Cohesion and Self-bias in Communities

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Abstract

This paper investigates the impact and desirability of segregated versus mixed communities in a society that contains two different types of individuals. We do so within the context of a global game, where individuals' payoffs depend on their personal preferences, and on the extent to which their actions are coordinated with those of others in the same community. Individual preferences contain both an idiosyncratic and a type-specific component. As type-specific components are not directly observable, individuals form expectations on them, based on the information at their disposal. We characterize the unique equilibrium of the game and describe how the composition of a community affects the equilibrium strategies of its members. We find that people always benefit from having a small fraction of individuals of another type in their community. Moreover, contrary to common intuition, mixed composition may increase community cohesion by decreasing the variance of the actions of its members. Although people prefer to live in communities where their type is majoritarian ("self-bias"), there exist mixed communities where both types are simultaneously better off than if they lived in segregated environments. **JEL Code:** D82, D62.

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1 Introduction

The socio-political debate of our times contains a lot of discussions that center around diversity in society. What happens when people of different types interact with one another? Are people better off in mixed communities or in homogeneous communities, where they only interact with others of their own type? The existing literature on the topic is vast, and the elements involved are numerous and multifaceted.[1] The aim of this paper is to provide a theoretical contribution to this literature, that may highlight some aspects that have so far been ignored. Starting from a set of "primitives" – namely, preferences that are not directly defined over community composition – we characterize the equilibrium behaviour of people as a function of their type and the make up of their community. From this, we derive induced preferences over community composition. In our model, therefore, people have preferences over the composition of the community they live in because this determines the behaviour of their fellow community-members.

We find a series of interesting results. First, people dislike to interact with individuals of a different type because they are less predictable than individuals of their same type. This is intuitively appealing. "Stranger" is synonymous of unknown, and this makes strangers somehow less desirable than people over whom we possess more information. Second, we find that their unpredictability is also the feature that makes strangers useful. The presence of individuals of a different type in an otherwise homogeneous community acts as a *coordination device* for the incumbent members of the community. Intuitively, the need to adapt to these "different" individuals induces all the incumbents to modify their behaviour in a coordinated manner. In the presence of strategic complementarities, this generates an indirect benefit that may outweigh the direct cost of having to interact with those different people in the first place. The key trade-off faced by communities is therefore the following. On one hand, heterogeneity introduces individuals over whom people possess little information – something that, keeping everything else equal, is not desirable. On the other hand, however, heterogeneity also increases the amount of coordination among the incumbent members of the community – a desirable feature. The comparison between mixed and homogeneous communities ultimately depends on the interplay between these two effects.

More precisely, we build a model where each individual wishes to minimize the distance

[1] See for instance Alesina and La Ferrara (2005) for a survey on the empirical literature on diversity in communities and economic performance, and the section on related literature below.

between his action and a weighted average of i) a taste parameter (capturing his preferences) and ii) the average action selected by those with whom he interacts (who are a representative sample of the individual's community). Actions may range from behaviour in public places, to punctuality, to alcohol consumption. Returns from social interaction are characterized by strategic complementarities, and are greater when an individual selects an action that is close to the average action of the group of people with whom he interacts. This is the rationale for individuals' concern about coordination.

Individuals also possess preferences over actions, captured by a taste parameter. Preferences are private information, and may vary according to the specific circumstances surrounding an interaction. Each interaction is characterized by a type-specific shock and an idiosyncratic shock. Type-specific shocks affect the preferences of all individuals of the same type in the same way. In contrast, idiosyncratic shocks vary from individual to individual. People cannot ditinguish between the type-specific and the idiosyncratic components of their preferences. The structure of the game is that of a private value global game, where individuals are uncertain about the actions of other people, because they are unable to observe their payoff parameters.

Individuals select their actions by making forecasts about the preferences of others. Consider an individual belonging to a type t. People belonging to type $t' \neq t$ are, by definition, different from him, and therefore possess preferences that are uncorrelated to his own. Hence, the best guess that a type t individual can make of the preferences of people belonging to type t' is based on a publicly known prior. In contrast, people belonging to type t are similar to him, and possess preferences that are drawn from the same distribution as his own. The prediction that the individual can make of the preferences of another type t individual is therefore *more precise* than that he can make of those of a person of type t'. This is because his own preferences provide him with an additional piece of information on which he can base his forecast. Notice however that while the predictions that people of type t can make of the preferences of others of their same type are quite precise, they also differ from individual to individual (as they are based on an individual's own preferences). In contrast, the information type t possess about the preferences of type t' is *less precise, but common.* This is the sense in which introducing type t' in a community of type t individuals has a coordination-enhancing effect for its incumbent members. As shown in the paper, when this cohesion-enhancing effect is sufficiently strong, mixed community may actually exhibit a lower variance of actions than homogeneous communities. This happens in spite of the variation that is generated by differences between types, and qualifies the commonly held view that diversity hinders community cohesion.[2]

We find that the utility that an individual expects to obtain when interacting in a community depends on the precision with which he can forecast the actions of others in the community. In other words, people are better off when they face little *strategic uncertainty.* The idea is that, when the actions of others are known, one can easily adapt to coordinate with them. In contrast, uncertainty over the actions of others is unpleasant, because it makes it harder to coordinate. Preferences over community composition therefore depend on the amount of strategic uncertainty faced by an individual in different types of communities.

