the post WWII cross-country data. This is also illustrated in Figure 9.

It is also interesting to notice that the turning points in the graphs in Figure 9 are different and they match well with the real world data in Figure 2. As in the real world data, the U-shaped relationship between female labor participation and economic development is more apparent (and with a turning point at higher level of economic development) than the inverted-U relationship between population growth and economic development in the cross-country comparison. This requires some explanation. If we separate countries into two subsets or clubs: one including only those that have modernized (countries 3, 4, 5) and the other including only those that have not modernized yet (countries 1 and 2), the negative association between female labor participation and population growth is apparent within each club. For the club of modernizers (3, 4 and 5), population growth is inversely related to economic development whereas female labor participation is positively related to economic development. For the club of non-modernizers (1 and 2), population growth is positively related to economic development while female labor participation is negatively related to economic development. The difference between the cross-country relationships stems from the relative position of country 2 (the better performer in the club of non-modernized countries) and country 3 (the latest modernizer) in terms of population growth and female labor participation. Suppose during the WWII period, the population growth of country 2 is higher than country 3, i.e. $n_2 > n_3$, our question is: is it possible for the female labor participation for country 2 to be higher than country 3 as well? The answer is yes. Notice that country 2 is in the pre-modern regime and there is no education choice. Thus, female labor participation of country 2 is given by $\tau_{p2}^f = 1 - zn_2$. On the other hand, even though population growth is lower in country 3 and hence the time allocated toward child-rearing is smaller, the remaining time is allocated between working and getting education. Female labor participation for country 3 is given by $\tau_{m3}^f = \frac{1}{2}\left(1 - zn_3 + q_3^f\right)$. Even though $n_2 > n_3$, it is plausible that $\tau_{p2}^f = 1 - zn_2 > \frac{1}{2}\left(1 - zn_3 + q_3^f\right) = \tau_{m3}^f$. If this is the case, the turning point in the inverted-U relationship between population growth and economic development in the cross-country data will take place at the level of economic development of country 2, whereas the turning point in the U-shaped relationship between female labor participation and economic development in the cross-country data will take place at the level of economic development of country 3.

In terms of economic performance, economic growth is meager for all countries when they are all at the pre-modern regime. Countries with better institutions performance slightly better and grow slightly faster. But the Great Divergence takes place when some of the countries modernize while others have not. Country 5, which represents the early modernizers such as Western Europe and North America, modernizes during the Industrial Revolution and their economies take off since, with economic growth increasing for a while and become roughly constant for a century or so. Meanwhile, other countries lag behind because they are still in the pre-modern regime. When the later modernizers, such as country 4 (Japan) and country 3 (East Asia), take off subsequently, they join the club of industrialized countries. Their economies take off with demographic transition and increase in education and labor force participation of women. Moreover, because of larger general equilibrium imbalance effect, these economies potentially grow at rates unprecedented in the Western European and North American history. Thus, the East Asian miracles result and the world divide itself into two clubs. Starting with a rather continuous spectrum of country performance at the beginning of the post WWII period, the world income distribution becomes bipolarized. This corresponds to the observation by Quah (1996). Moreover, the rank of country in terms of their living standard is monotonic with the rank of their degrees of property rights protection. This accords with the level regression findings of Hall and Jones (1999) and Acemoglu et al. (2001) that institutional index of diversion has strong explanatory power of the productivity across nations.

7. Conclusion

The present paper develops a model with endogenous fertility choice and structural change and with particular emphasis on the role of women in production and child-rearing. In the pre-modern regime, inequality in gender endowment is permanent. Women do participate initially in production but merely as unskilled labor. Eventually, however, when “skill”-complementary machines are accumulated, male dominates the production of the economy, as their relative wages increase even further. Gender inequality in earnings widens. Women gradually retreats from the production sector and spend most if not all of their time rearing children. Population growth rises to a pseudo steady state value. Because of the limited potential for physical human capital improvement, the engine of growth in the pre-modern sector is primarily physical capital accumulation, which eventually suffers from diminishing returns. Economic growth in the pre-modern sector is thus meager. As population keeps increasing, however, the population density will reach a point in which intergenerational externality on the productivity of education is strong enough and the economy takes off. In the modern regime, gender endowment in production is unequal initially. As a result, male start getting educated with more years of schooling than women do. Eventually, however, female education attainment increases with time and at faster pace. Net fertility drops as women decide to spend less time in child-rearing and their labor participation increases as the economy grows. The engine of growth in the modern regime is mental human capital accumulation. During the modern era, average human capital for both male and female increase indefinitely in time and the gender gap in earnings narrow. In the eventual balanced growth path, population growth remains at a low steady state level, while physical capital per labor, human capital per labor and output per labor all grow at the same constant rate, which depends on the endogenous steady state population growth rate. The model sheds light on the long-run time series development of Western Europe and North America, with population growth rising then falling with economic development and the demographic transition accompanied by economic take-off.

Focusing on the impact of property rights on the timing of modernization, this paper also provides an explanation for the Great Divergence since the Industrial Revolution and the more recent bipolarization of world income distribution. The model explains the income disparity by appealing to difference in institutional arrangements of property rights protection, which accords with the findings of Hall and Jones (1999) and Acemoglu et al. (2001). Moreover, the structural change aspect of the model implies that the "extortionary tax" due to poor property rights protection needs not be very high in order to induce huge income disparity. Similarly, since the difference between the rich and poor nowadays is mainly due to the inability of the poor to take off, marginal product of capital needs not be higher in the poor countries and therefore the lack of capital inflow to the poor countries a la Lucas (1990) is easily understandable in the present framework.

