Lobbying and Public Good Provision in a Federal Economy: A Dynamic Approach

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Abstract: *In this paper, we address the issue of dynamic lobbying within a federation that consists of a central government and two regional governments. Regions lobby for more central funds to produce a pure public good such as an improvement in environmental quality. Thus, benefits of lobbying are public, unlike what is traditionally assumed in the public choice literature. Lobbyists capture a part of the central grant; the other part is used to produce the public good. Allocation of central funds is dictated by two considerations: the level of lobbying as well as the effective use of central funds. Welfare to the consumer equals utility from the public good, net of lobbying cost. Unlike the regional governments, the central government is non strategic. We investigate steady state behaviour as well as welfare implications of different lobbying protocols: e.g. open and closed loop cooperative and non co-operative behaviour. We show that, for symmetric provinces, the steady state lobbying level and stock of public good are greater than the case when lobbyists are benevolent and cannot capture the rent. Second, introducing a rent-appropriating lobbyist may increase the welfare of the consumers compared to the case where the lobbyist is 'benevolent'. Third, for a wide range of parameter values, non-cooperative lobbying results in higher welfare for the consumer vis-à-vis cooperative lobbying protocols. Predictions of Fershtman and Nitzan (1991, hereafter FN) emerge as limiting cases of our model.*

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1 Introduction

Regional lobbying, in some form or other, is constitutionally valid in many federations, despite being denounced by public choice theorists (Tullock 1967; Krueger 1974; Posner 1975). The main argument against lobbying is that the benefits of lobbying are private. It leads to wastage of scarce resources. A direct policy implication is that in an ideal world, lobbying must be discouraged. In reality, the lobbying power of provinces often determines the pattern of centre-state transfers. Such a policy can be justified, on a normative basis, only if the benefits of federal grants are *public* in nature. At limit, the public good produced within the region is a pure public good within the region. Examples include environmental programme pursued by a certain province or provincial investment in knowledge.[2]

The objective of the present paper is to examine the dynamic provision of a public good in the presence of regional lobbying. The federation consists of a central government and two provinces. There exists one lobbyist (who is the local politician,say) in each province. Lobbying imposes a private cost on the residents of a region. A central grant flowing to a region is partly determined by regional lobbying. A fraction of the central grant is converted into a public good via a fixed coefficient technology. The rest might be captured by the lobbyist for his/her own benefit. Thus, the benefit of lobbying is public. Provinces differ in two dimensions: lobbying efficiency (proxied by cost of lobbying) and production efficiency (ability to convert one unit of central grant into the stock of public good). Lobbyists maximise the sum of the present values of the lifetime utility of the province and the lifetime rent accruing to the lobbyist.

We consider different types of strategies and behavioural protocols that can be adopted by the lobbyists. For example, lobbyists can undertake cooperative (maximising the sum of utilities of both the provinces and of rents accruing to both the lobbyists) or non co-operative (maximising the sum of utility of his/her own province and personal rent) protocols. Lobbyists can employ either open loop strategies, where they commit to a time path of efforts, or closed loop strategies where individual efforts at any given instant depends on the stock of public good at that moment (that is, Markov perfect, in the sense of Maskin and Tirole, 2001).[3] We compare and contrast our results with the scenario where

[2]The EPA Superfund is an example of federally funded provincial environmental programme. The Californian stem cell research programme, has it been partly funded by Federal government, would be a good example for knowledge investment.

[3]See appendix A1 for complete definitions. Economists are usually interested in properties of *closed loop*, *non cooperative* solutions for two reasons. First, cooperation is hard to sustain. Second, unlike open

the lobbyist cannot (or does not) capture the rent, and acts as a benevolent politician. Specifically, we ask three questions. What are the level of lobbying and public good stock at the steady state? What lobbying protocol is beneficial for the provincial consumers? Are the consumers better off in the presence of a non-benevolent (i.e. rent-seeking) lobbyist?

We now turn to a discussion of our main results. First, at least in the symmetric case, lobbying levels and the stock of public good increase compared to the case with the benevolent lobbyist for *all* lobbying protocols. We also found conditions such that the lobbying level is positive, i.e. the benefit from the public good and the rent outweighs the free rider problem. Second, using numerical analysis, we show that, at least for some parameter values, welfare accruing to the *consumer* is higher for noncooperative lobbying protocols than cooperative protocols. This result is in contrast with all standard models of dynamic public good provision (e.g. Fershtman and Nitzan, 1991, hereafter FN). Third, we show that non cooperative strategies with a rent capturing lobbyist can bestow higher welfare to the *consumer* compared to a benevolent lobbyist. This provides us with a normative yardstick against which to evaluate different lobbying protocols (e.g. as in List and Mason, 2001). Fourth, we show that, using asymmetric provinces, higher efficiency in public good production might lead to perverse effects on equalisation.

In a static setting, lobbying associated with public good is considered in Katz *et al.* (1990). They seek to investigate the level of lobbying when two localities lobby for a public good (such as removal of pollution). Removal of pollution is a pure public good within a certain locality and a private good among the jurisdictions. Their basic results and intuitions are very similar to the Tullock (1980) rent-seeking game in the sense that the lobbying efforts are less than the utility (value) of the local public good. In a recent paper, Cheikbossian (2006) considers the case where public goods within each region have an interjurisdictionary spillover effect. The aim of his paper is to identify the conditions under which decentralised outcome (when regions lobby for and produce a public good) provides higher surplus to the federation compared to the centralised outcome (when central government itself produces a public good in each region). However, in our opinion, these models, being static in nature, can not capture the dynamic reality of many environmental public good.

Provision of public good in a dynamic setting has been first systematically studied by FN. Extending the framework provided by Bergstrom *et al.* (1986) to a dynamic set up, they compare two different types of strategies adopted by the players. The basic result is

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loop equilibrium, a closed loop equilibrium is always *subgame perfect*, making it a desirable outcome for this class of games.

a pessimistic one: compared to an open loop case, with Markov perfect non cooperative strategies, each agent provides less effort and thus generates a lower amount of public good in the long run. Second, FN find that individual efforts are strategic substitutes: if one agent provides less effort, the opponent will provide more. It is also obvious that in such a setting, a cooperative strategy (if possible) will yield the highest level of consumer benefit. However, the FN results are not robust to alternative specifications. Exploiting the technique provided by Tsutsui and Mino (1990), Wirl (1996) showes that among the alternative equilibriums (static, open loop, cooperative), linear[4] Markov strategy provides the worst outcome (i.e. lowest effort and public good stock) vis-à-vis non linear ones.[5] Thus the 'pessimistic result' in FN depends on the assumption of linear strategies. Kessing (2007) considers the discrete good case (i.e. a public good to be provided in the future when the sum of contribution meets the cost) and, unlike FN, finds that individual efforts are strategic complements rather than substitutes.

Wirl (1994) considers lobbying in a dynamic set up. In order to explain why lobbying expenditure is smaller than the prize itself, he shows that lobbying expenditure is strictly less in the Markovian set up. In his analysis, lobbying is a constant sum game : thus a benefit to one group is a loss to the other. Other related studies include Tornel and Lane (1999), Tornel and Velasco (1994) and Long and Sorger (2006). In these models, competing interest groups (which can be interpreted as competing regions) lobby for and grab productive resources (owned by central government) for *private* benefit, thus affecting growth potentials of the economy.

To sum up, we abstract away from a world of pure private benefits of lobbying, as in the public choice literature, since it may not be appropriate for many public goods. On the other hand, we no longer believe that a lobbyist is necessarily benevolent, as a straightforward reading of the standard models of dynamic provision of public good suggests. Adopting a position somewhere in between, we show that noncooperative, closed loop lobbying may provide more welfare to the consumer than other lobbying protocols. And consumers are better off with a rent seeking lobbyist compared to a benevolent lobbyist under certain protocols.

The paper is divided into the following parts. In section 2, we set up the model. In section 3, we discuss the open loop case. The effects of closed loop or Markovian strategy is discussed in the fourth section. Section 5 analyses the symmetric equilibrium. In section

[4] That is, efforts are linear functions of the stock of public good.

[5] For a criticism of Tsutsui and Mino's methodology as well as a discussion of some recent issues on non-linear Markov strategies, see Rowat (2006).

6, we show some numerical results. Asymmetric provinces are discussed in section 7 and section 8 concludes.

2 The Model

The central government distributes funds p_i ($i = 1, 2$) to produce pure public good to two regions. Given the fund, the production of public good in each region is $\alpha_i p_i$ ($\alpha_i < 1$). There is no cost of provision of this good for the local government. However, the regions lobby for the fund, and lobbying is costly. Lobbying expenditure of region i is L_i. The overall public good stock is E. This evolves through time by the following equation:

$$\dot{E} = \Sigma \alpha_i p_i - mE \tag{2.1}$$

where m is the natural rate of 'decay' of overall stock of public good. This is a fairly standard representation of dynamic evolution of public good.[6]

For a real life example, $\alpha_i p_i$ might be expenditure on pure public good such as purification of air. Improved quality of air will benefit the federation as a whole due to symmetric spillover. Note that, α_i can be thought as an efficiency parameter.

As we have mentioned before, the central government's transfer to region i depends on both L_i and α_i. For simplicity, let us assume the linear transfer formula:

$$p_i = L_i + \lambda \alpha_i. \tag{2.2}$$

The parameter λ is the relative weight between lobbying and efficiency.[7] Thus, the central transfer formula consists of both discretionary and formulaic parts. The central decision maker matches the lobbying effort (discretionary), and provides an unconditional block grant (based on efficiency) to the regions.[8] In our formulation, central government is not a strategic player. This is also a somewhat standard simplifying assumption, e.g. in Tornel and Lane (1999) as well as Wirl (1994).[9]

[6] Our main results will follow if the spillover effect is less than 1, i.e. if we are not dealing with a pure public good.

[7] A more general specification is $p_i = \psi(L_i, \alpha_i)$, with $\psi_{L_i} > 0, \psi_{\alpha_i} > 0$

[8] In the above formulation, it seems that the central government does not face a budget constraint. However, if we change the funding equation by $p_i = 2\sqrt{L_i} + \lambda \alpha_i$, then this would reflect that central funds have some alternative use: the marginal reward for lobbying is decreasing. Incorporating this does not change our results significantly.

[9] A key feature of the fiscal federalism literature (e.g. Koethenbuerger, 2006) is that central government acts as a strategic agent. Assuming a naïve central governement, we abstract away from this issue.

The portion of the central fund which is not spent ('lost in transit') is appropriated by the local politicians (lobbyist) as rent. The rent thus equals $(1-\alpha_i)p_i$. We assume that the lobbyist maximises his/her own rent *plus* the net utility of the population. In other words, his/her behaviour is partly benevolent and partly selfish, and reasonably approximates the behaviour of a local politician. We further assume that the regions derive identical benefit ($U(E)$) from the pure public good E. The lifetime utility function of a lobbyist is given by:

$$\int_0^\infty e^{-\rho t} \left(U(E) + (1-\alpha_i)p_i - C_i(L_i) \right) dt \tag{2.3}$$

For the sake of tractability, we use the following assumptions of convenience:

(a) The utility function is linear-quadratic: $U(E) = E - \frac{\gamma}{2}E^2$. Here, γ is a small positive number such that $E_{\max} = \frac{1}{\gamma}$ is the satiation level of the public good. γ determines the rate at which the marginal utility falls.

(b) The cost function is convex and has the following simple structure $C_i(L_i) = \frac{1}{2}c_i L_i^2$, c_i being a constant.

These two assumptions imply that we are dealing with a standard linear quadratic game. Apart from tractability, another defence that has been put forward for such a game is that it might represent "Good Taylor approximation of more general games" (Fudenberg and Tirole, 1991).

Note that (2.3) can be decomposed into

$$\int_0^\infty e^{-\rho t} \left(E - \frac{\gamma}{2}E^2 - \frac{1}{2}c_i L_i^2 \right) dt + \left[\frac{\lambda \alpha_i (1-\alpha_i)}{\rho} + (1-\alpha_i) \int_0^\infty e^{-\rho t} L_i dt \right] \tag{2.3a}$$

The first term represents the consumer's benefit, while the second component (within squared bracket) equals the total rent appropriated by the lobbyist. This representation highlights the importance of inclusion of the cost term in lobbyists' utility. If the lobbyist does not care for the cost, he/she would demand infinite amount of fund.[10]

3 Open Loop Strategy

Open loop strategy requires that the players commit to a strategy from the beginning, and the path is dependent on time. In other words, the strategy space is described by

[10] If we put $\alpha_i = 1$, we are back to the FN model (1991). Thus, it is a special case of our own model.

$L_i = f(t) \forall t$. Such a strategy may work if both players make a binding agreement of adhering to it.

3.1 Open Loop Cooperative Behaviour

If the lobbyists maximise their joint utility,[11] the maximand becomes

$$\int_0^\infty e^{-\rho t} \left(2E - \gamma E^2 + \Sigma(1-\alpha_i)p_i - \frac{1}{2}\Sigma c_i L_i^2 \right) dt$$

Let μ be the costate variable. Then the current value Hamiltonian for the problem can be written as (plugging in the values for p_i)

$$\begin{aligned} H^1 &= 2E - \gamma E^2 + (1-\alpha_1)(L_1 + \lambda\alpha_1) + (1-\alpha_2)(L_2 + \lambda\alpha_2) - \frac{1}{2}c_1 L_1^2 - \frac{1}{2}c_2 L_2^2 \\ &\quad + \mu\left(\alpha_1(L_1 + \lambda\alpha_1) + \alpha_2(L_2 + \lambda\alpha_2) - mE\right) \\ &= 2E - \gamma E^2 + (1-\alpha_1)L_1 + (1-\alpha_2)L_2 + A_3 - \frac{1}{2}c_1 L_1^2 - \frac{1}{2}c_2 L_2^2 \\ &\quad + \mu\left(\alpha_1 L_1 + \alpha_2 L_2 + A_2 - mE\right) \end{aligned}$$

Where, $A_2 = \lambda(\alpha_1^2 + \alpha_2^2)$

$A_3 = \lambda(1-\alpha_1)\alpha_1 + \lambda(1-\alpha_2)\alpha_2$

The SOC is satisfied since $\dfrac{\partial^2 H^1}{\partial L_i^2} = -2c_i < 0$

Let us assume an interior equilibrium ($L_i > 0$).