Since people possess more accurate information about the preferences of people of their same type rather than people of a different type, one may conjecture that individuals should be biased towards homogeneous communities. We find that this conjecture is only partially correct. Although people would always rather live in communities where their type is majoritarian (what we call "self-bias"), this bias is rather mild, and does not imply that they favor homogeneity. To see this, consider a community initially composed of only type t, in which a fraction of type t' is introduced. On one hand, as mentioned above, this increases the amount of strategic uncertainty faced by type t agents, as they possess little information about the newcomers. On the other hand, however, the presence of type t' also induces type t agents to modify their actions in a coordinated manner. The actions selected by type t now place less weight on their personal preferences, and more weight on the publicly known priors. Hence, the actions of type t individuals are now *more predictable.* This decreases the amount of strategic uncertainty faced by type t when trying to forecast the actions of others of their same type. Moreover, this effect may be substantial even when the faction of type t' introduced in the community is very small. Each type t individual forecasts that, in response to the presence of type t', all type t will put less weight on their personal preferences, when selecting their actions. Coordination concerns then induce him to put *even less* weight on his personal preferences, and so on. This multiplier effect ensures that the presence of an arbitrarily small share of t' in the community generates a first-order gain for its incumbent members, and only a second-order

[2]The British media and political discourse on the eroding effects of diversity on social cohesion is an example. See D. Goodhart, "Too Diverse" *Prospect*, February 2004 and "The Kindness of Strangers. A reprt on Multiculturalism", *The Economist*, 28 February 2004, pp. 3-4 for illustations of the current debate on diversity in British society.

cost. Indeed, we show that the ideal community for any type t individual always includes a positive (but minoritarian) fraction of type t'.

Finally, we question the sustainability of mixed communities, by asking whether it is possible that *both* types t and t' may be better off when coexisting in the same community, rather than in a homogeneous community of their own. We show that this may indeed be the case, and identify the necessary and sufficient conditions for the result to hold.

Overall, therefore, our model allows us to gain some novel insights into the effects of community composition on individual behaviour, and the reasons why people may like/dislike the presence of people of a different type in their community. While the arrival of "strangers" always generates negative *direct effects* – as one is confronted with people who are different, and therefore less predictable – it also generates positive *indirect effects*, by increasing coordination among the incumbent members of the community. Although individuals do exhibit a form of self-bias, in that they would rather live in communities in which their type is majoritarian, the indirect benefits of diversity are sufficiently strong to ensure that people always gain from having a small fraction of individuals of a different type around. Moreover, cohabitation may simultaneoulsy benefit all types. This suggests that mixed communities may not only be desirable, but also sustainable.

Related Literature Our paper is connected with several strands of literature.

Thematic Connection First, the themes we address are clearly related with the literature on diversity and economic performance, as well as the literature on discrimination. One part of this literature – such as Becker (1957), Arrow (1973) and Alesina and La Ferrara (2000) – assumes the individuals have direct preferences over the composition of society, characterized by a "taste for discrimination". Here, diversity is never desirable, as it simply reduces individuals' welfare without generating any benefits. Another branch – such as Lazear (1999) – argues that diversity may be beneficial, because it may generate productivity gains. Essentially, the rationale is that individuals belonging to different types possess complementary sets of skills, abilities, or information; as a result, heterogeneous teams benefit from a competitive advantage, stemming from greater productivity.

Our contribution to this literature is twofold. First, we provide a novel rationale of why diversity may be desirable. Second, rather than postulating exogenously given preferences over community composition, we derive these preferences *endogenously.* This allows us to gain a deeper understanding of individuals' attitudes, and of the desirability of diversity within communities.

Conceptual Connection Our paper is also connected with the literature on social interactions – see Glaeser and Scheinkman (2002) for a survey, as well as Akerlof (1997). In common with this literature, we assume that an individual's utility depends on his chosen action, the actions chosen by agents in his group, and on his taste paramenter, and investigate the nature of the equilibria under these assumptions. Our novelty with respect to this literature is that we allow for different types of individuals to interact within the same group.

Technical Connection Finally, our paper is connected with the literature on global games; indeed, our model falls in the same category as those analyzed by Morris and Shin (2002) and Angeletos and Pavan (2004, 2007). More specifically, we study what Morris and Shin (2005) call a *private value global game.* The only type of uncertainty facing a player is the strategic uncertainty over the opponents' actions, which in turn is attributable to the uncertainty over the opponents' payoff parameters. Our contribution to this literature is to show how the introduction of different types of agents may affect the amount of strategic uncertainty in the game, in a non-monotonic way.

The remainder of the paper is organized as follows. In Section 2, we present the model, while in Section 3 we derive and discuss our results. Section 4 concludes. All the proofs can be found in the Appendix.

2 Model

Background and Utility Functions A community contains a continuum of interacting individuals. In each interaction, an individual is matched with a representative sample of people from his community, to form an interacting group. Each group contains a continuum of individuals of unit mass, indexed by the unit interval $[0, 1]$. The actions that people select affect their returns from social interaction. Actions may range from punctuality (amount of effort that exerted at being punctual), to alcohol consumption, to dress codes.

Social interactions exhibit strategic complementarities. That is, the returns from social exchange for an individual are greater if the action that he selects is close to the actions selected by others with whom he is matched.[3] Moreover, individuals possess personal

[3]For instance, in Akerlof (1997) social interaction corresponds to mutually beneficial trade between

preferences, that also affect their utility of undertaking a certain action. The utility of an individual i who selects action $a_i \in \mathbb{R}$ is

$$u_i(a_i, \overline{a}_j, \alpha_i) = -(a_i - \theta\overline{a}_j - (1-\theta)\alpha_i)^2 \quad (1)$$

where $\overline{a}_j = \int_0^1 a_j dj$ is the average action of the individuals j with whom i is matched, $\theta \in (0,1)$ is a constant that takes the same value for all individuals, and $\alpha_i \in \mathbb{R}$ is a taste parameter that reflects i's preferences.[4] The parameter θ measures the weight given to $\overline{a}_j$ and α_i in the utility function. When θ is high (close to one) individuals are almost exclusively concerned with selecting an action that is close to the average action selected in their group. When θ is low (close to zero), individuals care almost exclusively about the extent to which their actions match their preferences. Notice that our utility specification (1) implies that a person's choice of action affects the utility of all those to whom he is matched, although he doesn't take this into account when making his choice. Hence, there are externalities among individuals, which are not internalized.