The emphasis on the women role in production during the pre-modern and modern regime also provides the theoretical underpinning of the U-shaped female labor participation function with economic development as explored in Goldin (1994). If the premise of the paper on the gender difference in human capital endowment is correct, though female participation is high in both very poor and very rich countries, women's role in production is much more significant in the rich countries that have modernized. The observation by Young (1995) that rising women labor participation is a key determinant of the extraordinary growth in East Asia can therefore be understood in this light. Further investigation into the gender role in the East Asian miracles is another fruitful area of research.

Lastly, the endogenous growth mechanism in the present model is mental human capital accumulation as in Lucas (1988). Since there is no gender difference in mental human capital endowment, women's participation in the modern regime is an important fuel for such human capital accumulation. Alternatively, one could model the endogenous growth mechanism as explicit R&D efforts that promote technological change. Particularly, directed technology change as in Acemoglu (2002) might interact with the female role in modern production in some interesting ways. On the one hand, increasing female years of schooling during the modern era might spur skill-biased technological change by the market size effect. On the other hand, skill-biased technological change that rises due to other reasons (say increase in male years of schooling) will also spur female to spend more time in education by raising the skill premium. The empirical exercise by Blau and Kahn (1997) supports this line of reasoning. Traditionally, women work relatively more in the unskilled sector. Thus, the recent rise in wage premium would at first sight seem detrimental to gender earnings inequality. But in fact the gender wage gap falls along with the rise in wage premium. This is likely because the rising wage premium induces women to become skilled. Thus, the rise of professional women in recent decades could be seen as both an effect and a cause of skill-biased technological change. Furthermore, the passive role of women in the pre-modern or early modern era might also imply that there is not enough market size for skill-biased change then. Thus, the "unskilled"-biased technology change that took place in the late eighteenth and early nineteenth centuries (Goldin and Katz, 1998) could be related to the gender role in production suggested in the present paper. A challenging extension is therefore to relate the history of directed technological changes with the changing female role in production.

References

Acemoglu D. Directed technical change. Review of Economic Studies 2002;69; 781-809.

Acemoglu D, Johnson S, Robinson JA. The colonial origins of comparative development: An empirical investigation. American Economic Review 2001;91; 1369-1401.

Barro RJ, Becker GS. Fertility choice in a model of economic growth. Econometrica 1989;57; 481-501.

Barro RJ, Lee JW. Data set for a panel of 138 countries. 1994.

Becker GS, Murphy KM, Tamura RF. Human capital, fertility, and economic growth. Journal of Political Economy 1990;98; Pt. 2; S12-S37.

Blau FD, Kahn LM. Swimming upstream: Trends in the gender wage differential in the 1980s. Journal of Labor Economics, 1997;15; Pt. 1; 1-42.

Ehrlich I, Lui FT. Intergenerational trade, longevity, and economic growth. Journal of Political Economy 1991;99; 1029-1059.

Galor O, Weil DN. The gender gap, fertility, and growth. American Economic Review 1996;86; 374-387.

Galor O, Weil DN. Population, technology, and growth: From Malthusian stagnation to the demographic transition and beyond. American Economic Review 2000;90; 806-828.

Goodfriend M, McDermott J. Early development. American Economic Review 1995;85; 116-133.

Goldin C. Understanding the gender gap: An economic history of American women. New York: Oxford University Press; 1990.

Goldin C. The U-Shaped female labor force function in economic development and economic history. NBER Working Paper No. 4707; 1994.

Goldin C. The rising (and then declining) significance of gender. NBER Working Paper No. 8915; 2002.

Goldin C, Katz KF. The origins of technology-skill complementarity. Quarterly Journal of Economics 1998;113; 693-732.

Hall RE, Jones CI. Why do some countries produce so much more output per worker than others. Quarterly Journal of Economics 1999;110; 83-116.

Hansen GD, Prescott EC. Malthus to Solow. American Economic Review 2002;92; 1206-1217.

Jones CI. R&D-based models of economic growth. Journal of Political Economy 1995;103; 759-784.

Jones CI. Was the industrial revolution inevitable? Economic growth over the very long run. Advances in Macroeconomics 2001;1; 1-43.

Jones CI. Sources of U.S. economic growth in a world of ideas. American Economic Review 2002;92; 220-239.

Kalemli-Ozcan S. Does the mortality decline promote economic growth? Journal of Economic Growth 2002;7; 411-439.

Kremer M. Population growth and technological change: One Million B.C. to 1990. Quarterly Journal of Economics 1993;108; 681-716.

Lagerlof NP. Gender equality and long-run growth. Journal of Economic Growth 2003;8; 403-26.

Lucas RE Jr. On the mechanics of economic development. Journal of Monetary Economics 1988;22; 3-42.

Lucas RE Jr. Why doesn't capital flow from rich to poor countries? American Economic Review 1990;80; 92-96.

Lucas RE Jr. The industrial revolution: Past and future. in Lectures on Economic Growth. Cambridge: Harvard University Press; 2002.

Maddison A. Monitoring the world economy, 1820-1992. Paris: OECD; 1995.

Maddison, A. The world economy - A millenial perspective. Paris: OECD; 2001.

Mammen K, Paxson C. Women's work and economic development. Journal of Economic Perspectives 2000;14; 141-164.

Quah DT. Twin peaks: Growth and convergence in models of distribution dynamics. Economic Journal 1996;106; 1045-1055.

Romer P. Endogenous technological change. Journal of Political Economy 94;1990; S71-S103.

Tamura RF. Human capital and the switch from agriculture to industry. Journal of Economic Dynamics and Control 2002;27; 207-242.