The necessary conditions are

$$(1-\alpha_i) - c_i L_i + \alpha_i \mu = 0 \tag{3.1.1}$$

$$\dot{\mu} = (\rho + m)\mu - 2(1 - \gamma E) \tag{3.1.2}$$

$$\dot{E} = \alpha_1 L_1 + \alpha_2 L_2 + A_2 - mE \tag{3.1.3}$$

Proposition 1. *The open loop co-operative behaviour results in the following steady state values of E and L_i*

$$E_s^{Oc} = \frac{2A_1 + (s_1\alpha_1\beta_1 + s_2\alpha_2\beta_2 + A_2)(m+\rho)}{m(\rho+m) + 2\gamma A_1}$$

[11] In Mason and List (2001), the cooperative, or centralised outcome entails equal efforts for both regions.

$$L_{is}^{Oc} = \beta_i \frac{m(2+\rho s_i + m s_i) - 2\gamma(A_2 - d_i\alpha_j\beta_j)}{m(\rho+m) + 2\gamma A_1}$$

Where, $\beta_i = \dfrac{\alpha_i}{c_i}$, $A_1 = \Sigma \dfrac{\alpha_i^2}{c_i}$, $s_i = \dfrac{1-\alpha_i}{\alpha_i}$, $d_i = s_i - s_j$

The steady state is stable, in a saddlepoint sense. And, given certain restriction on parameters, $E_s^{Oc} < \dfrac{1}{\gamma}$.

Proof. See appendix 1. ■

Note that, $\dfrac{1}{c_i}$ is a measure of efficiency of region i's lobbyist (the higher is $\dfrac{1}{c_i}$, the lower is the effort cost),and α_i is a measure of region $i's$ efficiency of use of the public fund. So, β_i, the product of these two measures, is an overall measure of efficiency of region i.

Comment 1: Lobbying may be zero if γ or A_2 is too high. From the neccesary condition (3.1.3), A_2 is the 'autonomous' growth of E when lobbying is zero. γ is the rate at which MU of public good goes down. If these are high, the province may not altogether engage in costly lobbying. Lobbying is highest when $\lambda = 0$, i.e.when the centre allocates only on the basis of lobbying. In that case, $A_2 = A_3 = 0$.

Comment 2: Compared to the benevolent world[12], it seems that $E_s^{Oc} > E_{sB}^{Oc}$. However, it is not necessarily true that $L_{is}^{Oc} > L_{isB}^{Oc}$. Here the subscript B signifies associated variables in the model with benevolent lobbyists.

Comment 3: Unlike the benevolent case, the ratio of steady state lobbying levels is no longer represented by ratio of relative efficiency

We can also represent the open loop behaviour as a 'closed loop solution' , where L_i is a function of E. To do this, we must solve the differential equations in terms of E and L explicitly, and make the necessary substitutions. The following proposition summarizes our main findings with regard to the 'closed loop' form.

Proposition 2. *The equilibrium open loop, cooperative lobbying can be represented in the closed loop form as*

$$L_i = a_{0i} - b_{0i}E$$

where,

$$b_{0i} = -\frac{\beta_i}{2A_1}\left(2m + \rho - \sqrt{8\gamma A_1 + (2m+\rho)^2}\right) > 0$$

$$a_{0i} = L_{is}^{Oc} + b_{0i}E_s^{Oc} > 0$$

[12] See appendix 2.

Proof. See appendix 1. ■

Comment 4: The positive value of b_{0i} parallels the FN result that efforts are strategic substitutes(although, in case of cooperation, the word 'strategic' is somewhat misleading). If one region lobbies less, the other one will lobby more.

3.2 Open loop Non cooperative behaviour:

If the regions play a non co-operative game, let π_i be costate variable of region i.Then the relevant Hamiltonian is

$$\bar{H}_i = E - \frac{\gamma}{2}E^2 + (1-\alpha_i)(L_i + \lambda\alpha_i) - \frac{1}{2}c_i L_i^2 + \pi_i\left(\alpha_1(L_1 + \lambda\alpha_1) + \alpha_2\left(L_2 + \lambda\alpha_2\right) - mE\right)$$

This is concave in control variable.

The necessary conditions are

$$(1-\alpha_i) - c_i L_i + \pi_i\alpha_i = 0 \tag{3.2.1}$$

$$\dot{\pi}_i = (\rho + m)\pi_i - (1-\gamma E) \tag{3.2.2}$$

$$\dot{E} = A_2 + \Sigma\alpha_i L_i - mE \tag{3.2.3}$$

In this case, we can summarise the steady state properties[13] in the following theorem:

Proposition 3. *The steady state , stable values of public good stock and lobbying is given by the following expressions*

$$E_s^{Onc} = \frac{A_1 + (s_1\alpha_1\beta_1 + s_2\alpha_2\beta_2 + A_2)(m+\rho)}{m(\rho+m) + \gamma A_1}$$

$$L_{is}^{Onc} = \beta_i \frac{m(1 + s_i(m+\rho)) - \gamma\left(A_2 - d_i\alpha_j\beta_j\right)}{m(\rho+m) + \gamma A_1}$$

where the superscript nc implies no cooperation, O refers to open loop and the subscript s stands for steady state.[14]

Proof. See appendix 1. ■

As usual, we can represent the open loop strategies into a closed loop form in the following proposition :

[13] Comments analogous to 1,2, 3 apply here as well.

[14] Given some restrictions on the parameters, there is no "over-accumulation" of steady state stock.

Proposition 4. *The closed loop form of open loop ,non-cooperative lobbying can be represented as*

$$L_i = a_{1i} - b_{1i}E$$

where,

$$b_{1i} = -\frac{\beta_i}{2A_1}\left(2m + \rho - \sqrt{4\gamma A_1 + (2m+\rho)^2}\right) > 0$$

$$a_{1i} = L_{is}^{Onc} + b_{1i}E_s^{Onc} > 0$$

Proof. See appendix 1. ■

Now we can compare the cooperative and non cooperative strategies. The steady state behaviours are reported in the following proposition:

Proposition 5. *For the open loop case, if γ is small enough (to prevent overprovision of public good), then cooperative strategy induces more public good as well as higher lobbying expenditure at the steady state.*

Proof. See appendix 1. ■

Comment 5: The necessary and sufficient conditions for $E_s^{Oc} > E_s^{Onc}$ and $L_{is}^{Oc} > L_{is}^{Onc}$ turn out to be that of preventing overaccumulation of public good.

Comment 6: The proposition parallels the fact that in noncooperative case, agents try to free ride and thus provide less effort. Therefore, the noncooperative stock of public good and efforts (here, lobbying is analogous to effort) will be less than the cooperative case

The following lemmas compare the closed loop forms of co-operative and non cooperative lobbying.

Lemma 1 $b_{0i} > b_{1i}$: *as E goes up, the marginal reduction in lobbying is higher in cooperaive case than in non cooperative case.*

This follows immediately from comparing

$$b_{1i} = -\frac{\beta_i}{2A_1}\left(2m + \rho - \sqrt{4\gamma A_1 + (2m+\rho)^2}\right)$$

and

$$b_{0i} = -\frac{\beta_i}{2A_1}\left(2m + \rho - \sqrt{8\gamma A_1 + (2m+\rho)^2}\right)$$

The above inequality implies that, given a decrease in opponent's lobbying, a province will lobby for more in case of cooperative lobbying. Therefore the degree of strategic substitutability goes down.

Lemma 2 $a_{0i} > a_{1i}$: *when* E *is small (i.e. almost* 0*), the effort is higher in cooperative case than in non co-operative case.*

This is a corollary to proposition 5 and the above lemmas : $L_{is}^{Oc} > L_{is}^{Onc}$ and $E_s^{Oc} > E_s^{Onc}$ as long as $\dfrac{m}{A_2 + A_1 s_1 - d_1 \alpha_2 \beta_2} > \gamma$. Thus, $\left(L_{is}^{Oc} + b_0\ E_s^{Oc}\right) > \left(L_{is}^{Onc} + b_1\ E_s^{Onc}\right)$

As before, if we write $L_i^{Oc} = a_0 - b_0 E$ and $L_i^{Onc} = a_1 - b_1 E$, then solving this set of linear equations, we can see that the unique solution is at $E_A = \dfrac{a_0 - a_1}{b_0 - b_1}$ and $L_A = \dfrac{1}{b_0 - b_1} (a_1 b_0 - a_0 b_1)$. Since $a_0 > a_1$ and $b_0 > b_1$, there exists a positive E_A such that these equations are satisfied. However, its not certain whether L_A is positive or not. The denominator is always positive. The numerator can be written as

$b_0 L_{is}^{Onc} - b_1 L_{is}^{Oc} + b_0 b_1 (E_s^{Onc} - E_s^{Oc})$, which is negative if $\left(b_0 L_{is}^{Onc} - b_1 L_{is}^{Oc}\right) < 0$. It is not possible to compare these two analytically.

These two possibilities are summarised in the following two graphs. On the horizontal axis, we have the state variable, i.e. stock of public good, and the vertical axis represents lobbying.

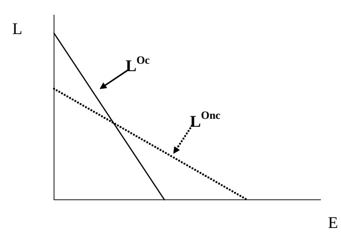

Figure 1: $L_A > 0$; open loop strategies

and

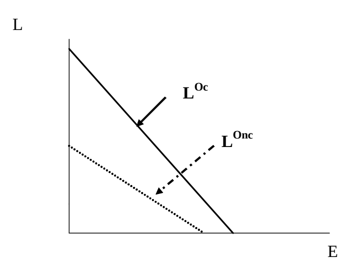

Figure 2: $L_A < 0$; open loop lobbying

The discussion can be summarised in the following proposition:

Proposition 6. *For open loop case, if E is small, cooperative lobbying dominates non cooperative effort. On the other hand, when E is large, it may or may not dominate the latter. However, given an increase in E, the marginal decrease in lobbying effort is higher in cooperative case.*

4 Closed loop Strategies

Stationary closed loop strategies employ $L_i = F_i(E)$, i.e. the control variable is a function of the state variable . Each lobbyist understands that lobbying creates more public good, which is related with utility. Therefore, it is reasonable to base costly lobbying effort on the current value of stocks. As before, we begin with cooperative strategy.

4.1 Closed Loop Cooperative Strategy

For co-operative solution the utility is the sum of two regions . The relevant HJB equation is,

$$\rho V(E) = \max_{L_i} \left[2E - \gamma E^2 + \Sigma(1-\alpha_i)(L_i + \lambda\alpha_i) - \frac{1}{2}\Sigma c_i L_i^2 + V'(E)(\Sigma\alpha_i p_i - mE) \right]$$

Where $V(E)$ is the value function.

Maximisation of the LHS implies

$$L_i = \beta_i(s_i + V'(E)) \tag{4.1.1}$$

Where $\beta_i = \frac{\alpha_i}{c_i}$

Substituting this back into the RHS, the HJB equation reduces to

$$\rho V(E) = 2E - \gamma E^2 + \frac{1}{2}A_1 + A_3 + \frac{1}{2}(V'(E))^2 A_1 + V'(E)(A_1 + A_2) - mEV'(E)$$

Given the linear quadratic structure of the game, we posit that the value function is quadratic. Let $V(E) = a + bE + \frac{c}{2}E^2$ be the trial solution

Then

$$V'(E) = b + cE$$

and the lobbying effort is

$$L_i = \beta_i(s_i + b + cE) \tag{4.1.1a}$$

The cooperative structure implies that we are solving a dynamic optimisation problem, and there is no difference between open and closed loop linear outcomes. The difference, if any, occurs when the lobbyists employ non-linear strategy.

4.2 Closed Loop Non Cooperative Strategy

For non cooperative strategy, each region maximises its own benefit. Consequently, each of the regions has its own value function. Let the function be denoted as $W_i(E)$. The HJB equation of lobbyist is give by,

$$\rho W_i(E) = \max_{L_i}\left[E - \frac{\gamma}{2}E^2 + (1-\alpha_i)(L_i + \lambda\alpha_i) - \frac{1}{2}c_i L_i^2 + W_i'(E)(\Sigma\alpha_i p_i - mE)\right]$$

Again, we postulate a quadratic value function (to ensure linear strategies)

$$W_i = d_i + k_i E + \frac{f_i}{2}E^2$$

Proposition 7. *The coefficients of the value function are determined by the following equations:*

$$f_i = \frac{1}{2\alpha_i\beta_i}\left(2m + \rho - 2f_j\alpha_j\beta_j \pm \sqrt{4\gamma\alpha_i\beta_i + \left(2m + \rho - 2f_j\alpha_j\beta_j\right)^2}\right)$$

$$k_i = \frac{A_2 f_i + f_i\beta_i - f_i\alpha_i\beta_i + k_j f_i\alpha_j\beta_j + f_i s_i\alpha_i\beta_i + f_i s_j\alpha_j\beta_j - c_i f_i s_i\beta_i^2 + 1}{m + \rho - 2f_i\alpha_i\beta_i - f_j\alpha_j\beta_j + c_i f_i\beta_i^2}$$

$$\begin{aligned}\rho d_i \quad = \quad & \lambda\alpha_i + k_i\beta_i + s_i\beta_i - k_i\alpha_i\beta_i - s_i\alpha_i\beta_i + k_i k_j\alpha_j\beta_j + k_i s_i\alpha_i\beta_i \\ & + k_i s_j\alpha_j\beta_j - \lambda\alpha_i^2 + k_i^2\alpha_i\beta_i - k_i c_i s_i\beta_i^2 - \frac{1}{2}k_i^2 c_i\beta_i^2 - \frac{1}{2}c_i s_i^2\beta_i^2\end{aligned}$$

The negative value of f_i is chosen to ensure stability. The steady state value of E is given by

$$E_s^{CNc} = \frac{\beta_1 + \beta_2 - A_1 + A_2 + \alpha_1\beta_1 k_1 + \alpha_2\beta_2 k_2}{m - f_1\alpha_1\beta_1 - f_2\alpha_2\beta_2}$$

Proof. See appendix 1. ■

The lobbying effort is given by $L_i = \beta_i(s_i + k_i + f_i E)$. For stability, we need that $f_i < 0$.

