Bond Liquidity Premia

Jean-Sébastien Fontaine
Université de Montréal

René Garcia
Université de Montréal, CIRANO and CIREQ.

February 28, 2007

Abstract

Bond traders know very well that both risk and liquidity affect bond prices. Our paper introduces a liquidity factor within a term structure model, complementing the level, slope and curvature factors. Estimation from coupon bonds proceeds with a non-linear filter and produces a persistent liquidity factor, common to all maturities which captures the on-the-run premium. Moreover, we find that increases of this liquidity factor predict lower future returns for *both* on-the-run and off-the-run bonds. This highlights the presence of a systematic liquidity component in the prices of all bonds.

JEL Classification: E43, H12.

Address for coresspondence: Département de Sciences Économiques, Université de Montréal, C.P. 6128, Succ. Centre-Ville, Montréal, Québec, H3C 3J7, Canada, *E-mail:* `js.fontaine@umontreal.ca` and `rene.garcia@umontreal.ca`. The first author gratefully acknowledges financial support from the Institut de Finance Mathématique de Montréal (IFM^2) and the Banque Laurentienne. The second author gratefully acknowledges financial support from the Fonds Québécois de la recherche sur la société et la culture (FQRSC), the Social Sciences and Humanities Research Council of Canada (SSHRC), the Network of Centres of Excellence (MITACS), Hydro-Québec, and the Bank of Canada.

"... a part of the interest paid, at least on long-term securities, is to be attributed to uncertainty of the future course of interest rates."
(p.163)

"... the imperfect 'moneyness' of those bills which are not money [...] causes the trouble of investing in them and [...] to stand at a discount."
(p.166)

"... In practice, there is no rate so short that it may not be affected by speculative elements; there is no rate so long that it may not be affected by the alternative use of funds in holding cash."
(p.166)

John R. Hicks, Value and Capital, 2nd edition, 1948.

1 Introduction

Bond traders know very well that government bonds with the same maturity and coupon rate may sell at different prices. This difference in price is attributed to liquidity risk. Typically, recently issued (on-the-run) bonds sell at a premium with respect to seasoned (off-the-run) bonds of similar characteristics. In fact, the U.S Treasury recognizes and takes advantages of this price differential[1].

Term structure models are almost exclusively estimated with bootstrapped zero-coupon yields that sidestep the liquidity issue. Moreover, in these models, deviations of observed bond prices from the expectation hypothesis are rationalized by time-varying risk premia (see Dai and Singleton [15] and Piazzesi [47] for affine models.) On the other hand, the liquidity premium has been measured in empirical studies highlighting correlations between prices and returns on one side and microstructure measures of liquidity on the other[2].

The main contribution of this paper is to introduce liquidity in the framework of a term structure model. We sidestep credit risk and pre-processing issues altogether by

[1] "In addition, although it is not a primary reason for conducting buy-backs, we may be able to reduce the government's interest expense by purchasing older, "off-the-run" debt and replacing it with lower-yield "on-the-run" debt." [Treasury Assistant Secretary for financial markets Lewis A. Sachs, Testimony before the House Committee on Ways and Means]

[2] see Amihud and Mendelson [3], Warga [57] and, more recently, Goldreich, Hanke and Nath [29] for evidence of the liquidity premia and its link with transaction costs.

focusing on government coupon bonds. We find that a liquidity factor complements the level, slope and curvature factors as determinants of bond prices. Moreover, increases in this measure of on-the-run premia predicts lower returns for all bonds. Note that our identification scheme of this on-the-run premium relies on individual characteristics of on-the-run bonds. Nonetheless, its predictive power for returns extends to all bonds. This suggests that valuation increase for all U.S. Treasuries when aggregate liquidity is scarce: the demand for money affects asset bond prices.

We base our model on a dynamic Nelson-Siegel representation of the term structure proposed in Diebold and Li [18] and Diebold, Rudebusch and Aruoba [19]. This approach is parsimonious, delivers good in-sample fit and forecasts well. The smooth shape of Nelson-Siegel curves allows us to analyze small deviations, relative to an idealized curve, caused by variations in market liquidity.