Young A. The tyranny of numbers. Quarterly Journal of Economics 1995;110; 641-680.

Zhang J, Zhang J, Li T. Gender bias and economic development in an endogenous growth model. Journal of Development Economics 1999;59; 497-525.

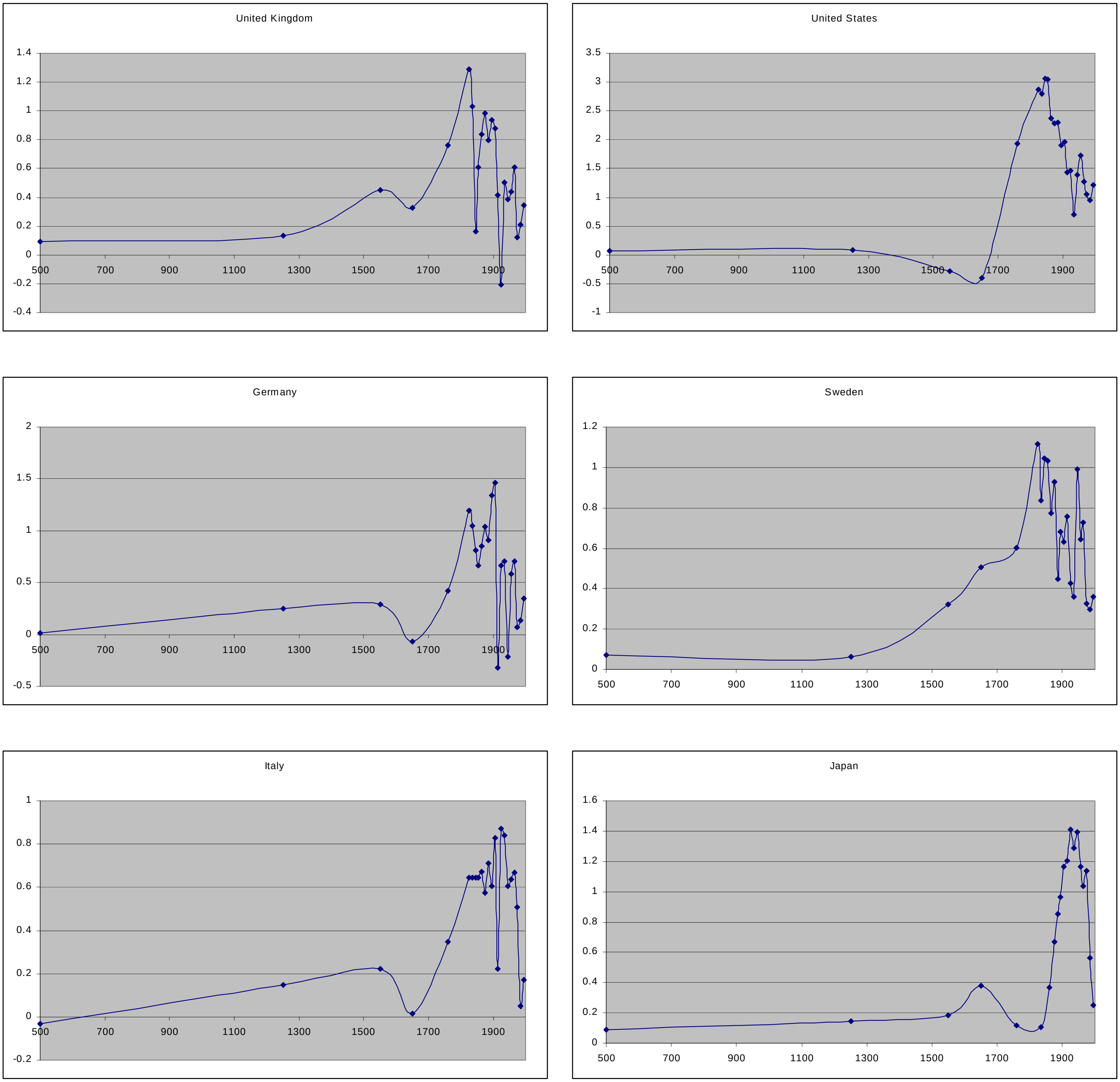


Figure 1: Demographic transitions in a representative sample of OECD countries. (Source: Maddison (2001))