Unfortunately, it is not possible to solve the equations explicitly. In the next section, we impose some restrictions on the parameter space in order to obtain analytical solutions.

5 Symmetric Equilibria

5.1 Steady State Public Good Stocks and Lobbying

We restrict ourselves to the set of symmetric solution: both regions have similar efficiency and cost parameters.

Let $\alpha_i = \beta x, c_i = x$, such that $\beta_i = \beta$

We have,

$A_1 = 2\beta^2 x$

$A_2 = 2\lambda\beta^2 x^2$

Open Loop Cooperative Outcome:

$$L_{1s}^{Oc} = \frac{\beta}{m(m+\rho) + 4x\beta^2\gamma}\left(m\left(\frac{(m+\rho)}{x\beta}(1-x\beta)+2\right) - 4x^2\beta^2\lambda\gamma\right)$$

$$E_s^{Oc} = \frac{4x\beta^2 + (m+\rho)\left(2\beta(1-x\beta)+2x^2\beta^2\lambda\right)}{m(m+\rho)+4\gamma x\beta^2}$$

There are two sources of increase in relative efficiency: either through increase in $\alpha(=\beta x)$ or through decrease in $c(=x)$.

Open Loop Noncoopearive Outcome:

$$L_{1s}^{Onc} = \frac{\beta}{m(m+\rho)+2\gamma x\beta^2}\left(m\left(\frac{1}{x\beta}(m+\rho)(1-x\beta)+1\right) - 2x^2\beta^2\lambda\gamma\right)$$

$$E_s^{Onc} = \frac{2x\beta^2 + (m+\rho)\left(2\beta(1-x\beta)+2x^2\beta^2\lambda\right)}{m(m+\rho)+2\gamma x\beta^2}$$

It is clear that[15] $E_s^{Oc} > E_{Bs}^{Oc}$;$E_s^{Onc} > E_{Bs}^{Onc}$; $L_{1s}^{Onc} > L_{1Bs}^{Onc}$; $L_{1s}^{Oc} > L_{1Bs}^{Oc}$ as long as $\alpha = x\beta < 1$.

Comment 7: The subscript B refers to the benevolent lobbyist case. Thus, at least for the symmetric outcome, the lobbying and public good stock will be greater than the world of benevolent lobbyist.[16]

Closed Loop Noncoperative Outcome:

Needless to say, we shall have $f_i = f, k_i = k$ and $d_i = d$

Proposition 8. *The symmetric solution implies the following value function*

$$W(E) = d + kE + \frac{f}{2}E^2$$

[15] See appendix 2.

[16] See proposition 1, comment 2.

where,

$$f = \frac{1}{6x\beta^2}\left((2m+\rho) - \sqrt{12x\beta^2\gamma + (2m+\rho)^2}\right)$$

$$k = \frac{2f\beta - 2fx\beta^2 + 2fx^2\beta^2\lambda + 1}{m+\rho-3fx\beta^2}$$

Proof. See appendix 1. ■

It can be seen that $\frac{\partial f}{\partial \gamma} < 0$ and $\frac{\partial f}{\partial A} > 0$, where $A = 2m+\rho$

The symmetric SS stock of public good is given by

$$E_s^{CNc} = \frac{2\beta - 2x\beta^2 + 2kx\beta^2 + 2x^2\beta^2\lambda}{m - 2fx\beta^2} \tag{5.1.1}$$

As before, if we posit the lobbying effort as $L^{Cnc} = a_2 - b_2 E$, then

$$b_2 = -\beta f = -\frac{1}{6x\beta}\left((2m+\rho) - \sqrt{12x\beta^2\gamma + (2m+\rho)^2}\right)$$

and

$$a_2 = \beta\,(s+k)$$

For the 'closed loop form' of open loop lobbying protocols,

$b_1 = -\frac{\beta_i}{2A_1}\left(2m+\rho - \sqrt{4\gamma A_1 + (2m+\rho)^2}\right) = -\frac{1}{4x\beta}\left(2m+\rho - \sqrt{8x\beta^2\gamma + (2m+\rho)^2}\right)$

$b_0 = -\frac{\beta_i}{2A_1}\left(2m+\rho - \sqrt{8\gamma A_1 + (2m+\rho)^2}\right) = -\frac{1}{4x\beta}\left(2m+\rho - \sqrt{16x\beta^2\gamma + (2m+\rho)^2}\right)$

The following lemma compares the slopes of closed loop forms non cooperative lobbying.

Lemma 3 *For symmetric case, $b_1 \geq b_2$. That is, the marginal reduction in lobbying for closed loop strategy is lower than open loop counterpart.*

Proof. As $x \to 0$, $b_2 \to \frac{\beta\gamma}{2m+\rho}$ and $b_1 \to \frac{\beta\gamma}{2m+\rho}$(applying L'Hôpital's rule) so $b_1 = b_2$

Now,

$b_1 \geq b_2$,

or, $-b_2 + b_1 \geq 0$

i.e.

$$\frac{1}{6x\beta}\left((2m+\rho) - \sqrt{12x\beta^2\gamma + (2m+\rho)^2}\right) - \frac{1}{4x\beta}\left(2m+\rho - \sqrt{8x\beta^2\gamma + (2m+\rho)^2}\right) \geq 0$$

when $x \neq 0$, this implies

$\Rightarrow 2\left(A - \sqrt{12x\beta^2\gamma + A^2}\right) - 3\left(A - \sqrt{8x\beta^2\gamma + A^2}\right) \geq 0 \;\mid\;$ where, $A = 2m + \rho$

If $x = \varepsilon > 0$ and very small, then the difference ≈ 0 and both expressions becomes equal.

Differentiating the expression w.r.t. x,

$$-2\frac{12\beta^2\gamma}{2\sqrt{12x\beta^2\gamma + A^2}} + 3\frac{8\beta^2\gamma}{2\sqrt{8x\beta^2\gamma + A^2}}$$

$$= -\frac{12\beta^2\gamma}{\sqrt{12x\beta^2\gamma + A^2}} + \frac{12\beta^2\gamma}{\sqrt{8x\beta^2\gamma + A^2}} > 0, \text{ since } \sqrt{8x\beta^2\gamma + A^2} < \sqrt{12x\beta^2\gamma + A^2}$$

Since this is an increasing function, the difference is strictly positive as x increases . In other words, $b_1 > b_2$ when $x > 0$. ■

Thus, given an decrease in L_j (and therefore,E), the increase in lobbying effort by region i is least in non cooperative closed loop case: the degree of strategic substitutability further goes down.

6 Numerical Analysis

We continue our analysis of symmetric equilibria. Unless otherwise mentioned, we willl take $\lambda = 1$, $\rho = m = .5$, and $\gamma = .01$. Sensitivity analysis shows that our results are robust.

6.1 Steady State Lobbying and Public Good

In this section, our concern will be to look at the steady state values of stock and lobbying levels under different lobbying protocols. Specifically, three possibilities arise.

6.1.1 β Constant, both c and α increases proportionally

The basic message is the following: keeping β constant, when both alpha and cost increases, lobbying effort goes down and steady state stock increases. Usually, the cooperative strategy entails highest stock of public good as well as highest lobbying. This is consistent with standard public good result: in a centralised environment, the efforts as well as public good stock would be highest.

We have plotted $x(= c)$ on the x-axis. It can be seen that E_s^{Onc} and E_s^{Cnc} almost overlap. In reality, the former is slightly greater throughout.

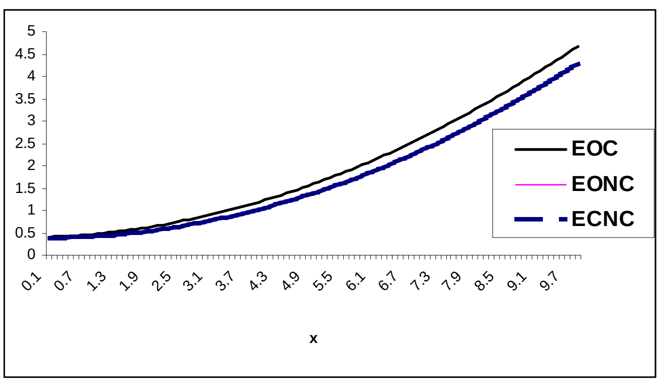

Figure 3: SS Stock for $\beta = .1$

We redraw the same variables for a higher value of β.

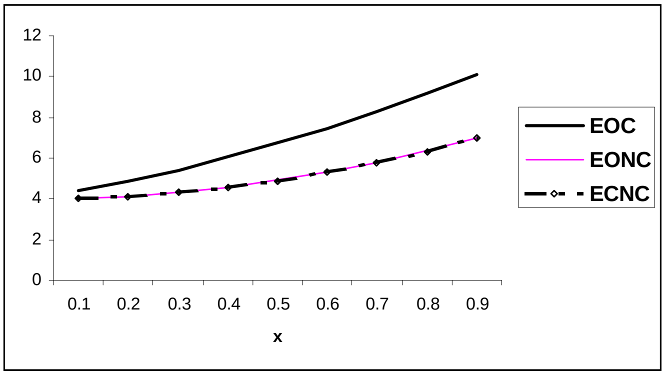

Figure 4: SS stock for $\beta = 1$

Therefore, as β increases, the steady state stock of public good will increase for a given α and c.

The following pair of diagrams depict SS lobbying for a given β and increasing x. As α and c increase, cost of lobbying

goes up and incentive for lobbying goes down *without any effect* on relative efficiency. This discourages lobbying.

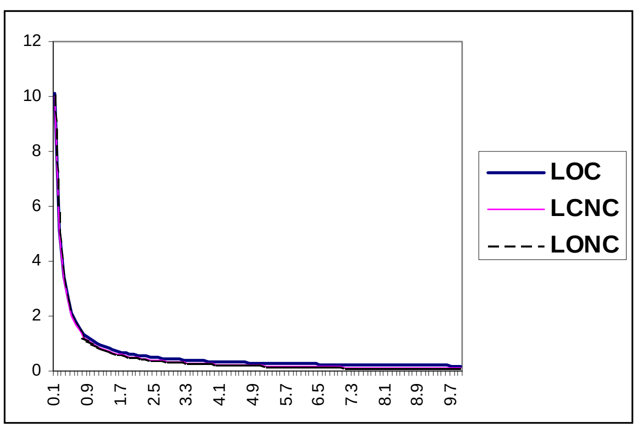

Figure 5: SS lobby with $\beta = .1$

It is somewhat hard to distinguish, but cooperative lobbying (LOC) is greater than both closed loop noncooperative ($LCNC$) and open loop cooperative ($LONC$) lobbying levels. $LONC$ is slightly greater than $LCNC$: effort in open loop is higher than the closed loop case.

And

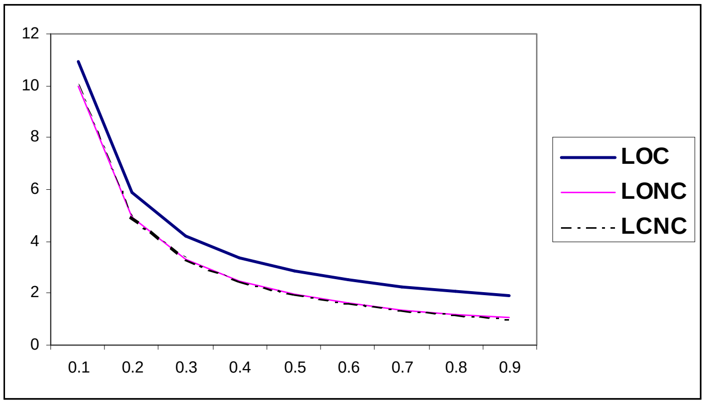

Figure 6: SS lobby with $\beta = 1$

Therefore, with increasing β, lobbying will be higher for a given x.

Result 1: *Keeping β constant, as α and c increase, SS lobbying falls and SS output goes up.*

6.1.2 Constant c, increase in efficiency through increase in α

Now we want to analyse the specific case when the increase in β comes through increase in α. As α goes up, the relative efficiency of the province increases, but also the share to the lobbyist goes down. So it is not clear whether the lobbyist will have enough incentive to lobby more.

It can be seen that co-operative lobbbying increases, but noncooperative lobbying actually decreases as β increases through increase in α. Here, we have taken $c = x = 1$ and plot β in the horizontal axis.

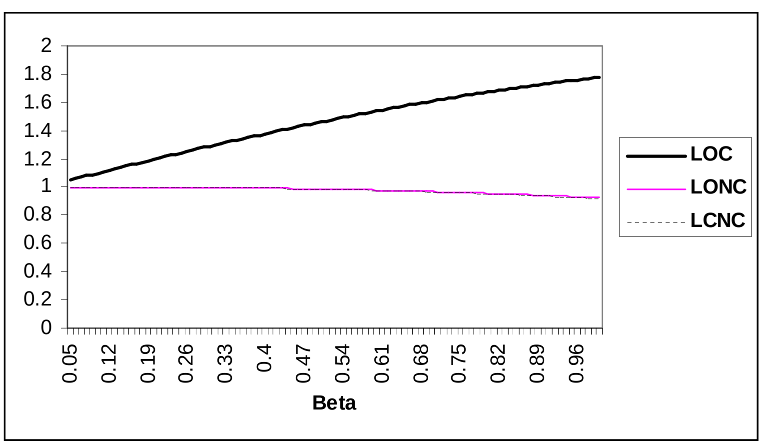

Figure 7: SS Lobby with $\rho = .5$

However, when ρ decreases, the trend is somewhat altered. With increased β, lobbying goes up for all lobbying protocols.