Types Individuals are divided into mutually exclusive categories, or types. Types may be thought of as reflecting cultural background ("Southerners" versus "Northerners"), religious beliefs (Christians versus Muslims), ethical attitudes (Puritans versus Libertarians) and so. We take a somehow crude – but, we believe, illustrative – position, and categorize individuals into two types, type A and type B. The share of individuals of each type in a community is common knowledge. Individuals belonging to the same type possess, on average, the same preferences. However, there exist some intra-type variation, in that, within the same type, different preferences may coexist. Individual preferences are given by the sum of two components: a type-specific component, and an idiosyncratic component. The taste parameter α_i^t of an individual i belonging to type $t = A, B$ is equal to

$$\alpha_i^t = \mu^t + e^t + \varepsilon_i \quad (2)$$

where $e^t \sim N(0,1)$ for $t = A, B$, $e^A \perp e^B$, and $\varepsilon_i \sim N(0,\sigma)$ for a positive constant σ, with $\varepsilon_i \perp \varepsilon_j$ for $i \neq j$ and $\varepsilon_i \perp e^t$ for any i and t; μ^A and $\mu^B \neq \mu^A$ are common knowledge.

individuals, and is assumed to increase in the proximity of individuals' choices of locations in some "social space". Strategic complementarities are a common feature of models of social interactions. See Glaeser and Scheinkman (2002) for a survey of this literature.

[4]In what follows, the term "preferences" will always be utilized to indicate α_i.

The variance of type-specific shocks is normalized to 1. The parameter σ measures the variance of idiosyncratic shocks *relative* to type-specific shocks.

The sum of the first two elements $\mu^t + e^t$ corresponds to the type-specific component. This is equal to the sum of a known mean μ^t, plus a random element e^t, which captures the common effect that specific circumstances have on all individuals of type t; ε_i corresponds to the idiosyncratic component (or idiosyncratic shock).

Information Individual preferences are private information. Each individual observes his own α_i^t, but he is unable to discriminate between e^t and ε_i, the type-specific and the idiosyncratic shocks to his preferences. What we have in mind is that the specific circumstances surrounding social interactions may vary, and attitudes or preferences may vary with them. An individual may be unsure of the attitude that people of a given type will tend to adopt in any given *precise* situation, although he may have a good idea of the attitudes that they adopt on average, when specific circumstances are averaged out.

Notice that the estimate that an individual i of type t can make of the preferences of a type-t' individual can only be based on public information, i.e., $\mu^{t'}$. In contrast, when predicting the preferences of another person of type t, individual i possesses additional information, since he knows *both* μ^t *and* α_i^t – which, from (2), is correlated with the preferences of others of the same type. This captures the intuitively-appealing feature that a person of type t is in a better position to make forecasts of the preferences of other type t individuals than of type $t' \neq t$. As will become clear in the main body of the paper, this feature plays an important role in our analysis.

Timing

Social exchanges occur at fixed intervals of time within communities. In each social exchange:

$t = 0$ Individuals are matched in interacting groups.

$t = 1$ For each group, nature selects e^A, e^B and individual idiosyncratic shocks.

$t = 2$ Each individual observes his personal preferences and selects his action.

$t = 3$ Payoffs are realized.

Notice that the realization of type–specific shocks is group–dependent. This reflects the notion that type–specific shocks arise from the specific circumstances surrounding an interaction. These circumstances are common for all individuals in the same group, but vary across different interacting groups.

3 Results

Having described our model, we are now in the position of introducing our results. All the proofs can be found in the appendix. We start off by characterizing the equilibrium of the game. First, utility maximization yields

$$a_i = \theta E(\overline{a}_j \mid \alpha_i) + (1-\theta)\alpha_i \tag{3}$$

Individuals select actions that are a weighted average of their expectations over the average action in their group, and their preferences. Suppose that the proportion of individuals of type $t = A, B$ in a given community is $1-\lambda$ (and that of individuals of type $t' = B, A$ is λ). Then:

Lemma 1 (Description of Equilibrium): *In the unique equilibrium of the game, the action of an individual i of type t operating in this community is equal to*

$$a_i^t = k^t(\lambda)\alpha_i^t + \lambda\theta\mu^{t'} + (1-\lambda\theta - k^t(\lambda))\mu^t$$

where $k^t(\lambda) = \frac{(\sigma+1)(1-\theta)}{\sigma+1-\theta(1-\lambda)}$.

Lemma 1 describes the equilibrium strategy followed by individuals, as a function of the composition of the community in which they interact. Each individual selects an action that is a weighted average of his preferences, the average preferences of individuals of his same type across interactions, and those of individuals of the other type. Notice that, substituting for $\alpha_i^t = \mu^t + e^t + \varepsilon_i$, the equilibrium action a_i^t played by individual i of type t can also be rewritten as

$$a_i^t = k^t(\lambda)\left(e^t + \varepsilon_i\right) + \lambda\theta\mu^{t'} + (1-\lambda\theta)\mu^t \tag{4}$$

As (4) illustrates, the only source of within-type heterogeneity of individual actions is given by $e^t + \varepsilon_i$.[5] The weight put by individuals on these sources of within-type variations is equal to $k^t(\lambda)$, namely the weight they put on their personal preferences when selecting their actions. The key feature here is $k^t(\lambda)$ is strictly *decreasing* in λ, the proportion of

[5]Within interacting groups, the type-specific shock is the same for all individuals of the same type, and the only source of within-type variation is the idiosyncratic shock. Across interacting groups, type-specific shocks take different values, so type-specific shocks also contribute to within-type variation in actions. Notice however that, when assessing individual welfare, this second element is absent, as individuals only care about variance of actions within their interacting group.

individuals of type t' in the community. Intuitively, the correlation between the preferences of an individual of type t and those of an individual of type $t' \neq t$ is zero. Hence, the introduction of type t' in the community reduces the usefulness for a type t of utilizing his private preferences for making predictions about the actions of others. Each type t individual decreases the weight he places on his preferences when selecting his action, and correspondingly increases the weight placed on $\mu^{t'}$ (and possibly[6] also μ^t). The following Lemma makes clear the consequences of this effect in terms of within–type variance of actions (or within–type cohesion).