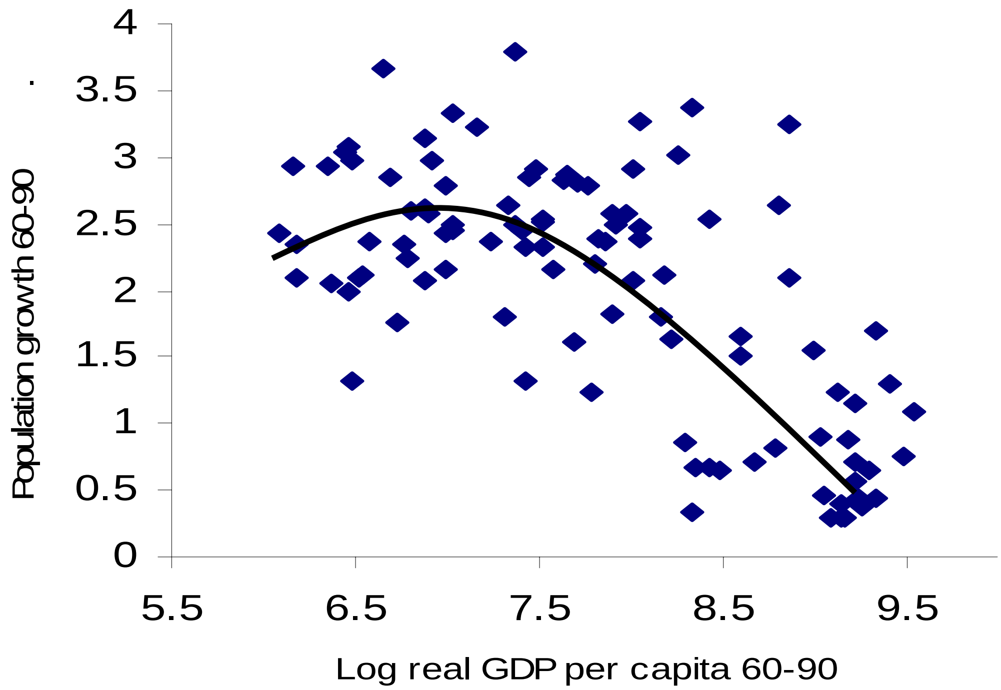


Figure 2a: Inverted U-shaped population growth with economic development in cross-country data. (Source: Barro and Lee (1994))

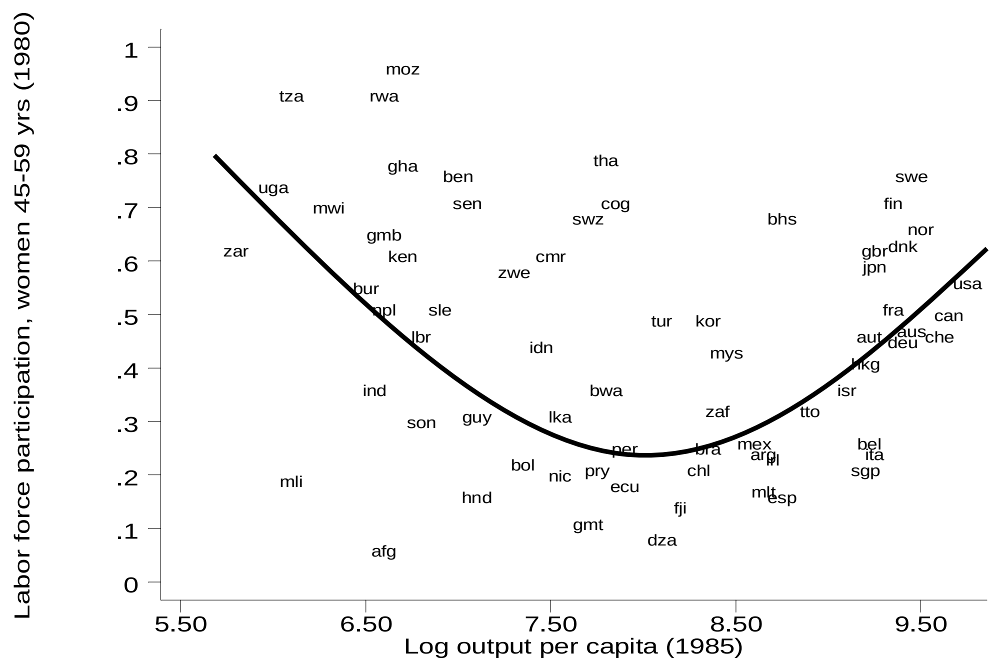


Figure 2b: U-shaped female labor participation with economic development in cross-country data. (Source: Goldin (1994))