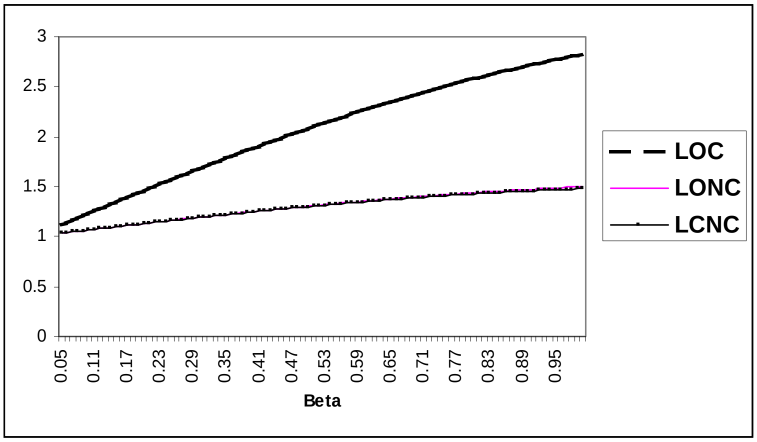

Figure 8: SS lobby with $\rho = .1$

The change in lobbying behaviour comes somewhere between $\rho = .3$ and $\rho = .4$. Thus, with non cooperative lobbying, decrease in share of rent actually reduces lobbying effort if ρ is 'high enough'.

In both cases though, SS stock of public good increases

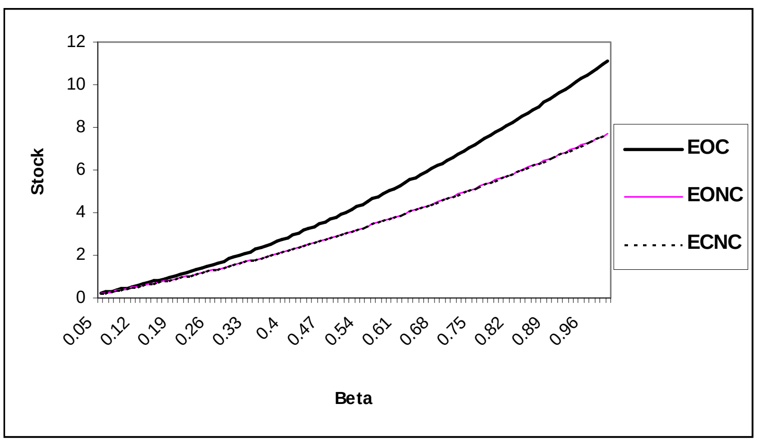

Figure 9: SS stock with $\rho = .5$

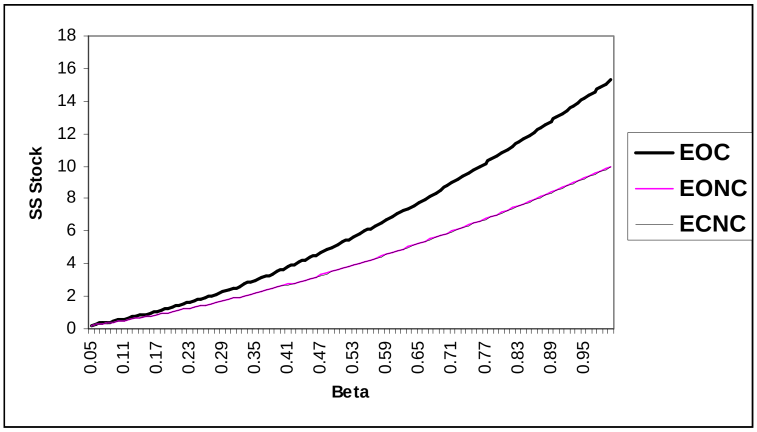

Figure 10: SS Stock with $\rho = .1$

It can be seen that lower ρ fosters higher lobbying as well as higher public good stock.

Result 2: *As β increases through increasing α, two opposite effects act on lobbying. Efficiency of lobbying goes up, but the rent of the lobbyist goes down. For cooperative lobbying, the first effect dominates the latter. For non cooperative lobbying, the second effect is dominant when agents are relatively impatient. On the other hand, public good stock increases, even for non cooperative lobbying. Thus increase in α outweighs decrease in L.*

6.1.3 Constant α, increase in efficiency through decrease in c

Both lobbying and steady state stock increases with decreasing c, as revealed by the following diagrams. For this example, $\alpha = .8$, $\rho = m = .5$, $\lambda = 1$. We have plotted β along the horizontal axis.[17]

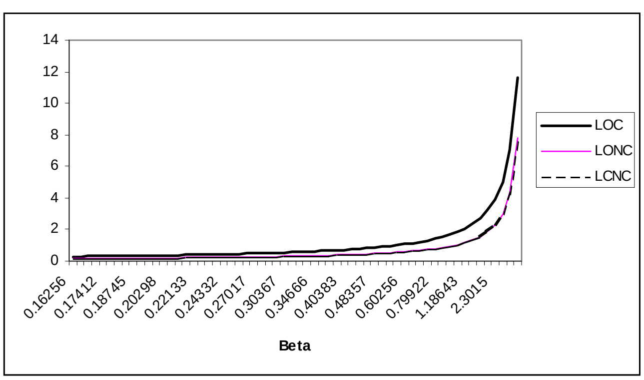

Figure 11: SS lobby with decreasing c

[17] Compare with Fig. A2.2

For output

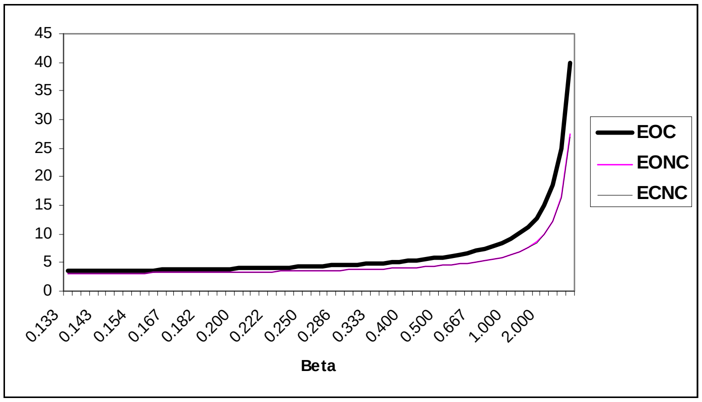

Figure 12: SS stock as c decreases

When c decreases, the efficiency of lobbying increases without any change in lobbying incentive. This will increase lobbying, ceteris paribus, and also increase steady state stock.

Result 3: *As c goes down, β increases without any effect on lobbying incentive. This leads to higher lobbying and higher public good stock.*

6.2 Welfare analysis: The Need for a Lobbyist

In this section, we perform the welfare[18] analysis in a symmetric world. We look at a specific example: when increase in efficiency comes through increase in α. Let us recall that increase in α will increase output, but will decrease lobbying for non co-operative outcomes. Therefore, the effect on welfare of the *consumer*(utility net of cost) becomes ambiguous in nature. For the following set of diagrams, we have taken $E_0 = 0, \rho = m = .5, \lambda = 1$, and $c = 2$, unless otherwise mentioned. β is plotted in the horizontal axis.

6.2.1 Components of Welfare with Rent dissipation: Different Lobbying Protocols

Total welfare(that of lobbyist and population) is depicted in the following figure:

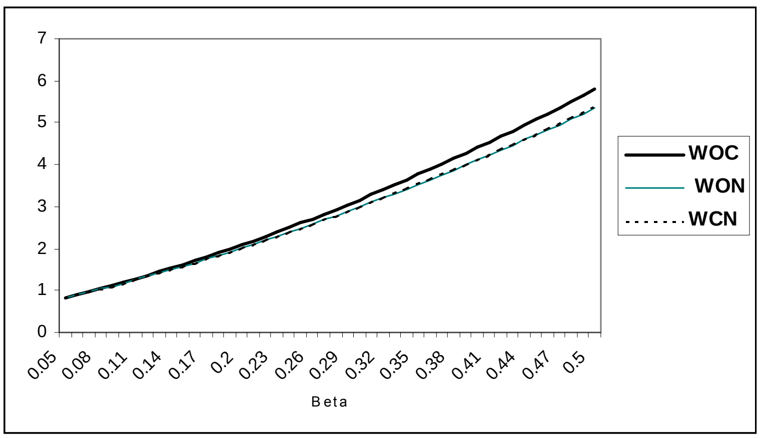

Figure 13: Total welfare as β increases

We can see that the cooperative welfare (WOC) is higher than non cooperative open loop (WON) or closed loop(WCN) welfare and all of them are increasing in β.The total welfare for non cooperative outcomes almost coincide.

As mentioned before, total welfare can be decomposed into two parts: lobbyists' welfare (or rent) and consumers' welfare (utility net of cost). In the following figures, we depict the decompositions.

[18]The relevant formulae have been documented in appendix 3.

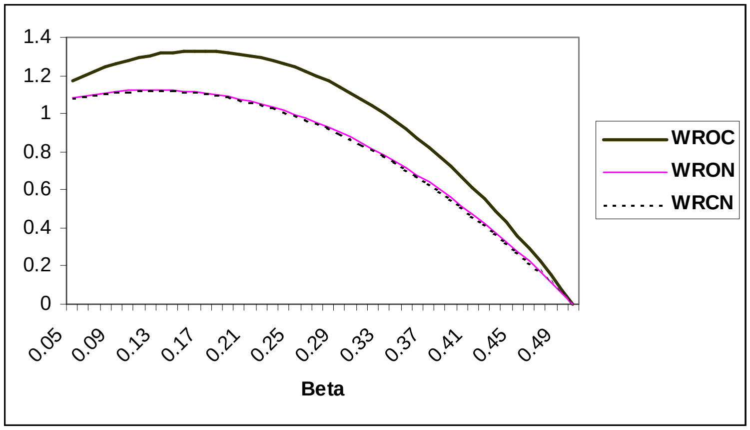

Figure 14: Lobbyists lifetime rent as β increases

As β goes up, and α goes to 1, the lifetime welfare of the lobbyists go to zero (mimicing the FN world). The lobbyist would get higher return ($WROC$) from co-operative lobbying.than the noncooperative open loop ($WRON$) or closed loop($WCON$) protocols. Again, welfare from non cooperative strategies almost coincide, although $WRON > WRCN$.

Now, regarding the consumer's welfare,[19]

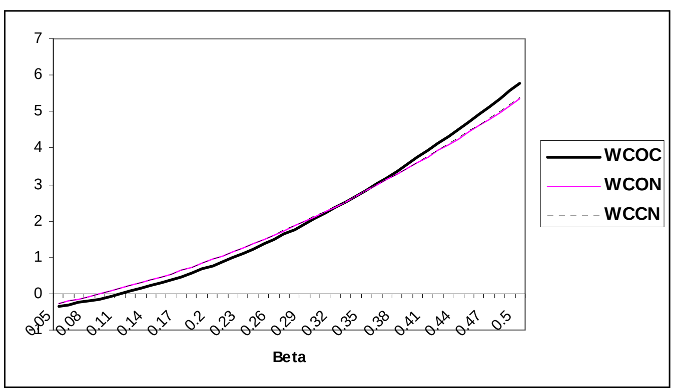

Figure 15: Consumers' welfare as β increases

It can be seen that for a range of β, welfare accruing to the consumer is higher for non cooperative lobbying (and closed loop strategy,$WCCN$ gives higher welfare than open loop, $WCON$) than cooperative lobbying ($WCOC$). Intitial welfare is negative, because utility from the public good is less than the cost of lobbying. For cooperative case, lobbying

[19]We are depicting U_C, in terms of appendix 3.

is sufficiently high for relatively lower values of β compared to other protocols. This makes it undesirable from consumers' point of view.The range of β where the consumer prefers non cooperative, closed loop lobbying (here, it is approximately $0-.35$) is higher for higher values of ρ and m. In other words, it goes down when people are more patient and/or the depreciation rate is low.

6.2.2 Need for a Not-so-benevolent (NB) Lobbyist

Closed loop non-cooperative lobbying: In a benevolent world there is no rent for lobbyist. In that case, the consumer gets total welfare: this is maximised with cooperative lobbying protocol. However, for a certain range of β, non cooperative closed loop protocol with NB lobbyist will provide highest benefit to consumer. This can be demonstrated in the following diagram:

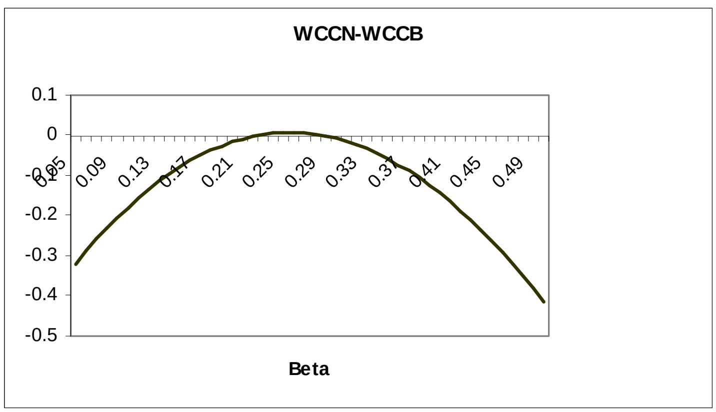

Figure 16: Difference in welfare levels

Here, $WCCB$ measures the cooperative welfare in a benevolent world. We have the result that at least, for certain values of β,noncooperative, closed loop welfare with NB lobbyist will be higher than *all* lobbying protocols within benevolent world. This justifies the use of a NB lobbyist. We have used $WCCN$ because it is hard to sustain cooperative outcome as well as any open loop strategy (which requires a power to commit).[20] However, this is not true if $\rho = m = .1$. Thus, as people gets more impatient or the rate of dissipation goes up, there is a rational for having a politicians who are not entirely benevolent and use *closed loop, non cooperative strategies.*

Open loop protocols: Let us assume that agents can commit to a time path of efforts. We can compare non cooperative protocols. For $\rho = m = .5$,

[20] See fn. 3

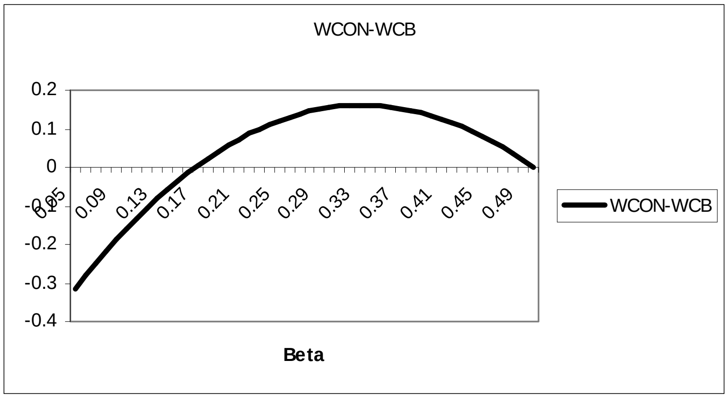

Figure 17: Difference in Welfare: $\rho = m = .5$

Thus, if we have open loop lobbying, for a wide range of β (approximately, $0.2 - 0.5$), NB non-cooperative lobbying (fosters $WCON$)will dominate corresponding strategy in benevolent world (generates WCB). Note that, as α goes to 1, the difference dies down.