Lemma 2 (Within–Type Variance of Actions): *The variance of the actions of individuals of type $t = A, B$ around the mean type t–action in a community is strictly decreasing in λ, the proportion of type $t' = B, A$ in the community.*

Lemma 2 illustrates how the presence of type t' acts as a *coordination (or cohesion-enhancing) device* on individuals of type t. This is because, in mixed communities, individual actions are less sensitive to idiosyncratic and interaction–specific variations. The idea is that the information that type t people possess about the preferences of others of type t is at least partially private, since it derives from each individual's personal preferences. So the forecasts that type t make of the actions of other agents of type t *differ,* according to the precise realization of their personal preferences. In contrast, the information that type t possess about the preferences of type t' is public, and thus the same for all. As a result, all individuals of type t make the *same* forecast of the actions of type t'. It follows that the introduction of type t' in the community decreases the heterogeneity that exists in people's forecasts over other individuals' actions. In turn, this generates greater cohesion between type t individuals.

Notice that the introduction of type t' in the community generates a *direct* and an *indirect* effect on individuals' actions. The *direct effect* arises because individuals forecast that they will now interact with individuals of type t', and accordingly adjust their actions, by putting less weight on their individuals preferences. The *indirect (or multiplier) effect* arises because each individual realizes that, through the direct effect, all individuals of type t will put less weight on their private preferences when selecting their actions. In turn, this decreases the weight put by each individual on his private preferences even further, and so on. This indirect effect ensures that even if the share of type t' introduced in the

[6] It is straightforward t verify that $\frac{d(1-\lambda\theta-k^t(\lambda))}{d\lambda}$ may be negative or positive depending on parameter values.

community is arbitrarily small (or second-order), the effect that this has on individuals' actions is non-negligeable (first order).[7] In other words, even the introduction of a very small share of type t' in a community will sensibly affect the actions of the incumbent members of the community. As will become clear below, this feature plays an important role when assessing the welfare effects of heterogeneity.

3.1 Community Cohesion

Governments in Europe and elsewhere have recently been promoting the idea that "Community Cohesion" should be fostered and encouraged.[8] It is therefore important to investigate whether we should expect mixed or homogeneous communities to be more cohesive. This is what we do in this Section. Since the concept of community cohesion is in itself quite vague, it is first necessay to make clear how we define it in the present context.

Definition 1: *We define community cohesion as the variance of the actions of individuals within a community around the community mean action.*

As seen in Lemma 2, introducing a fraction of type t' in a community increases the within–type cohesion of individuals of type t. However, it also introduces a new source of variance, arising from between–type variations in the choice of actions. Proposition 1 spells out the sufficient conditions for the latter effect to be weaker than the former.

Proposition 1 (Community Cohesion): *The sufficient condition for community cohesion to be greater in mixed communities is that*

$$\left(\mu^A - \mu^B\right)^2 < \theta \frac{\sigma + 1}{\left(\sigma - \theta + 1\right)^3} \left[\left(\sigma + 1\right)\left(4\left(\sigma + 1\right) - 3\theta\right) + \theta^2\right]$$

Proposition 1 shows that, for some parameter values, cohesion may be higher in mixed as opposed to homogeneous communities. Consider for instance $\theta = 3/4$, $\sigma = 1$. Then the condition for mixed communities to be more cohesive than homogeneous communities is that $\mid \mu^A - \mu^B \mid < 3$. More generally, keeping everything else equal, the righthandside of

[7]This can be seen by noticing that $\lim_{\lambda \to 0} \frac{dk^t(\lambda)}{d\lambda} = -\theta\left(1 - \theta\right) \frac{\sigma+1}{(\sigma-\theta+1)^2}$.

[8]For instance, in the UK a Community Cohesion Unit has recently been created within the Home Office, to oversee initiatives on building community cohesion. In addition to the ministerial group, there are several advisory groups and taskforces such as the Community Cohesion Review Team. In a similar vein, the Local Government Association (LGA), together with the Home Office, has recently issued a draft guidance to local authorities on mainstreaming and promoting community cohesion ("Community Cohesion: An Action Guide", available at www.lga.gov.uk).

the condition lain out in Proposition 1 is increasing in θ, and decreasing in σ. Intuitively, a greater value of θ implies that, when selecting their actions, people put a lot of weight on their forecasts of other people's preferences. Heterogeneity in individuals forecasts of others' preferences is then particularly disruptive for community cohesion, as it translates into large variations in individuals actions. Correspondingly, introducing individuals of a different type is very helpful for increasing community cohesion, as it decreases the amount of heterogeneity in individual forecasts of the preferences of other agents.

In contrast, a large σ decreases the usefulness of utilizing one's preferences for predicting the preferences of others. This increases cohesion in homogeneous communities, and decreases the extent to which the introduction of people of a different type enhances within-type cohesion.[9]

Proposition 1 illustrates how the often–heard argument that heterogeneity results in less cohesion may be incorrect. Although differences between types are indeed a source of greater variance in actions, in mixed communities individuals of the same type select actions that are closer to one another. For certain parameter values, this second effect may dominate, and result in greater community cohesion. Empirically, therefore, our model predicts that mixed communities may indeed exhibit lower variance of actions than homogeneous ones. This more likely to occur when (i) individuals are more concerned with selecting actions that are close to those of others than with accommodating their preferences and (ii) the variance of idiosyncratic shocks to preferences is small with respect to that of type-specific shocks.