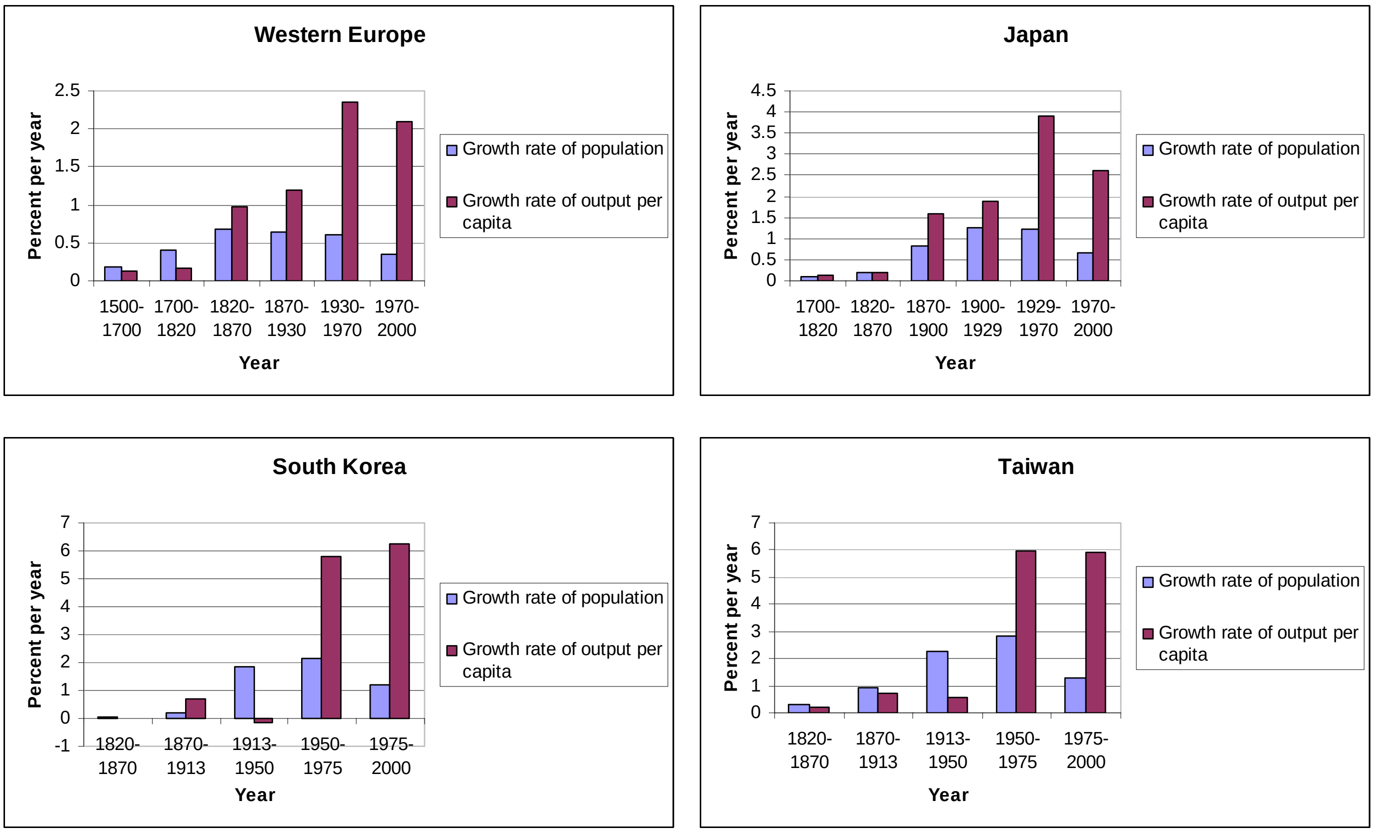
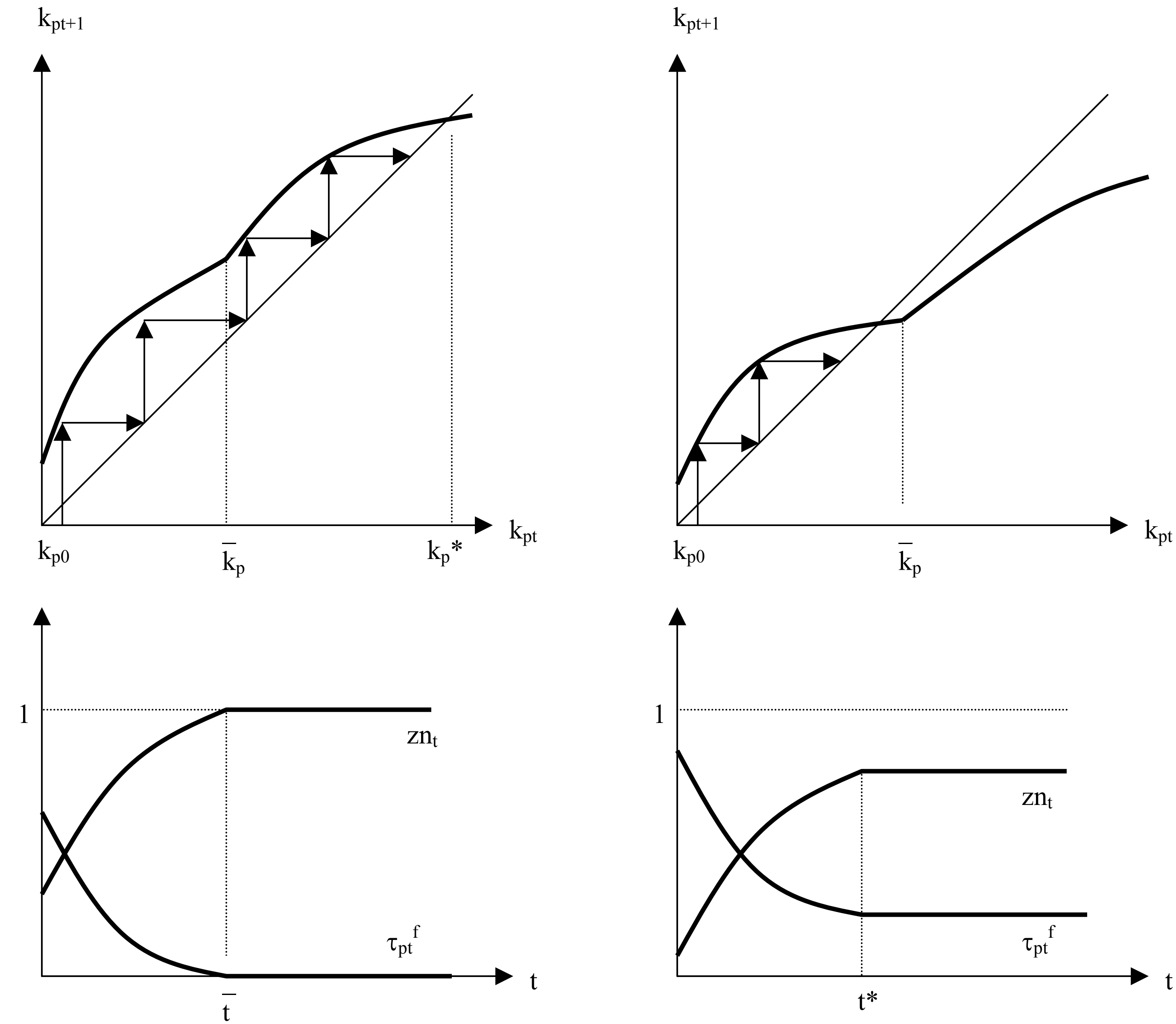


Figure 3: Demographic transitions and economic take-off in some representative early modernizers and newly industrialized countries (NICs) (Source: Maddison (2001))