On the other hand, as ρ and m goes down, then NB non cooperative strategy completely dominates the other counterpart.

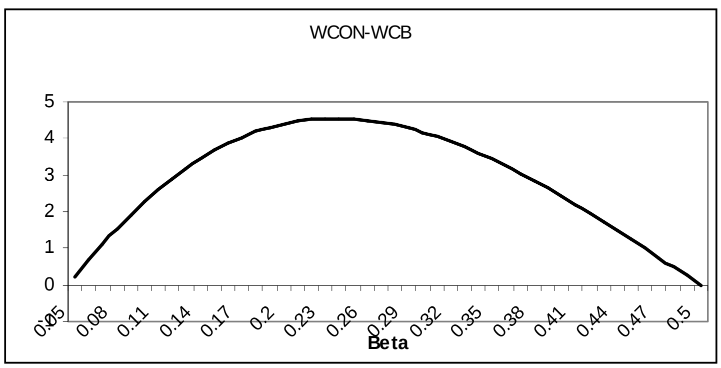

Figure 18: Difference in Welfare: $\rho = m = .1$

Here, as before, as α goes to 1, the difference ceases to be. For all other values of α, a politician who is only partly benevolent, will benefit the consumers.

7 Extension: Asymmetric Provinces

In this section, we investigate asymmetric provinces: i.e. assume that the provinces differ by β_i. A potential implication for such asymmetry is *welfare differential (or inequality)* among the provinces, an issue which can not be dealt in analysis of symmetric equilibrium. Let us recall that, the regional welfare to *consumers* is defined by

$$W_{ic} = \int_0^\infty e^{-\rho t} \left(U(E) - \frac{1}{2} c_i L_i^2 \right) dt$$

Since both regions reap identical benefit from the (pure) public good, it is clear that the region where the lobbying cost is higher will experience lower welfare. To investigate the effect of asymmetry on lobbying cost, we consider the case of open loop non cooperative lobbying. The oher lobbying protocols will have similar results.

The lobbying effort of each region is given by

$$L_i(t) = L_i^s + b_{1i}(E(t) - E_s)$$

Here,

$$b_{1i} = -\frac{\beta_i}{2A_1} \left(2m + \rho - \sqrt{4\gamma A_1 + (2m+\rho)^2} \right) = \beta_i k < 0$$

For non cooperaive, open loop case, the steady state lobbying efforts are given by following expressions

$$L_1^s = \beta_1 \frac{m(1 + s_1 (m+\rho)) - \gamma (A_2 - d_1 \alpha_2 \beta_2)}{m(\rho + m) + \gamma A_1}$$

$$L_2^s = \beta_2 \frac{m(1 + s_2 (m+\rho)) - \gamma (A_2 - d_2 \alpha_1 \beta_1)}{m(\rho + m) + \gamma A_1}$$

Then, the differential in lobbying is given by

$$L_1(t) - L_2(t) = \underbrace{(L_1^s - L_2^s)}_{\mathbf{A}} + \underbrace{(\beta_1 - \beta_2) k (E(t) - E_s)}_{\mathbf{B}}$$

Let $E_0 = 0$. Then $E(t) \leq E_s \Rightarrow k(E(t) - E_s) \geq 0$

There can be two sources of changing β_i: change in α_i and change in c_i . We take up each of them, one at a time.

7.1 Variation Through α:

Assume that all variation in β comes from variation in α_i . As α_i goes up, each province has higher productivity, but the lobbyist has less motivation for lobby. The result can be summarised in the following proposition:

Proposition 9 : *If $\alpha_i > \alpha_j$ (i.e. the region i is relatively more efficient in production of the public good) then $L_i(t) > L_j(t)$ if* α_i is not too high. *In other words, the more* *efficient region lobbies more vigorously, contributes more to public good ($\alpha_i p_i > \alpha_j p_j$, given the central rule $p_i = L_i + \lambda\alpha_i$), bears comparatively higher lobbying cost $\left(= \frac{cL_i^2}{2}\right)$ and, as a result, enjoys less lifetime welfare than the other region.*

Proof. See appendix 1 ■

7.2 Variation through c

Now assume that the variations come from $c_1 \neq c_2$, but $\alpha_1 = \alpha_2 = \alpha$.

We have a useful proposition:

Proposition 10 *The province which is more cost efficient pays higher lobbying cost and enjoys lower welfare, when all differences in relative efficiency $\left(\beta_i = \frac{\alpha}{c_i}\right)$ is explained through variations in cost.*

Proof. See appendix 1. ■

Discussion:

If one province bears higher lobbying cost than the other , we can say that there is a transfer of welfare, through the pure public good channel, from that province to the other. Normally, the concept of equalisation within a province implies that the direction of transfer of welfare occurs from the efficient province to the inefficient province. Here, if relative efficiency is based on cost differences, this is the case. The cost efficient region lobbies vigorously, adds more stock to the public good, but achieves smaller lifetime welfare than the inefficient province. On the other hand, if relative efficiency is based on productive efficiency (α_i), then results are ambiguous. If the productive efficiency parameter in region i (α_i) is high, then the steady state lobbying by region i goes down (due to lower incentive for lobbying) compared to the other region. This implies, at least in the vicinity of steady state, welfare is transfered from the inefficient province to the efficient province.

8 Conclusion

In this paper, we have explored the dynamic provision of a pure public good under various protocols of lobbying and a non benevolent lobbyist. Our basic results are summarised below.

First, individual lobbying efforts are strategic substitutes. The degree of strategic substitutability is highest under the cooperative case. It falls as we move to the open loop noncooperative case. Finally, it is lowest for the closed loop non cooperative case.

Second, if the lobbyists are non-benevolent, the levels of lobbying as well as the stock of public good will be higher in a symmetric steady state as compared to a world with benevolent lobbyists who do not capture any rent. This holds true for both co-operative and non cooperative lobbying protocols.

Third, the effect of increase in relative efficiency on lobbying is ambiguous. When cost decreases, lobbying goes down. On the other hand, if production efficiency increases, lobbying under non cooperative protocols goes down if the depreciation rate and/or discount factor is too high. With similar parameter values, this effect is absent under benevolent lobbying.

We find two results on *consumer* welfare. These lead to the policy prescription that, when the rate of depreciation and/or discount factor is high, it is better to follow the non-cooperative, closed loop protocol (that is, the protocol having the most desirable economic properties) and a non-benevolent lobbyist. This suggestion is based on two observations:

First, in a non benevolent world, *non cooperative, closed loop lobbying* yields higher welfare than *all other protocols* for a specific range of parameter values.

Second, *non cooperative, closed loop lobbying* with a *non benevolent lobbyist* may foster higher welfare compared to *all other protocols* in a benevolent world.

The latter result highlights the importance of having a non benevolent lobbyist.[21] This is further bolstered by the following result: welfare under the *open loop, non cooperative* protocol with a non benevolent lobbyist may be significantly higher than open loop, non cooperative protocol in a benevolent world.

These results are further reinforced if we allow for asymetric provinces: higher efficiency may lead to perverse effects on equalisation prospects.

Briefly, we mention some further issues which can be analysed using this framework. First, we have assumed that the central government is not a strategic agent. Relaxation of this assumption implies a hierarchical structure with leader-follower aspects, leading to a Stackelberg equilibrium. Second, introduction of re-election probability of the lobbyists that grab too much of resources will endogenise the choice of provincial efficiency. Thus there remain legitimate issues for future research.

[21] *New York Times* (02/07/2006) reports that small towns in US are actually hiring professional lobbyists for a fee to get federal funds for projects such as construction, reparation of bridges and sewer plants, which have a public good aspect. Our model is applicable to these situations.

Appendix 1
Definitions

Problem (A1.P)

Let $i \in \{1, 2, ...N\}$ denote an agent. Let $u_i(c(t), x(t), t)$ denote the instantaneous utility function of the agent i at time t, where $c =< c_i >_{i=1}^n$ is the set of control variables and $x =< c_i >_{i=1}^n$ the set of state variables. The objective of each agent is to choose c_i in order to maximise the lifetime utility

$$\max_{c_i} \int_0^T u_i(c(t), x(t), t) e^{-\rho_i t} dt$$

such that the transition equation

$$\dot{x}(t) = f(x(t), c(t), t)$$

and some boundary conditions are satisfied.

ρ_i is the rate of discount.

Definition D1: Markovian(or closed loop) Nash Equilibrium:

The n-tuple $(\xi_i)_{i=1}^n$of functions is called a Markovian Nash equilibrium if , for all $i \in \{1, 2, ...n\}$ an optimal control path of the problem (P) exists and given by

$c_i(t) = \xi_i(x(t), t)$. Such strategies ξ_i is called a Markovian or closed loop strategy.

Definition D2: Open loop Nash Equilibrium:

The n-tuple $(\xi_i)_{i=1}^n$of functions is called a open loop Nash equilibrium if , for all $i \in \{1, 2, ...n\}$ an optimal control path of the problem (P) exists and given by

$c_i(t) = \xi_i(t)$. Such strategies are called open loop strategies.

Theorem A1.1

For infinite horizon autonomous problems, the Markovian strategy collapses to stationary Markovian strategy, i.e. $\xi_i(x(t), t) = \xi_i(x(t))$

Proof. See Dockener et al (2000) ■

We have used these definitions and properties to formulate our problem.

Proof of Certain Propositions

Proof of Proposition 1:

Steady State Values of Open loop Cooperative Strategy

The necessary conditions are

$$(1-\alpha_i) - c_i L_i + \alpha_i \mu = 0$$

$$\dot{\mu} = (\rho + m)\mu - 2\left(1 - \gamma E\right)$$

$$\dot{E} = \alpha_1 L_1 + \alpha_2 L_2 + \lambda(\alpha_1^2 + \alpha_2^2) - mE$$

Dividing the first equation by c_i and let $\beta_i = \dfrac{\alpha_i}{c_i}$, we get

$L_i = (s_i + \mu)\beta_i$

Thus

$\mu = \dfrac{L_i}{\beta_i} - s_i$

And $L_2 = \dfrac{\beta_2}{\beta_1} L_1 + \beta_2 s_2 - \beta_2 s_1$

Thus,

$\dot{E} = \alpha_1 L_1 + \alpha_2 \left(\dfrac{\beta_2}{\beta_1} L_1 - \beta_2 s_1 + \beta_2 s_2 \right) + A_2 - mE$

Using (A) and (B)

$$\begin{aligned} \dot{L}_1 = \beta_1 \dot{\mu} &= \beta_1 \left((\rho + m)\mu - 2\left(1 - \gamma E\right) \right) \\ &= \beta_1 \left((\rho + m) \left(\frac{L_1}{\beta_1} - s_1 \right) - 2\left(1 - \gamma E\right) \right) \\ &= (\rho + m)\left(L_1 - s_1 \beta_1\right) - 2\beta_1 \left(1 - \gamma E\right) \end{aligned}$$

From these the SS values can be calculated, when $\left(\dot{E}, \dot{L}_1\right) = (0, 0)$

$E_s^{Oc} = \dfrac{2A_1 + \left(s_1 \alpha_1 \beta_1 + s_2 \alpha_2 \beta_2 + A_2\right)(m + \rho)}{m(\rho + m) + 2\gamma A_1}$

$L_{1s}^{Oc} = \beta_1 \dfrac{m(2 + \rho s_1 + m s_1) - 2\gamma(A_2 - d_1 \alpha_j \beta_j)}{m(\rho + m) + 2\gamma A_1}$

Here, $A_1 = \left(\dfrac{\alpha_1^2}{c_1} + \dfrac{\alpha_2^2}{c_2} \right)$ and $A_2 = \lambda \left(\alpha_1^2 + \alpha_2^2 \right)$

As for stability check, the coefficient matrix is

$$\begin{pmatrix} \rho + m & 2\beta_1 \gamma \\ \left(\dfrac{\alpha_1 \beta_1 + \alpha_2 \beta_2}{\beta_1} \right) & -m \end{pmatrix}$$

The trace is $\rho > 0$ and the determinant is $\left(-m\rho - 2\gamma \alpha_1 \beta_1 - 2\gamma \alpha_2 \beta_2 - m^2 \right) < 0$. Thus the system is stable in a saddlepoint sense.