3.2 Preferences over Community Composition

We now investigate the preferences of individuals with respect to community composition. We do so by taking an ex-ante perspective, evaluating the expected utility of an individual as a function of the composition of his community. The idea is to try to gain a better understanding of the welfare characteristics of different types of communities. Are individuals always better off in homogenoeus communities? Or are they willing to introduce some "different" people in their communities? In this latter case, how large is the share of "strangers" that people are willing to introduce? These are the types of questions that we wish to address in this section.

[9]This can be seen by noticing that $\frac{dk^t(\lambda)}{d\lambda}$ is increasing in σ.

Substituting for the optimal action (3) into the utility function, we see that an individual's expected utility can be written as

$$E\left[u_i\left(a_i, \overline{a}_j, \alpha_i\right)\right] = -\frac{1}{4}E\left[E\left(\left(\overline{a}_j - E\left(\overline{a}_j \mid \alpha_i\right)\right)^2 \mid \alpha_i\right)\right] = -\frac{1}{4}E\left(Var\left(\overline{a}_j \mid \alpha_i\right)\right) \qquad (5)$$

where $Var\left(\overline{a}_j \mid \alpha_i\right)$ indicates the conditional variance of $\overline{a}_j$ around $E\left(\overline{a}_j \mid \alpha_i\right)$, i's expectation of $\overline{a}_j$ conditional on α_i.

In words, $E\left(Var\left(\overline{a}_j \mid \alpha_i\right)\right)$ denotes the accuracy with which i is able to forecast the average action of others in his group. In this sense, (5) captures the amount of *strategic uncertainty* present in the game. An individual's expected utility is therefore inversely proportional to the amount of strategic uncertainty he faces.

The effect of community composition on strategic uncertainty is ambiguous. Consider a type A individual i, whose community changes from being entirely composed of type A to also including a fraction of type B. Because the preferences of B individuals are entirely uncorrelated to his own, the change in the composition of i's community introduces people, over whom i possesses little information (in particular, less information than over the actions of other type A individuals). Keeping everything else equal, this *direct effect* increases strategic uncertainty. Notice however that, as seen in the previous section, the move from a homogeneous to a heterogeneous community also decreases the weight put by other type A agents on their preferences when selecting their actions. Hence, it is now easier for i to predict the actions of other type A individuals with whom he interacts. Keeping everything else equal, this *indirect effect* decreases strategic uncertainty.

Overall, therefore, people face a trade-off. In order to decrease the strategic uncertainty they face with respect to other individuals of their same type, they must introduce in their communities individuals of other, different types, over whom they possess little information. Put differently, individuals dislike people of a different type from their own because their preferences are drawn from a different distribution, and are therefore *less predictable.* Yet, it is precisely this feature that also makes them desirable! In response to the presence of type t', the actions of individuals of type t put less weight on personal preferences and are therefore *more predictable.*

Denote the share of individuals of a type different from i's as λ. It is straightforward to verify that $E\left(Var\left(\overline{a}_j \mid \alpha_i\right)\right)$ is strictly convex in λ and reaches a minimum at $\lambda^* \in (0, 0.5)$. Hence, a small amount of heterogeneity unambiguously decreases the amount of strategic uncertainty faced by individual i. As λ increases, however, the direct effect described above becomes gradually more important, and eventually takes over. The strategic uncertainty faced by i is minimized when the share of individuals belonging to a different type to i is somewhere between zero and one-half.

Proposition 2 (Preferences over Community Composition): *Agents always prefer to interact in mixed as opposed to homogeneous communities. However, individuals exhibit a self-bias, in that their expected utility is maximized when the share of people of their type in the community is greater than one-half.*

Proposition 2 establishes that individuals of each type would ideally like to include a positive but minoritarian fraction of the other type in their communities. The rationale for the first result – namely, that individuals would like to include a positive fraction of the other type in their community – derives from two observations. First, there exists externalities among individuals, which are not internalized. Each individual selects his action without taking into account its effect on the utility of others. Hence, from an aggregate welfare perspective, at equilibrium people put too much weight on their individual preferences. This generates excessive variance of actions, or, equivalently, insufficient cohesion. Second, as seen above, the introduction of an arbitrarily small share of individuals of another type in a homogeneous community generates a first order cohesion-enhancing effect. This first-order effect is beneficial, as it reduces the excessive heterogeneity in actions connected with the presence of externalities. In contrast, the drawback generated by the presence of people of another type, namely the fact that they select actions that are less predictable, is only second-order (as it is proportional to their share in the community). Hence, introducing an arbitrarily small share of the other type in a homogeneous community is unambiguously welfare-improving: it generates a first-order benefit, and only a second-order cost.

Notice that $k^t(\lambda)$ – the weight put by individuals of type t on their preferences when a share λ of people in the community is of type t' – is convex in λ. Hence, the cohesion-enhancing effect of type t' decreases as their share in the community increases. For type t, the benefits introduced by the presence of t' exhibit decreasing returns to scale. In contrast, the costs imposed by the presence of t' is proportional their share in the community. As a result, when λ is equal to one-half, type t would actually gain from a decrease in λ. More generally, there exist a value λ^* of λ (strictly smaller than one-half) such that, when $\lambda \geq \lambda^*$, type t would benefit from a reduction of the share of type t' in the community. This self-bias effect shares some similarity with the taste for discrimination parameter that features in some of the literature on discrimination (such as Becker 1957). However, while in this literature the taste for discrimination is imposed exogenously, here the self-bias effect is derived endogenously. Moreover, the self-bias effect only kicks in when the share of people of different type present in the community is sufficiently high. As shown in Proposition 1, communities unambiguously benefit from having a few "strangers" around. Hence, our model predicts that homogeneous communities should generally welcome the arrival of individuals of a different type. As the share of those "strangers" in the community grows, however, attitudes will become less welcoming. This latter prediction is a consequence of the self-bias effect.

We now explore whether it is possible to construct communities that are simultaneously preferred by both types over homogeneous communities, composed exclusively of individuals of their own type. This is important, as it gives a measure of the sustainability of mixed communities in practice.