Case 1: when B_p/A_p is sufficiently high

Case 2: when B_p/A_p is sufficiently low

Figure 4: Dynamics of k_{pt}, child-rearing time (zn_t) and female labor participation (τ_{pt}^f) in the pre-modern regime

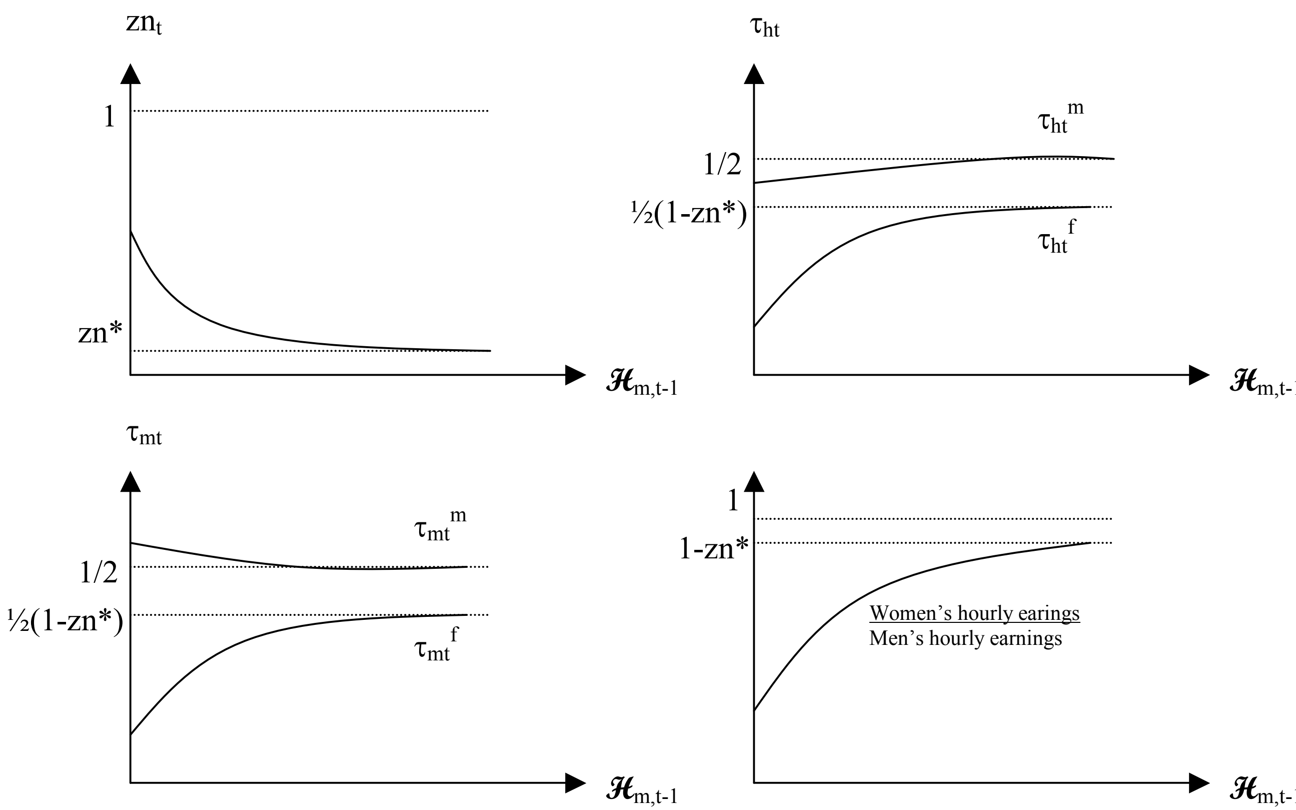


Figure 5: Dynamic paths for child-rearing time (zn_t) and female and male years of schooling (τ_{ht}^f and τ_{ht}^m), female and male labor participation (τ_{mt}^f and τ_{mt}^m) and gender gap in earnings in the modern regime