The Eigenvalues are $\frac{1}{2}\rho \pm \frac{1}{2}\sqrt{8\gamma A_1 + (2m + \rho)^2}$

And,

$\dfrac{2A_1 + \left(s_1 \alpha_1 \beta_1 + s_2 \alpha_2 \beta_2 + A_2\right)(m + \rho)}{m(\rho + m) + 2\gamma A_1} < \dfrac{1}{\gamma}$

$\gamma\left(2A_1 + \left(s_1 \alpha_1 \beta_1 + s_2 \alpha_2 \beta_2 + A_2\right)(m + \rho)\right) - m(\rho + m) - 2\gamma A_1 < 0$

$\Leftrightarrow$

$\gamma\,(m+\rho)\,(A_2+s_1\alpha_1\beta_1+s_2\alpha_2\beta_2) < m(m+\rho)$

$\Leftrightarrow$

$\gamma < \dfrac{m}{A_2+s_1\alpha_1\beta_1+s_2\alpha_2\beta_2} \Rightarrow \dfrac{m}{A_2+\beta_1+\beta_2-A_1} > \gamma$

If this condition is maintained, we can prevent 'over-accumulation' of public good.

Proof of Proposition 2

Closed Loop Representation of Open Loop Cooperative Strategy:

The homogeneous part of the equations can be written as

$\dot{E} = \left(\dfrac{\alpha_1\beta_1+\alpha_2\beta_2}{\beta_1}\right) L_1 - mE$

and

$\dot{L}_1 = (\rho+m)L_1 + 2\beta_1\gamma E$

After substitution, we have the following equation in E

$\ddot{E} - \rho\dot{E} - (m(\rho+m)+2\beta_1\gamma C)\,E = 0$, where $C = \dfrac{\alpha_1\beta_1+\alpha_2\beta_2}{\beta_1}$

Solving this, we get

$E(t) = C_1\exp(r_1 t) + C_2\exp(r_0 t)$

Here,

$r_i = \frac{1}{2}\rho \pm \frac{1}{2}\sqrt{8\gamma A_1 + (2m+\rho)^2}$ $(i=1,0)$

The complete solution for E is thus

$E = C_1\exp(r_1 t) + C_0\exp(r_0 t) + E_s^{Oc}$

When $t=0$, $C_1 + C_2 = E_0 - E_s^{Oc}$

Along the steady path, when $t\to\infty$, $E = E_s^{Oc} \Rightarrow C_1 = 0$

Thus $C_2 = E_0 - E_s^{Oc}$

$\Rightarrow E = E_s + (E_0 - E_s^{Oc})\exp(r_0 t)$

Substituting back in the homogeneous equation of L_1,

$\dot{L}_1 = (\rho+m)L_1 + 2\beta_1\gamma\,\{(E_0-E_s)\exp(r_0 t)\}$

Solving this, we get the homogeneous solution.

$L_1 = C_3 e^{t(m+\rho)} - \dfrac{2\beta_1\gamma}{r_0-\rho-m}\left(E_s^{Oc}-E_0\right)\exp(r_0 t)$

and the complete solution

$L_1 = C_3 e^{t(m+\rho)} - \dfrac{2\beta_1\gamma}{r_0-\rho-m}\left(E_s^{Oc}-E_0\right)\exp(r_0 t) + L_{1s}$

When $t=0$,

$C_3 - \dfrac{2\beta_1\gamma}{r_0-\rho-m}\left(E_s^{Oc}-E_0\right) = L_1(0) - L_{1s}^{Oc}$

When $t\to\infty$, then, along the stable path, $L_1 = L_{1s}^{Oc}$. Thus $C_3 = 0$

The stable path for the $i-th$ province is thus characterised by

$L_i^{Oc} = L_{is}^{Oc} - \dfrac{2\beta_i\gamma}{r_0-\rho-m}\left(E_s^{Oc}-E_0\right)\exp(r_0 t)$

$= L_{is}^{Oc} + \frac{2\beta_i\gamma}{\rho + m - r_0}(E_s^{Oc} - E)$
$= L_{is}^{Oc} + \frac{2\beta_i\gamma}{\rho + m - \frac{1}{2}\rho + \frac{1}{2}\sqrt{8\gamma A_1 + (2m+\rho)^2}}(E_s^{Oc} - E)$
$= L_{is}^{Oc} + \frac{2\beta_i\gamma}{m + \frac{1}{2}\rho + \frac{1}{2}\sqrt{8\gamma A_1 + (2m+\rho)^2}}(E_s^{Oc} - E)$
$= L_{is}^{Oc} + \frac{4\beta_i\gamma}{2m + \rho + \sqrt{8\gamma A_1 + (2m+\rho)^2}}(E_s^{Oc} - E)$
$= L_{is}^{Oc} + \frac{\beta_i}{2A_1}\left(2m + \rho - \sqrt{8\gamma A_1 + (2m+\rho)^2}\right)(E - E_s^{Oc})$
Thus $\frac{\partial L_i^{Oc}}{\partial E} = \frac{\beta_i}{2A_1}\left(2m + \rho - \sqrt{8\gamma A_1 + (2m+\rho)^2}\right) < 0$

The intercept is
$L_{is}^{Oc} - \frac{\beta_i}{2A_1}\left(2m + \rho - \sqrt{8\gamma A_1 + (2m+\rho)^2}\right) E_s^{Oc} > 0$

Proof of Proposition 3:

Steady State Values of Open Loop, Non Cooperative Strategy

The necessary conditions are

$$\alpha_i - c_i L_i + \pi_i \alpha_i = 0$$

$$\dot{\pi}_i = (\rho + m)\pi_i - (1 - \gamma E)$$

$$\dot{E} = \Sigma \alpha_i L_i - mE + A_2$$

Using the fact that $L_i = \beta_i(s_i + \pi_i)$, we can write the system of differential equation as

$\dot{L}_i = (\rho + m)L_i + \beta_i\gamma E - \beta_i(1 + s_i\rho + s_i m)$

and

$\dot{E} = \Sigma \alpha_i L_i - mE + A_2$

The steady state of the system reveals that

At SS, the system of equations are

$$\begin{aligned}
(\rho + m)(L_1 - s_1\beta_1) + \beta_1\gamma E - \beta_1 &= 0 \\
(\rho + m)(L_2 - s_2\beta_2) + \beta_2\gamma E - \beta_2 &= 0 \\
\alpha_1 L_1 + \alpha_2 L_2 - mE + A_2 &= 0
\end{aligned}$$

Solving this,one gets

$L_{is}^{Onc} = \beta_i \dfrac{m(1+s_i(m+\rho)) - \gamma\left(A_2 - d_i\alpha_j\beta_j\right)}{m(\rho+m)+\gamma A_1}$

and

$E_s^{Onc} = \dfrac{A_1 + (s_1\alpha_1\beta_1 + s_2\alpha_2\beta_2 + A_2)(m+\rho)}{m(\rho+m)+\gamma A_1}$

where the superscript nc implies no cooperation,O refers to closed loop and the subscript s stands for steady state.

The relevant coefficient matrix is

$\begin{pmatrix} \rho+m & 0 & \beta_1\gamma \\ 0 & \rho+m & \beta_2\gamma \\ \alpha_1 & \alpha_2 & -m \end{pmatrix}$, trace: $m+2\rho>0$

Determinant: $-m\gamma\alpha_1\beta_1 - m\gamma\alpha_2\beta_2 - \gamma\rho\alpha_1\beta_1 - \gamma\rho\alpha_2\beta_2 - m^3 - m\rho^2 - 2m^2\rho < 0$

Trace: $m+2\rho>0$

Eigenvalues : $m+\rho, \frac{1}{2}\rho \pm \frac{1}{2}\sqrt{4\gamma A_1 + (2m+\rho)^2}$

Thus the system is stable, in a saddlepoint sense.

and, to prevent over accumulation,

$\dfrac{A_1 + (s_1\alpha_1\beta_1 + s_2\alpha_2\beta_2 + A_2)(m+\rho)}{m(\rho+m)+\gamma A_1} < \dfrac{1}{\gamma}$

$\Leftrightarrow$

$\gamma(m+\rho)(A_2 + s_1\alpha_1\beta_1 + s_2\alpha_2\beta_2) < m(m+\rho)$:

$\Leftrightarrow$

$\gamma < \dfrac{m}{A_2 + s_1\alpha_1\beta_1 + s_2\alpha_2\beta_2}$

Proof of Proposition 4:

Closed Loop Form of Open Loop Non-Cooperative Strategy

The stable path of E is

$E(t) = E_s^{Onc} + (E_0 - E_s^{Onc})\exp(r_3 t)$

Where, r_3 is the negative eigenvalue of the coefficient matrix and is given by

$r_3 = \frac{1}{2}\rho - \frac{1}{2}\sqrt{4\gamma A_1 + (2m+\rho)^2}$

The non cooperative, 'closed loop form' of open loop lobbying would be

$$\begin{aligned} L_i^{Onc} &= L_{is}^{Onc} - \left(E - E_s^{Onc}\right)\frac{\gamma\beta_i}{\rho+m-r_3} \\ &= L_{is}^{Onc} - \left(E - E_s^{Onc}\right)\frac{\gamma\beta_i}{m+\frac{1}{2}\rho+\frac{1}{2}\sqrt{4\gamma A_1+(2m+\rho)^2}} \\ &= L_{is}^{Onc} + \frac{\beta_i\left(2m+\rho-\sqrt{4\gamma A_1+(2m+\rho)^2}\right)}{2A_1}\left(E - E_s^{Onc}\right) \end{aligned}$$

The slope is

$\dfrac{\partial L_i^{Onc}}{\partial E} = \dfrac{\beta_i}{2A_1}\left(2m+\rho-\sqrt{4\gamma A_1+(2m+\rho)^2}\right) < 0$

and the intercept

$$L_i^{Onc}|_{E=0} = L_{is}^{Onc} - \frac{\beta_i\left(2m+\rho-\sqrt{4\gamma A_1+(2m+\rho)^2}\right)}{2A_1}E_s^{Onc}$$

Proof of Proposition 5:

Comparison of Steady States: Open Loop Cooperative and Non-Cooperative Strategies:

$$E_s^{Oc} = \frac{2A_1 + (s_1\alpha_1\beta_1 + s_2\alpha_2\beta_2 + A_2)(m+\rho)}{m(\rho+m)+2\gamma A_1} > E_s^{Onc} = \frac{A_1 + (s_1\alpha_1\beta_1 + s_2\alpha_2\beta_2 + A_2)(m+\rho)}{m(\rho+m)+\gamma A_1}$$

$\Leftrightarrow$

$$(2A_1 + (s_1\alpha_1\beta_1 + s_2\alpha_2\beta_2 + A_2)(m+\rho))(m(\rho+m)+\gamma A_1) \\ - (A_1 + (s_1\alpha_1\beta_1 + s_2\alpha_2\beta_2 + A_2)(m+\rho))(m(\rho+m)+2\gamma A_1) > 0$$

$\Leftrightarrow$

$m\rho A_1 - m\gamma A_1 A_2 - \gamma\rho A_1 A_2 - m\gamma A_1 s_1\alpha_1\beta_1 - m\gamma A_1 s_2\alpha_2\beta_2 - \gamma\rho A_1 s_1\alpha_1\beta_1 - \gamma\rho A_1 s_2\alpha_2\beta_2 + m^2 A_1 > 0$

$\Leftrightarrow$

$$A_1 m(m+\rho) > A_1\gamma(m+\rho)(A_2 + s_1\alpha_1\beta_1 + s_2\alpha_2\beta_2)$$

$\Leftrightarrow$

$$\frac{m}{A_2 + s_1\alpha_1\beta_1 + s_2\alpha_2\beta_2} > \gamma$$

Also, for region 1,

$$L_{1s}^{Oc} = \frac{m(2+\rho s_1 + m s_1) - 2\gamma(A_2 - d_1\alpha_2\beta_2)}{m(\rho+m)+2\gamma A_1} > L_{1s}^{Onc} = \frac{m(1+\rho s_1 + m s_1) - \gamma(A_2 - d_1\alpha_2\beta_2)}{m(\rho+m)+\gamma A_1}$$

$\Leftrightarrow$

$$(m(2+\rho s_1 + m s_1) - 2\gamma(A_2 - d_1\alpha_2\beta_2))(m(\rho+m)+\gamma A_1) \\ - (m(\rho+m)+2\gamma A_1)(m(1+\rho s_1 + m s_1) - \gamma(A_2 - d_1\alpha_2\beta_2)) > 0$$

$\Leftrightarrow$

$$m^3 + m^2\rho > \gamma m(m+\rho)(A_2 + A_1 s_1 - d_1\alpha_2\beta_2)$$

$\Leftrightarrow$

$$m > \gamma(A_2 + A_1 s_1 - d_1\alpha_2\beta_2)$$

or

$$\frac{m}{A_2 + A_1 s_1 - d_1\alpha_2\beta_2} > \gamma$$

The denominator is equal to $A_2 + s_1\alpha_1\beta_1 + s_2\alpha_2\beta_2$(since $d_1 = s_1 - s_2$). This is the same condition of preventing over accumulation.