Proposition 3 (Simultaneous Preference for Mixed Versus Homogeneous Communities): *The necessary and sufficient condition for the existence of mixed communities that are simultaneously preferred by both types over homogeneous communities (composed only of their own type) is that* $\theta > 1 - \sqrt{\sigma + 2\sigma^2}$.

As mentioned when discussing Proposition 1, a large θ increases the heterogeneity of actions in homogeneous communities, making them less attractive. Hence, a large θ makes it more likely that both types may simultaneously prefer to live in a mixed community. Interestingly, note that the condition lain out in Proposition 3 is easier to satisfy for a larger σ. This may at first appear counterintuitive. As seen above, a greater σ reduces the cohesion-enhancing effect of introducing individuals of a different type, and this should make the result harder to obtain. Notice however that a large σ also decreases the weight that *all* individuals put on their preferences when selecting their actions. Hence, when σ is large, between-type differences in actions are correspondingly smaller. This makes mixed communities relatively more desirable.

Proposition 3 establishes that mixing in the same community may simultaneously benefit both types. Although individuals would ideally like to live in communities where their type is majoritarian, the benefits generated by mixing may be sufficiently large to ensure that people prefer mixed over homogeneous communities even when their type is *not* majoritarian. This suggests that policies aimed at encouraging greater mixing in society may be welfare enhancing for all parties involved.

4 Concluding Remarks

What happens when different types of people are mixed in the same community? Is there any reason why different types of people should wish to be mixed in the same community? Is it true that individuals exhibit a "taste for discrimination", in that they always prefer to interact with other individuals of their same type? We believe that our analysis may have provided a useful theoretical contribution to these debates. Starting from preferences that are not directly defined over community composition, we have shown that individuals may indeed behave differently in mixed as opposed to homogeneous environments. In the presence of strategic complementarities, heterogeneity may then have "hidden" or indirect benefits, by increasing the amount of cohesion that exists within each type. Although, keeping everything else equal, people would always prefer to interact with agents of their same type – as they are somehow more predictable – the hidden benefits of heterogeneity are sufficiently strong to ensure that communities always benefit from introducing a few "different" individuals in their midst, and that individuals of both types may simultaneously be better off in mixed as opposed to homogeneous communities.

Although our analysis has confined itself to a situation in which there are only two types of individuals, with symmetric utility functions, it would be interesting to explore the implications of relaxing these assumptions. What happens when we allow for n types to possibly interact to one another? Is there an "optimal" number of types that should cohexits in a community? What if some types are "more dogmatic" – i.e., they are more concerned with selecting actions that match their preferences – than others? Another direction in which the analysis could be expanded concerns the nature of the strategic interaction among agents. What would happen if the agents' actions were strategic substitutes (as may for instance be the case in the presence of congestion effects)? Would individuals be better off in heterogeneous, or homogeneous environments? We believe that these questions may provide a stimulating agenda for future research.

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Appendix

Proof of Lemma 1:
We first characterize the linear equilibrium of the game. We then argue that this is also the unique equilibrium.

Linear Equilibrium
An individual i of type t selects his action to minimize

$$E\left[\left(a_i - \theta\overline{a}_j - (1-\theta)\alpha_i^t\right)^2 \mid \alpha_i^t\right] \tag{6}$$

The first order condition yields:

$$a_i = \theta E\left(\overline{a}_j \mid \alpha_i^t\right) + (1-\theta)\,\alpha_i^t \tag{7}$$

Suppose that the share of individuals of type $t' \neq t$ in the community is λ, and that of type t is $1-\lambda$. In a linear equilibrium, the actions of a generic individual of type t can be written as

$$a_j^t = k^t\alpha_j^t + \delta^t \tag{8}$$

and those of an individual of type t' can equivalently be written as

$$a_j^{t'} = k^{t'}\alpha_j^{t'} + \delta^{t'} \tag{9}$$

For simplicity, denote k^t as k, $k^{t'}$ as K, δ^t as δ and $\delta^{t'}$ as Δ. Hence:

$$\begin{aligned} a_j^t &= k\alpha_j^t + \delta \\ a_j^{t'} &= K\alpha_j^{t'} + \Delta \end{aligned}$$

(7) can therefore be written as

$$a_i^t = \theta\left[\lambda\left(KE\left(\alpha_j^{t'} \mid \alpha_i^t\right) + \Delta\right) + (1-\lambda)\left(kE\left(\alpha_j^t \mid \alpha_i^t\right) + \delta\right)\right] + (1-\theta)\,\alpha_i^t \tag{10}$$

Now, because the preferences of individuals of different types are uncorrelated, we have

$$E\left(\alpha_j^{t'} \mid \alpha_i^t\right) = \mu^{t'} \tag{11}$$

In contrast, the preferences of individuals of the same type are correlated. Hence:

$$\begin{aligned} E\left(\alpha_j^t \mid \alpha_i^t\right) &= \mu^t + E\left(e^t \mid \alpha_i^t\right) + E\left(\varepsilon_j \mid \alpha_i^t\right) \\ &= \mu^t + E\left(e^t \mid \alpha_i^t\right) \text{ as } E\left(\varepsilon_j \mid \alpha_i^t\right) = 0 \end{aligned} \tag{12}$$

where

$$E\left(e^t \mid \alpha_i^t\right) = \frac{1}{\sigma+1}\left(\alpha_i^t - \mu^t\right) \tag{13}$$

Substituting in (10) we obtain

$$a_i^t = \theta\left[\lambda\left(K\mu^{t'} + \Delta\right) + (1-\lambda)\left(k\left(\frac{\sigma}{\sigma+1}\mu^t + \frac{1}{\sigma+1}\alpha_i^t\right) + \delta\right)\right] + (1-\theta)\,\alpha_i^t \tag{14}$$

We therefore have.