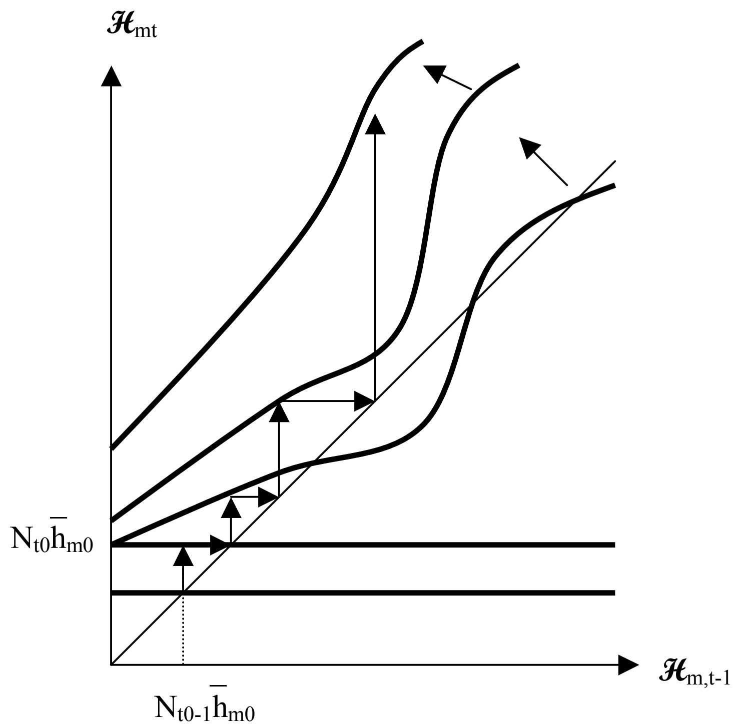


Figure 6: Dynamics of $\mathcal{H}_{mt}$ in the modern regime

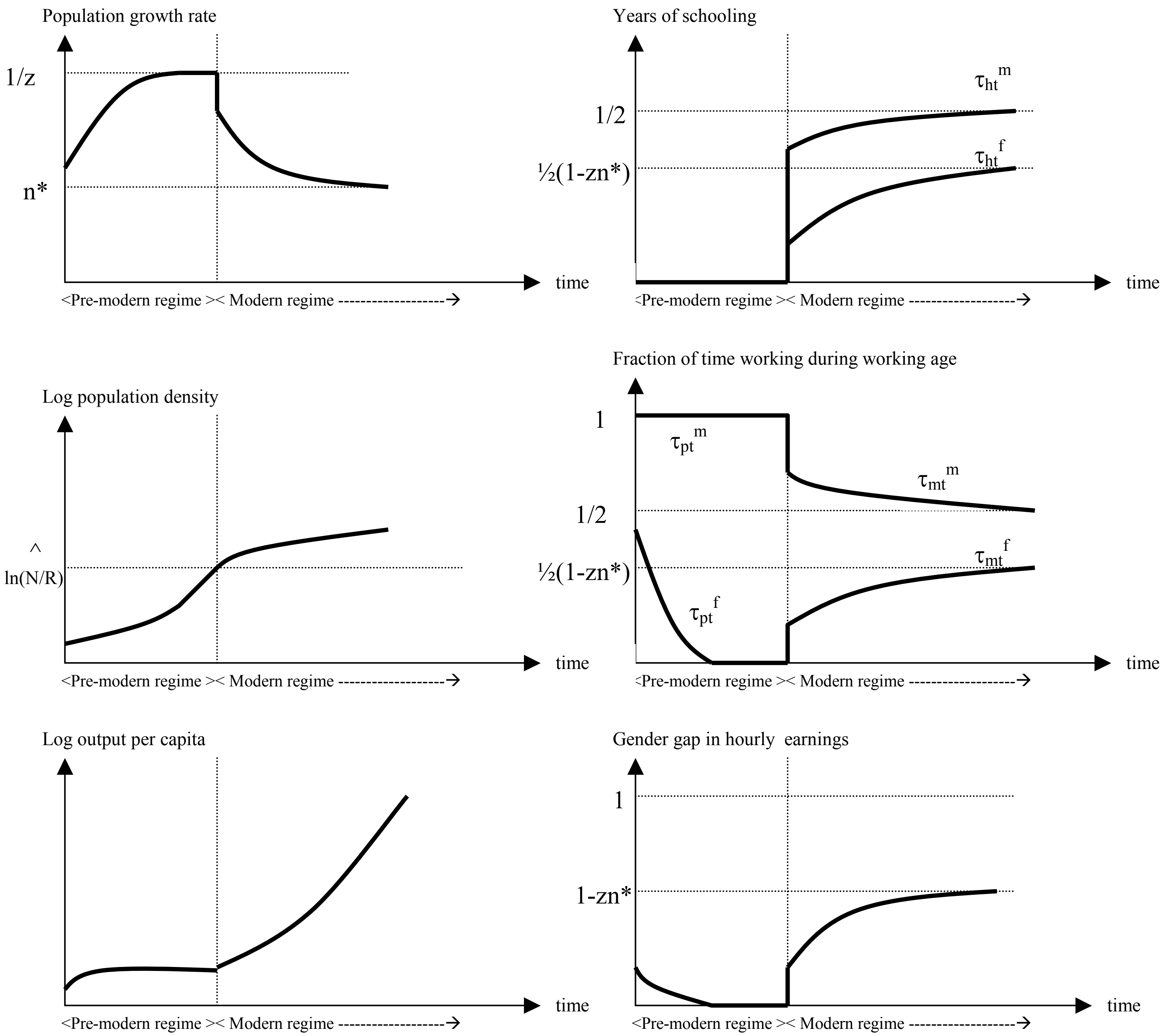


Figure 7: Time series implications of the model on population growth, population density, output per capita, years of schooling, labor participation and gender gap in earnings from the pre-modern regime to the modern regime