Proof of Proposition 7:

Value Function and Steady State Output for Closed Loop Non Cooperative Strategy:

The HJB equation is

$\rho W_i(E) = \max_{L_i} \left[E - \frac{\gamma}{2}E^2 + (1-\alpha_i)(L_i + \lambda\alpha_i) - \frac{1}{2}c_i L_i^2 + W_i'(E)(\Sigma\alpha_i p_i - mE) \right]$

The FOC yields

$L_i = \beta_i (s_i + W_i'(E))$

Let $W_i(E) = d_i + k_i E + \frac{f_i}{2}E^2$

Then $W_i' = k_i + f_i E$

HJB equation is

$\rho d_i + \rho k_i E + \rho \frac{f_i}{2} E^2 =$

$\lambda\alpha_i + A_2 k_i + k_i\beta_i + s_i\beta_i - k_i\alpha_i\beta_i - s_i\alpha_i\beta_i + k_i k_j \alpha_j \beta_j$

$+k_i s_i \alpha_i \beta_i + k_i s_j \alpha_j \beta_j - \lambda\alpha_i^2 + k_i^2\alpha_i\beta_i - k_i c_i s_i \beta_i^2 - \frac{1}{2}k_i^2 c_i \beta_i^2 - \frac{1}{2}c_i s_i^2 \beta_i^2 +$

$+E(A_2 f_i - m k_i + f_i\beta_i - f_i\alpha_i\beta_i + 2k_i f_i \alpha_i\beta_i + k_i f_j \alpha_j \beta_j$

$+k_j f_i \alpha_j \beta_j + f_i s_i \alpha_i \beta_i + f_i s_j \alpha_j \beta_j - k_i c_i f_i \beta_i^2 - c_i f_i s_i \beta_i^2 + 1)$

$+E^2 \left(f_i f_j \alpha_j \beta_j - m f_i - \frac{1}{2}\gamma + f_i^2 \alpha_i \beta_i - \frac{1}{2} c_i f_i^2 \beta_i^2 \right)$

Equate coefficients of E^2

$\rho \frac{f_i}{2} = f_1 f_2 \alpha_j \beta_j - m f_i - \frac{1}{2}\gamma + \frac{1}{2} f_i^2 \alpha_i \beta_i$

Equate coefficients of E

$A_2 f_i - m e_i + f_i\beta_i - f_i\alpha_i\beta_i + 2e_i f_i \alpha_i \beta_i + k_i f_j \alpha_j \beta_j$

$+k_j f_i \alpha_j \beta_j + f_i s_i \alpha_i \beta_i + f_i s_j \alpha_j \beta_j - k_i c_i f_i \beta_i^2 - c_i f_i s_i \beta_i^2 + 1 = \rho k_i$

Equate constants:

$\rho d_i = \lambda\alpha_i + A_2 k_i + k_i\beta_i + s_i\beta_i - k_i\alpha_i\beta_i - s_i\alpha_i\beta_i + k_i k_j \alpha_j \beta_j$

$+k_i s_i \alpha_i \beta_i + k_i s_j \alpha_j \beta_j - \lambda\alpha_i^2 + k_i^2 \alpha_i \beta_i - k_i c_i s_i \beta_i^2 - \frac{1}{2}k_i^2 c_i \beta_i^2 - \frac{1}{2} c_i s_i^2 \beta_i^2$

Thus the equations under considerations are

$f_i = \frac{1}{2\alpha_i\beta_i}\left(2m + \rho - 2f_j\alpha_j\beta_j - \sqrt{4\gamma\alpha_i\beta_i + \left(2m + \rho - 2f_j\alpha_j\beta_j\right)^2} \right)$

Similarly,

$k_i = \frac{A_2 f_i + f_i\beta_i - f_i\alpha_i\beta_i + k_j f_i \alpha_j \beta_j + f_i s_i \alpha_i \beta_i + f_i s_j \alpha_j \beta_j - c_i f_i s_i \beta_i^2 + 1}{m + \rho - 2f_i\alpha_i\beta_i - f_j\alpha_j\beta_j + c_i f_i \beta_i^2}$

$d_i = \frac{1}{\rho}(\lambda\alpha_i + k_i\beta_i + s_i\beta_i - k_i\alpha_i\beta_i - s_i\alpha_i\beta_i + k_i k_j \alpha_j \beta_j + k_i s_i \alpha_i \beta_i$

$+k_i s_j \alpha_j \beta_j - \lambda\alpha_i^2 + k_i^2 \alpha_i \beta_i - k_i c_i s_i \beta_i^2 - \frac{1}{2}k_i^2 c_i \beta_i^2 - \frac{1}{2}c_i s_i^2 \beta_i^2)$

The lobbying is $L_i = \beta_i(1 + k_i + f_i E)$

Now,

$\dot{E} = \alpha_1 p_1 + \alpha_2 p_2 - mE$

$= \alpha_1 L_1 + \alpha_2 L_2 + A_2 - mE$

$= \alpha_1\beta_1(s_1 + k_1 + f_1 E) + \alpha_2\beta_2(s_2 + k_2 + f_2 E) + A_2 - mE$

$= \alpha_1\beta_1 s_1 + \alpha_2\beta_2 s_2 + A_2 + \alpha_1\beta_1 k_1 + \alpha_2\beta_2 k_2 + E\left(f_1\alpha_1\beta_1 + f_2\alpha_2\beta_2 - m\right)$

This will be stable if $f_1\alpha_1\beta_1 + f_2\alpha_2\beta_2 - m < 0$

For stability, we assume the negative values of f_i

and the SS value is given by

$$E_s^{CNc} = \frac{\alpha_1\beta_1 s_1 + \alpha_2\beta_2 s_2 + \alpha_1\beta_1 k_1 + \alpha_2\beta_2 k_2}{m - f_1\alpha_1\beta_1 - f_2\alpha_2\beta_2}$$

$\alpha_1\beta_1 s_1 + \alpha_2\beta_2 s_2 = \beta_1 + \beta_2 - A_1$

Proof of Proposition 8:

Symmetric Solution for Closed Loop Non Cooperative Value Function

for symmetric solution, $f_1 = f_2 = f$ (say). Thus,

$f = \frac{1}{x\beta^2}\left(m + \frac{1}{2}\rho - x\beta^2 f - \frac{1}{2}\sqrt{4m\rho + 4m^2 + \rho^2 + 4x\beta^2\gamma - 8mx\beta^2 f - 4x\beta^2\rho f + 4x^2\beta^4 f^2}\right)$

Solving this, we get

$f = \dfrac{1}{6x\beta^2}\left((2m+\rho) \pm \sqrt{(2m+\rho)^2 + 12x\beta^2\gamma}\right)$

One of the roots of f is negative, the other one is positive. We choose the negative root, i.e.

$f = \dfrac{1}{6x\beta^2}\left((2m+\rho) - \sqrt{12x\beta^2\gamma + (2m+\rho)^2}\right)$

For symmetric case,

$k_i = k$ Thus

$k = \frac{1}{m+\rho-2fx\beta^2}\left(f\beta - fx\beta^2 + fx\beta^2 k + f\beta(1 - x\beta) + 2fx^2\beta^2\lambda + 1\right)$

Solving this,we get

$k = \dfrac{1}{m+\rho-3fx\beta^2}\left(2f\beta - 2fx\beta^2 + 2fx^2\beta^2\lambda + 1\right)$

Similarly, the value of d is obtained.

Proof of Proposition 9:

Effect of Increasing α_i on Provincial Lobbying Effort:

The differential in lobbying is given by

$$L_1(t) - L_2(t) = \underbrace{(L_1^s - L_2^s)}_{A} + \underbrace{(\beta_1 - \beta_2)k(E(t) - E_s)}_{B}$$

Let $E_0 = 0$. Then $E(t) \le E_s \Rightarrow k(E(t) - E_s) \ge 0$

Let $\alpha_1 > \alpha_2$, while $c_1 = c_2 = c$. This implies $\beta_1 > \beta_2$ and $B \ge 0$. Thus, for non cooperative solution, region 1 will lobby more, and incur higher lobbyisng costs $\left(= \dfrac{c}{2}L_1(t)^2\right)$ than region 2 if $A > 0$.

Using the definitions, we have

$L_1^s = \beta_1 \dfrac{m(1 + s_1(m+\rho)) - \gamma(A_2 - d_1\alpha_2\beta_2)}{m(\rho+m) + \gamma A_1}$

$$= \frac{cm\rho + cm\alpha_1 - cm\rho\alpha_1 - \gamma\alpha_1\alpha_2 + cm^2 - cm^2\alpha_1 + \gamma\alpha_2^2 - c\gamma\alpha_1^3 - c\gamma\alpha_1\alpha_2^2}{c^2m\rho + c\gamma\alpha_1^2 + c\gamma\alpha_2^2 + c^2m^2}$$

$$L_2^s = \beta_2 \frac{m(1 + s_2(m+\rho)) - \gamma(A_2 - d_2\alpha_1\beta_1)}{m(\rho+m) + \gamma A_1}$$

$$= \frac{cm\rho + cm\alpha_2 - cm\rho\alpha_2 - \gamma\alpha_1\alpha_2 + cm^2 - cm^2\alpha_2 + \gamma\alpha_1^2 - c\gamma\alpha_2^3 - c\gamma\alpha_1^2\alpha_2}{c^2m\rho + c\gamma\alpha_1^2 + c\gamma\alpha_2^2 + c^2m^2}$$

Note that, the denominators are equal. The difference, if any, will come from the numerators. Thus,

$L_1^s - L_2^s > 0$

$\Leftrightarrow$

$$\left(cm\rho + cm\alpha_1 - cm\rho\alpha_1 - \gamma\alpha_1\alpha_2 + cm^2 - cm^2\alpha_1 + \gamma\alpha_2^2 - c\gamma\alpha_1^3 - c\gamma\alpha_1\alpha_2^2\right) - \left(cm\rho + cm\alpha_2 - cm\rho\alpha_2 - \gamma\alpha_1\alpha_2 + cm^2 - cm^2\alpha_2 + \gamma\alpha_1^2 - c\gamma\alpha_2^3 - c\gamma\alpha_1^2\alpha_2\right) > 0$$

$\Leftrightarrow$

$$(\alpha_2 - \alpha_1)(\alpha_1 + \alpha_2) + c(\alpha_2 - \alpha_1)\left(\alpha_1^2 + \alpha_2^2\right) + \tfrac{mc}{\gamma}(\alpha_2 - \alpha_1)(m + \rho - 1) > 0$$

Now,

$\alpha_1 > \alpha_2 \rightarrow \alpha_2 - \alpha_1 < 0 \Leftrightarrow$

$(\alpha_1 + \alpha_2) + c\left(\alpha_1^2 + \alpha_2^2\right) + \tfrac{mc}{\gamma}(m + \rho - 1) < 0$

Let α_M be the mean of the two efficiency parameters:

$$\alpha_M \equiv \frac{\alpha_1 + \alpha_2}{2}$$

Then we can write

$$\alpha_1 = \alpha_M + \varepsilon \text{ where } \varepsilon > 0$$

and

$$\alpha_2 = \alpha_M - \varepsilon$$

$$\alpha_1 + \alpha_2 = 2\alpha_M$$

$$\alpha_1^2 + \alpha_2^2 = (\alpha_M + \varepsilon)^2 + (\alpha_M - \varepsilon)^2 = 2\alpha_M^2 + 2\varepsilon^2$$

The condition becomes

$$2\alpha_M + c\left(2\alpha_M^2 + 2\varepsilon^2\right) < \frac{mc(1 - \rho - m)}{\gamma}$$

So, if $\varepsilon > 0$ but not too great, then $L_1^s - L_2^s > 0$. But the sign reverses when ε is greater than a certain threshold. Note that, if either ρ or m are too high (such that the sum > 1), then the sign will be reversed for all ε.

Note that, if ε is sufficiently high, then $\alpha_1 > \alpha_2 \rightarrow L_1^s < L_2^s$, but nothing can be said about the path of $L_1(t) - L_2(t)$.

Proof of Proposition 10:

Effect of Higher Cost on Provincial Lobbying Effort:

We have,

$$L_1^s = \beta_1 \frac{m(1 + s_1(m+\rho)) - \gamma(A_2 - d_1\alpha_2\beta_2)}{m(\rho+m) + \gamma A_1}$$

$$= \frac{c_2\left(m\alpha + m\rho - m\alpha\rho + m^2 - m^2\alpha - 2\alpha^3\gamma\right)}{m\rho c_1 c_2 + \alpha^2\gamma c_1 + \alpha^2\gamma c_2 + m^2 c_1 c_2} = \frac{c_2\xi}{\psi(c_1, c_2)}$$

$$L_2^s = \beta_2 \frac{m(1 + s_2(m+\rho)) - \gamma(A_2 - d_2\alpha_1\beta_1)}{m(\rho+m) + \gamma A_1}$$

$$= \frac{c_1\left(m\alpha + m\rho - m\alpha\rho + m^2 - m^2\alpha - 2\alpha^3\gamma\right)}{m\rho c_1 c_2 + \alpha^2\gamma c_1 + \alpha^2\gamma c_2 + m^2 c_1 c_2} = \frac{c_1\xi}{\psi(c_1, c_2)}$$

It is clear that if $c_1 < c_2$, then $L_1^s > L_2^s$

$L_i(t) = L_i^s + \beta_i k(E(t) - E_s)$

Thus, cost to region i is

$$TC_i(t) = \frac{c_i}{2}\left(L_i(t)\right)^2$$

$$= \frac{c_i}{2}\left(L_i^s\right)^2 + \frac{c_i}{2}\beta_i^2 k^2 (E(t) - E_s)^2 + c_i L_i^s \beta_i k(E(t) - E_s)$$

$$= \frac{c_i}{2}\left(L_i^s\right)^2 + \frac{1}{2}\frac{\alpha^2}{c_i} k^2 (E(t) - E_s)^2 + \alpha L_i^s k(E(t) - E_s) \quad \text{(putting in } \beta_i = \frac{\alpha}{c_i}\text{)}$$

Since

$$\frac{L_1^s}{L_2^s} = \frac{c_2}{c_1}$$

$$\frac{\frac{1}{2}c_1\left(L_1^s\right)^2}{\frac{1}{2}c_2\left(L_2^s\right)^2} = \frac{c_1\left(c_2\right)^2}{c_2\left(c_1\right)^2} = \frac{c_2}{c_1}$$

If $c_2 > c_1$, then $\frac{1}{2}c_1\left(L_1^s\right)^2 > \frac{1}{2}c_2\left(L_2^s\right)^2$

Maintaining the same assumption,

$$\frac{1}{2}\frac{\alpha^2}{c_1}k^2(E_s - E(t))^2 > \frac{1}{2}\frac{\alpha^2}{c_2}k^2(E_s - E(t))^2$$

and

$\alpha L_1^s k(E(t) - E_s) > \alpha L_2^s k(E(t) - E_s)$, since $c_2 > c_1 \Rightarrow L_1^s > L_2^s$

Thus, $(c_2 > c_1) \Rightarrow (TC_1(t) > TC_2(t))$

Appendix 2

Lobbying Behaviour and Public Good Stock in the 'Benevolent Lobbyist' Environment

The original FN paper does not consider lobbying. Their problem (using our notation), is the following:

$$\int_0^\infty e^{-\rho t}\left(E - \frac{\gamma E^2}{2} - \frac{1}{2}c_i x_i^2\right) dt$$

$$s.t.\dot{E} = \Sigma\ x_i - mE$$

x_i represents individual contributions (efforts) for provision of public good.