$$k = \left(\frac{1-\lambda}{\sigma+1}\theta k + 1 - \theta\right) \text{ i.e. } k = \frac{(\sigma+1)(1-\theta)}{\sigma+1-\theta(1-\lambda)} \tag{15}$$

and

$$\delta = \theta\left(\lambda\left(K\mu^{t'} + \Delta\right) + (1-\lambda)\left(k\frac{\sigma}{\sigma+1}\mu^t + \delta\right)\right) \tag{16}$$

Now consider an individual of type t'. By analogy, we have

$$E\left(\alpha_j^t \mid \alpha_i^{t'}\right) = \mu^t \tag{17}$$

and

$$E\left(\alpha_j^t \mid \alpha_i^{t'}\right) = \frac{\sigma\mu^{t'} + \alpha_i^{t'}}{\sigma+1} \tag{18}$$

Hence:

$$a_i^{t'} = \theta\left[\lambda\left(K\left(\frac{\sigma}{\sigma+1}\mu^{t'} + \frac{1}{\sigma+1}\alpha_i^{t'}\right) + \Delta\right) + (1-\lambda)\left(k\mu^t + \delta\right)\right] + (1-\theta)\,\alpha_i^{t'} \tag{19}$$

It follows that

$$K = \left(\frac{\lambda}{\sigma+1}\theta K + 1 - \theta\right) \text{ i.e. } K = \frac{(\sigma+1)(1-\theta)}{\sigma+1-\theta\lambda} \tag{20}$$

and

$$\Delta = \theta\left(\lambda\left(K\frac{\sigma}{\sigma+1}\mu_{t'} + \Delta\right) + (1-\lambda)\left(k\mu^t + \delta\right)\right) \tag{21}$$

We now have a system, composed of (10) and (21), where k and K are given by (15) and (20). It is straightforward to verify that the solution of the system is

$$\begin{aligned} \delta &= \mu^{t'}\lambda\theta + \mu^{t}\left(1 - k - \lambda\theta\right) \\ \Delta &= \mu^{t'}\left(1 - K - \left(1 - \lambda\right)\theta\right) + \mu_{t}\left(1 - \lambda\right)\theta \end{aligned} \tag{22}$$

Uniqueness

Although we consider a private value environment, our game shares the same structure as the class of games analyzed in Morris and Shin (2002) and Angeletos and Pavan (2004, 2007). As explained in Angeletos and Pavan (2007), and following the same argument as in Morris and Shin (2002), uniqueness of equilibrium is ensured whenever $-\frac{\partial^2 u_i/\partial a_i \partial \overline{a}_j}{\partial^2 u_i/\partial a_i \partial a_i} < 1$. In our model, $-\frac{\partial^2 u_i/\partial a_i \partial \overline{a}_j}{\partial^2 u_i/\partial a_i \partial a_i} = \theta < 1$. Hence, the linear equilibrium identified above is unique.■

Proof of Lemma 2: The mean type t–action in a community is

$$\begin{aligned} E\left(a_i^t\right) &= k^t\left(\lambda\right) E\left(\alpha_i^t\right) + \theta\lambda\mu^{t'} + (1 - \theta\lambda - k^t(\lambda))\mu^t \\ &= \left(1 - \theta\lambda\right)\mu^t + \theta\lambda\mu^{t'} \text{ because } E\left(\alpha_i^t\right) = \mu^t \end{aligned} \tag{23}$$

The variance of the action of an individual of type t around mean type t–action in a community is

$$E\left[\left(a_i^t - E\left(a_i^t\right)\right)^2\right] \tag{24}$$

where

$$a_i^t - E\left(a_i^t\right) = k^t\left(\lambda\right)\left(\alpha_i^t - \mu^t\right) = k^t\left(\lambda\right)\left(e^t + \varepsilon_i\right) \tag{25}$$

Hence, we have

$$E\left[\left(a_i^t - E\left(a_i^t\right)\right)^2\right] = (k^t\left(\lambda\right))^2(1 + \sigma) \tag{26}$$

The derivative of the righthandside of (26) with respect to λ is

$$-2\theta\left(1 - \theta\right)^2 \frac{\left(\sigma + 1\right)^3}{\left(\sigma + 1 - \theta(1 - \lambda)\right)^3} < 0 \tag{27}$$

■

Proof of Proposition 1: Continue to denote k^t as k, $k^{t'}$ as K, δ^t as δ and $\delta^{t'}$ as Δ (as in the proof of Lemma 1). The average action in the community composed of a share λ of type t' and a share $(1-\lambda)$ of type t is

$$\overline{a} = \lambda\left(K\mu^{t'} + \Delta\right) + (1-\lambda)(k\mu^{t} + \delta) \tag{28}$$

The variance of actions in the community is

$$E\left[(a_j - \overline{a})^2\right] = \lambda E\left[\left(K\left(\mu^{t'} + e^{t'} + \varepsilon_j\right) + \Delta - \overline{a}\right)^2\right] + (1-\lambda)E\left[\left(k\left(\mu^{t} + e^{t} + \varepsilon_j\right) + \delta - \overline{a}\right)^2\right] \tag{29}$$

Solving out, we obtain

$$E\left[(a_j - \overline{a})^2\right] = \lambda(1-\lambda)\left(1-\theta\right)^2 (\mu^t - \mu^{t'})^2 + (1+\sigma)(\lambda K^2 + (1-\lambda)k^2) \tag{30}$$

Evaluated at $\lambda \to 0$, the derivative of the righthandside of (30) with respect to λ is negative when

$$\left(\mu^t - \mu^{t'}\right)^2 < \theta\frac{\sigma+1}{\left(\sigma-\theta+1\right)^3}\left[\left(\sigma+1\right)\left(4\left(\sigma+1\right) - 3\theta\right) + \theta^2\right] \tag{31}$$

Hence, (31) ensures that by introducing a small share of individuals of type $t' = B, A$ in a community of individuals of type $t = A, B$, community variance decreases. ■