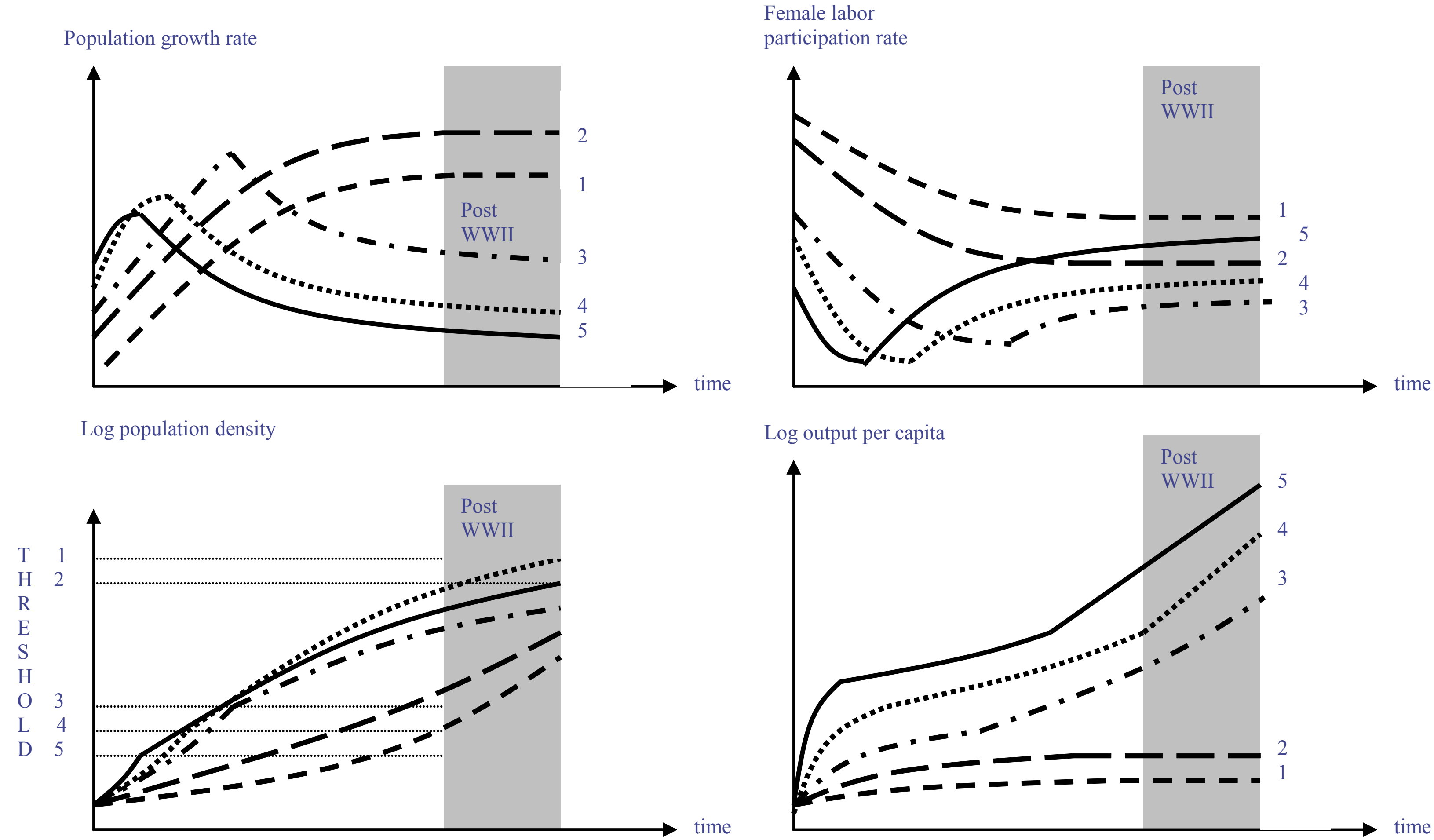


Figure 8: The timing of modernization, demographic transition and economic takeoff - A five-country comparison

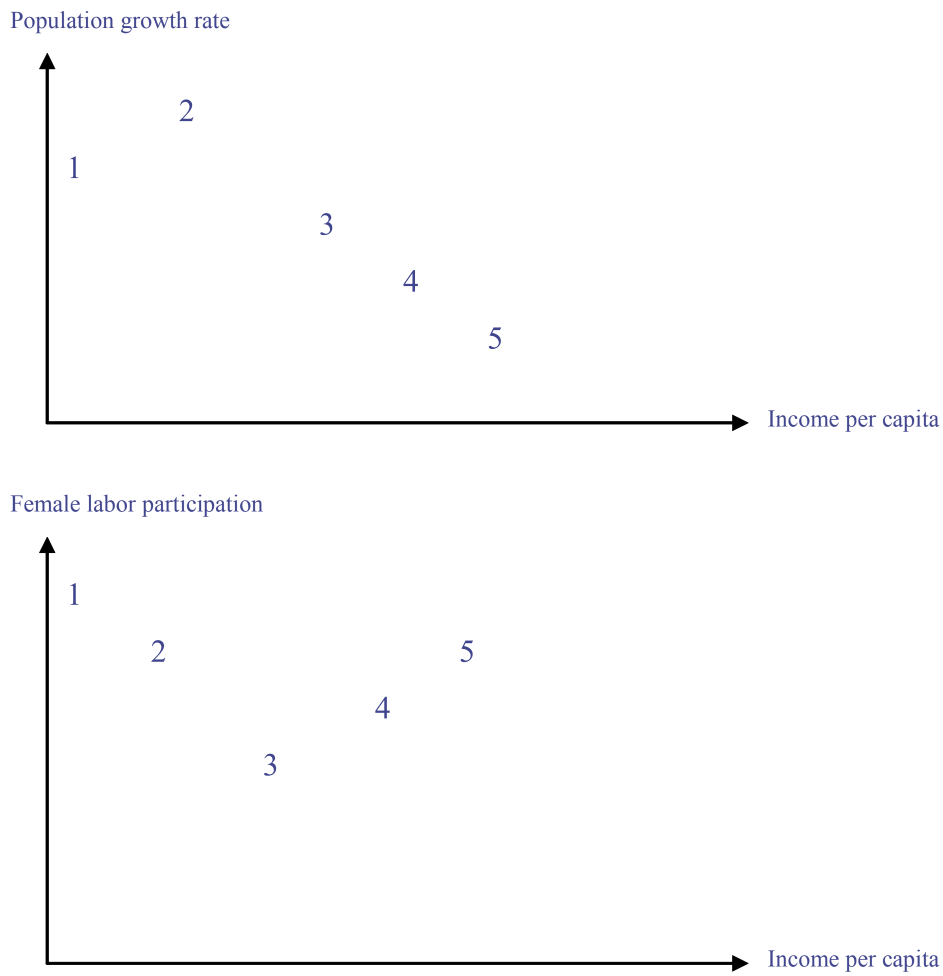


Figure 9: Cross-country implications of the model on the inverted-U relationship between population growth and economic development and the U-shaped female participation with economic development