We have re-interpreted the model in terms of federal lobbying. We replace individuals by regions, efforts by lobbying, and add the assumption of a federal government that cares for lobbying as well as efficiency in production. The problem of a lobbyist in modified model becomes,

$$\int_0^\infty e^{-\rho t}\left(E - \frac{\gamma E^2}{2} - \frac{1}{2}c_i L_i^2\right) dt$$

$$s.t.\dot{E} = \Sigma \alpha_i p_i - mE$$

The cooperative lobbying scenario yields the following pair of stable steady state lobbying (for the first region) and public good stock:

$$E_{Bs}^{Oc} = \frac{(m+\rho)A_2 + 2A_1}{m(\rho+m) + 2\gamma A_1} \tag{A2.1}$$

$$L_{1Bs}^{Oc} = 2\beta_1 \frac{m - \gamma A_2}{m(\rho+m) + 2\gamma A_1}$$

The subscript B indicates the relevant environment.

For the open loop non co-operative scenario,

$$E_{Bs}^{Onc} = \frac{(m+\rho)A_2 + A_1}{m(\rho+m) + \gamma A_1}$$

$$L_{1Bs}^{Onc} = \beta_1 \frac{m - \gamma A_2}{m(\rho+m) + \gamma A_1} \tag{A2.2}$$

It is clear that in both protocols, the ratio of steady state lobbying equals 'relative efficiency', i.e.

$$\frac{L^k_{1Bs}}{L^k_{2Bs}} = \frac{\beta_1}{\beta_2}$$

Here, k denotes different protocols. As β_i goes up, region i will lobby more . This particular result parallels Dixit's(1987) assertion that a player who has a strategic incentive, will overexert.

Given relevant condition to prevent over accumulation $\left(\gamma < \dfrac{m}{A_2}\right)$, it can be shown that $E^{Oc}_{Bs} > E^{Onc}_{Bs}$ and $L^{Oc}_{1Bs} > L^{Onc}_{1Bs}$

For a symmetric solution, with $\alpha_i = \beta x, c_i = x$ and $\gamma = .01$,we can write

Cooperative SS:

$$E^{Oc}_{Bs} = \frac{(m+\rho)A_2 + 2A_1}{m(\rho+m) + 2\gamma A_1} = \frac{4x\beta^2 + 2x^2\beta^2\lambda\,(m+\rho)}{m\,(m+\rho) + 0.04x\beta^2}$$

$$L^{Oc}_{1B} = 2\beta_1 \frac{m - \gamma A_2}{m(\rho+m) + 2\gamma A_1} = \frac{\beta\left(2m - 0.04x^2\beta^2\lambda\right)}{m\,(m+\rho) + 0.04x\beta^2}$$

Non co-operative SS:

$$E^{Onc}_{Bs} = \frac{mA_2 + \rho A_2 + \alpha_1\beta_1 + \alpha_2\beta_2}{m\rho + \gamma\alpha_1\beta_1 + \gamma\alpha_2\beta_2 + m^2} = \frac{2x\beta^2 + 2\lambda\beta^2x^2\,(m+\rho)}{m\,(m+\rho) + 0.02x\beta^2}$$

$$L^{Onc}_{1Bs} = \beta_1 \frac{m - \gamma A_2}{m(\rho+m) + \gamma A_1} = \frac{\beta\left(m - 0.02x^2\beta^2\lambda\right)}{m\,(m+\rho) + 0.02x\beta^2}$$

The following figures plot the steady lobbying when β increases through two different channels. We have kept the same parameter values as figure 7 and figure 11, respectively. Here, we have analysed only the open loop cases (since closed loop NC behaviour follows the open loop counterpart). $LONB$: non cooperative lobbying, $LOCB$: cooperative lobbying.

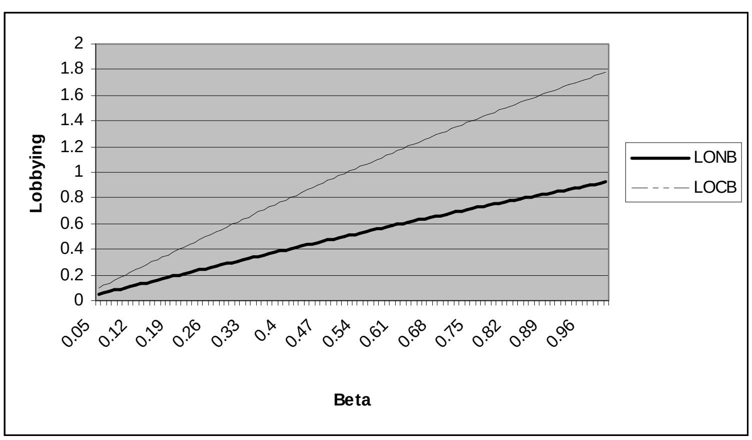

Figure A2.1: Increase in lobbying as α increases

Thus, as beta increases through increase in α, both types of lobbying increases. For the figure A.2.1, $\rho = m = .5, \lambda = 1$, and $c = 1$.

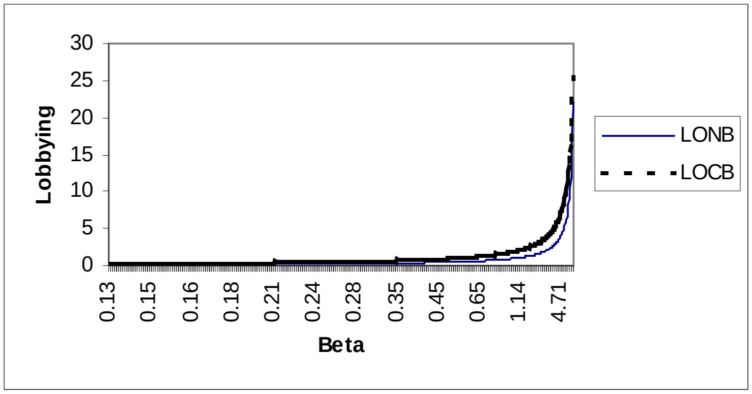

Figure A2.2: Increase in lobbying as c decreases

Here, lobbying increases with decreasing c, same as the model with rent dissipation. For the figure A2.2 , we have $\rho = m = .5, \lambda = 1, \alpha = .8.$ initial $c = 6$.

Results can be further generalised if we consider another version of (2.3a), i.e.

$$\int_0^\infty e^{-\rho t}\left(E - \frac{\gamma}{2}E^2 - \frac{1}{2}c_i L_i^2\right) dt + \Gamma\left[\frac{\lambda\alpha_i(1-\alpha_i)}{\rho} + (1-\alpha_i)\int_0^\infty e^{-\rho t} L_i dt\right]$$

Here, $\Gamma \geq 0$ is a weight between lobbyists' rent and consumers' welfare.Throughout the main discussion, we have assumed $\Gamma = 1$, while $\Gamma = 0$ correspondes to the world of a benevolent lobbyist.

Appendix 3

Welfare Analysis

The welfare of the lobbyist as well as the regions are maximised for co-operative strategy : this is trivially true. However, in real world, it is not possible to enforce cooperative strategy. Let us recall that the welfare of a province could be decomposed as

$$\int_0^\infty e^{-\rho t}\left(E - \frac{\gamma E^2}{2} - \frac{1}{2}c_i L_i^2\right)dt + \left[\frac{\lambda \alpha_i (1-\alpha_i)}{\rho} + (1-\alpha_i)\int_0^\infty e^{-\rho t} L_i dt\right]$$

The first term gives the lifetime utility of the population, while the second term represents the rent accruing to the lobbyist. .

For all protocols, public good stock and lobbying evolve according to

$E = E_s + (E_0 - E_s)\exp(rt)$

and

$L = L_s + b\,(E_0 - E_s)\exp(rt)$

where, r is the negative eigenvalue, E_0 is the initial stock, E_s is the final stock. The subscripts of L are explained in the same way.

Let $E_0 - E_s = \theta$

$$\int_0^\infty e^{-\rho t} E dt = \int_0^\infty e^{-\rho t}\left(E_s + \theta \exp(rt)\right)dt = \frac{E_s}{\rho} + \frac{\theta}{\rho - r}$$

$E^2 = E_s^2 + \theta^2 \exp(2rt) + 2E_s\theta \exp(rt)$

$$\int_0^\infty e^{-\rho t} E^2 dt = \frac{E_s^2}{\rho} + \frac{\theta^2}{\rho - 2r} + \frac{2E_s\theta}{\rho - r}$$

$L^2 = L_s^2 + b^2\theta^2 \exp(2rt) + 2L_s b\theta \exp(rt)$

$$\int_0^\infty e^{-\rho t} L^2 dt = \frac{L_s^2}{2} + \frac{b^2\theta^2}{\rho - 2r} + \frac{2L_s b\theta}{\rho - r}$$

And,

$$\int_0^\infty e^{-\rho t} L dt = \frac{L_s}{\rho} + \frac{b\theta}{\rho - r}$$

Thus, the consumers' welfare is

$$U_C = \frac{E_s}{\rho} + \frac{\theta}{\rho - r} - \frac{\gamma}{2}\left(\frac{E_s^2}{\rho} + \frac{\theta^2}{\rho - 2r} + \frac{2E_s\theta}{\rho - r}\right) - \frac{c}{2}\left(\frac{L_s^2}{2} + \frac{b^2\theta^2}{\rho - 2r} + \frac{2L_s b\theta}{\rho - r}\right)$$

and the lobbyists's welfare is

$$U_R = \frac{\lambda\alpha(1-\alpha)}{\rho} + \frac{L_s}{\rho} + \frac{b\theta}{\rho - r}$$

For all examples, we have taken $E_0 = 0$, so that $\theta = -E_s$

Bibliography

Bergstrom, T., Blume, L., and Varian, H. (1986), "On the Private Provision of Public Goods", *Journal of Public Economics*, **29**, pp 25-49

Cheikbossian, G. (2006), "Rent-seeking, Spillovers and the Benefits of Decentralization", *GREMAQ Working Papers,* University of Toulouse 1

Dixit, A.(1987), "Strategic Behavior in Contests", American Economic Review, **77**, pp 891-898.

Dockener, E. J., Jorgensen, S., Long, N.V., and Sorger, G. (2000), *Differential Games in Economics and Management Science*, Cambridge University Press, U K

Fershtman, C., and Nitzan, S. (1991), "Dynamic Voluntary Provision of Public Goods", *European Economic Review*, **35**, pp 1057-1067.

Fudenberg, D., and Tirole, J. (1991), *Game Theory*, MIT Press, Cambridge, Massachusetts

Katz, E., Nitzan, S., and Rosenberg, J. (1990), "Rent-seeking for Pure Public Goods", Public Choice, **65**, pp49-60

Kessing, S. G. (2007), "Strategic Complementarity in the Dynamic Private Provision of a Discrete Public Good", J*ournal of Public Economic Theory*, Forthcomming

Koethenbuerger, M. (2006), "Ex-Post Redistribution in a Federation: Implications for Corrective Policy", *Journal of Public Economics*, Forthcoming

Krueger, A.O. (1974), "The Political Economy of Rent-Seeking Society", American Economic Review, **64**, pp 291-303

List, J., and Mason, C. (2001), "Optimal Institutional Arrangements for Transboundary Pollutions in a Second Best World: Evidence from a Differential Game", *Journal of Environmental Economics and Management*, **42**, pp 277-296.

Long, N.V., and Sorger, G. (2006), "Insecure Property Rights and Growth: The role of Appropriation Costs, Wealth Effect, and Heterogeneity", *Economic Theory*, **28**, pp 513-29.

Maskin, E., and Tirole, J. (2001), "Markov Perfect Equilibrium", *Journal of Economic Theory*, **100,** pp 191-219.

Posner, R. (1975), "The Social Cost of Monopoly and Regulation", *Journal of Political Economy*, **83**, pp 322-340

Rowat, C. (2006), "Nonlinear Strategies in a Linear Quadratic Differential Game", *Department of Economics Discussion Paper*, University of Birmingham

Rudorn, J., and Pilhofer, A (2006), "Hiring Lobbyists for Federal Aid, Towns Learn That Money Talks", *The New York Times*, 2 July

Tornel, A., and Lane, P. (1999), "The Voracity Effect", *American Economic Review*, **89**, pp 22-46.

Tornel, A. and Velasco, A. (1994), "The Tragedy of the Commons and Voracity Effect: Why does Capital Flow from Poor to Rich Countries ?" *Journal of Political Economy*, **100**, pp 1208-1231.

Tsutsui, S. and Mino, K. (1990), "Nonlinear Strategies in Dynamic Duopolistic Game with Sticky Prices", *Journal of Economic Theory*, **52**, pp 136-61.

Tullock, G. (1967), "The Welfare Costs of Tariifs, Monopolies, and Theft", *Western Economic Journal*, **5**, pp 224-232.

Tullock, G. (1980), "Efficient Rent-seeking", in Buchanan, Tollison and Tullock (eds), *Towards a Theory of Rent-seeking Society*, College Station, Texas

Wirl, F. (1994), "The Dynamics of Lobbying: A Differential Game", *Public Choice*,**80**, pp 307-323

Wirl, F. (1996), "Dynamic Voluntary Contribution to Public Good: Extension to Non-Linear Strategies", *European Journal of Political Economy*, 1**2**, pp 555-560