Proof of Proposition 2:
We wish to prove that $E\left[E\left(\left(\overline{a}_j - E\left(\overline{a}_j \mid \alpha_i\right)\right)^2 \mid \alpha_i\right)\right]$ is strictly convex in λ and reaches a minimum at $\lambda^* \in (0, 0.5)$. Suppose without loss of generality that individual i is of type t, and denote as λ the share of individuals of type t' in the community. We then have

$$\overline{a}_j = \lambda\left(K\left(\mu^{t'} + e^{t'}\right) + \Delta\right) + (1-\lambda)\left(k\left(\mu^t + e^t\right) + \delta\right)$$

and

$$\begin{aligned} E\left(\overline{a}_j \mid \alpha_i\right) &= \lambda\left(K\mu^{t'} + \Delta\right) + (1-\lambda)\left(\tfrac{k}{\sigma+1}\left(\sigma\mu^t + \alpha_i^t\right) + \delta\right) \\ &= \lambda\left(K\mu^{t'} + \Delta\right) + (1-\lambda)\left(k\left(\mu^t + \tfrac{e^t+\varepsilon_i}{\sigma+1}\right) + \delta\right) \end{aligned} \tag{32}$$

after substituting for $\alpha_i^t = \mu^t + e^t + \varepsilon_i$

Hence, we can write

$$\overline{a}_j - E\left(\overline{a}_j \mid \alpha_i\right) = \lambda K e^{t'} + \frac{1-\lambda}{\sigma+1}k(\sigma e^t - \varepsilon_i) \tag{33}$$

so that

$$E\left[E\left(\left(\overline{a}_j - E\left(\overline{a}_j \mid \alpha_i\right)\right)^2 \mid \alpha_i\right)\right] = \lambda^2 K^2 + \frac{(1-\lambda)^2 k^2 \sigma}{\sigma+1} \tag{34}$$

Now,

$$\frac{d^2\left(\lambda^2 K^2\right)}{d\lambda^2} = 2\left(1-\theta\right)^2 \frac{\left(\sigma+1\right)^3}{\left(\sigma-\theta\lambda+1\right)^4}\left(\sigma+2\theta\lambda+1\right) > 0 \tag{35}$$

and

$$\frac{d^2\left(\left(1-\lambda\right)^2 k^2 \sigma\right)}{d\lambda^2} = 2\sigma\left(1-\theta\right)^2 \frac{\left(\sigma+1\right)^3}{\left(\sigma-\theta(1-\lambda)+1\right)^4}\left(1+\sigma+2\theta\left(1-\lambda\right)\right) > 0 \tag{36}$$

Hence, $E\left[E\left(\left(\overline{a}_j - E\left(\overline{a}_j \mid \alpha_i\right)\right)^2 \mid \alpha_i\right)\right]$ is undoubtedly convex in λ. Moreover:

$$\lim_{\lambda\to 0} \frac{d\left(\lambda^2 K^2 + \frac{(1-\lambda)^2 k^2 \sigma}{\sigma+1}\right)}{d\lambda} = -2\sigma\left(1-\theta\right)^2 \frac{\left(\sigma+1\right)^2}{\left(\sigma-\theta+1\right)^3} < 0 \tag{37}$$

and

$$\lim_{\lambda\to 0.5} \frac{d\left(\lambda^2 K^2 + \frac{(1-\lambda)^2 k^2 \sigma}{\sigma+1}\right)}{d\lambda} = 8\left(\sigma+1\right)^2 \frac{\left(1-\theta\right)^2}{\left(2\sigma-\theta+2\right)^3} > 0 \tag{38}$$

This proves that $E\left[E\left(\left(\overline{a}_j - E\left(\overline{a}_j \mid \alpha_i\right)\right)^2 \mid \alpha_i\right)\right]$ reaches a minimum at $\lambda^* \in (0, 0.5)$.■

Proof of Proposition 3: A *sufficient* condition for the proposition to hold is that both types are better off in a mixed community with 50/50 type composition than in a homogeneous community. This is the case if

$$\lim_{\lambda\to 0}\left(\lambda^2 K^2 + \frac{(1-\lambda)^2 k^2 \sigma}{\sigma+1}\right) - \lim_{\lambda\to 0.5}\left(\lambda^2 K^2 + \frac{(1-\lambda)^2 k^2 \sigma}{\sigma+1}\right) > 0 \tag{39}$$

where, as seen above, $\left(\lambda^2 K^2 + \frac{(1-\lambda)^2 k^2 \sigma}{\sigma+1}\right)$ is the value of $E\left[u_i\left(a_i, \overline{a}_j, \alpha_i\right)\right]$ when a share λ of individuals in the community is of a different type than i. Notice that (39) is also *necessary* for the proposition to hold. From Proposition 2, we know that $\lambda^2 K^2 + \frac{(1-\lambda)^2 k^2 \sigma}{\sigma+1}$ reaches a minimum at $\lambda^* \in (0, 1)$. If (39) is not satisfied, this therefore implies that individuals prefer to live in homogeneous communities rather than mixed communities in which their type is not majoritarian. In this case, then, Proposition 3 cannot hold.

Now,

$$\lim_{\lambda \to 0}\left(\lambda^2 K^2 + \frac{(1-\lambda)^2 k^2 \sigma}{\sigma+1}\right) - \lim_{\lambda \to 0.5}\left(\lambda^2 K^2 + \frac{(1-\lambda)^2 k^2 \sigma}{\sigma+1}\right)$$
$$= (1-\theta)^2 \frac{(\sigma+1)^2\left(2\theta+\sigma-\theta^2+2\sigma^2-1\right)}{(\sigma-\theta+1)^2(2\sigma-\theta+2)^2} \; > 0 \text{ if } \left(2\theta+\sigma-\theta^2+2\sigma^2-1\right) > 0 \tag{40}$$

It is straightforward to verify that $\left(2\theta+\sigma-\theta^2+2\sigma^2-1\right) > 0$ if $\theta > 1-\sqrt{\sigma+2\sigma^2}$.■