We extend the benchmark model and include one latent factor designed to capture on-the-run liquidity premia between recently issued and seasoned securities. Our strategy is to consider pairs of bonds with similar characteristics but different ages. The identification scheme relies on deterministic loadings for the liquidity factor. We assume that liquidity affects the price of all bonds, but its loadings varies with the age of a bond and its time to maturity.

The Fama-Bliss data set (FF) uses the bootstrap method to construct discount bond data from observed coupon bond prices (see Fama and Bliss [28]. This method smooths the liquidity premium away and leads us to consider coupon bonds directly. However, the pricing equation for coupon bonds is highly non-linear and does not belong to the affine class. We estimate the models with quasi-maximum likelihood using the Unscented Kalman Filter (UKF), an extension of the Kalman Filter for non-linear state-space systems.

The most common approach used for the estimation of nonlinear systems is the extended Kalman filter (EKF). This method linearizes the state-space system and makes the the traditional Kalman filter available. Duan and Simonato [20] uses the EKF to estimate term structure models from the FF data set.

The UKF, described in more detail below (see also appendix A), is based on a relatively new method for calculating statistics of a random variable which undergoes a nonlinear transformation (see Julier, Uhlmann and Durrant-White [37]). It starts with a well-chosen set of points with given sample mean and covariance. The nonlinear function is then applied to each point in turn. This approach has a Monte Carlo flavor but the sample is drawn according to a specific deterministic algorithm (see Julier and Uhlmann [36]). This reduces the computational burden considerably but still provides greater accuracy than linearisation. The UKF has been used recently in the term structure literature by Leippold and Wu [42] and foreign exchanges literature by Bakshi, Carr and Wu [6].

Estimation yields a persistent liquidity factor, common to all maturities, that captures differences between prices of recently issued bonds and prices of older bonds : the *on-the-run premia*. The effect of the liquidity factor increases with maturity. However, its impact decays with time from issuance as bonds become seasoned.

Increases in the systematic part of on-the-run premia predict lower future returns for both on-the-run and off-the-run bonds. Controlling for the predictive power of forward rates (see Fama and Bliss [28], Campbell and Shiller [12] and Cochrane and Piazzesi [11]) does not affect this predictive power. This result is most striking when looking at Figure 1b which shows the liquidity factor and excess returns computed from our model. Periods of high liquidity premia are associated with lower excess returns. Again, current valuation increases for all bonds when demand for on-the-run bonds increases.

This paper makes some ancillary contributions. First, we confirm the presence in coupon-bond data of a forward rate factor summarizing the predictive power of the yield curve as in Cochrane and Piazzesi [11]. Also, our results show that pre-processing bond prices with the bootstrap method does not affect properties of filtered level, slope and curvature factors. In fact, the predictability from our liquidity factor obtains in the FF discount data set as well.

Our results relate to other works documenting the on-the-run liquidity premium. Warga [57] documents lower average returns from holding on-the-run bonds compared to duration-matched portfolios. However, Krishnamurty [38] shows that the absence of arbitrage prevails when the cost of shorting bonds is properly accounted for. The seminal contribution of Amihud and Mendelson [2] suggests that differences in transaction costs may cause the premium. Goldreich, Hanke and Nath [29] evaluate the link between the premium and expected transaction costs and find a weak but significant effect. Amihud and Mendelson [3] examine bills and notes with less than 6 months to maturity and reach a similar conclusion.

Duffie [21] provides a theoretical link between on-the-run premia and lower financing costs on the repo market. Vayanos and Weil [55] extend this view and include search frictions explicitly[3]. Intuitively, the repo market provides the required heterogeneity between assets with identical payoffs. An investor with a short position

[3]Kyotaki and Wright [40] introduced search frictions in monetary theory and Shi [50] extends this framework to include bonds. Shi [52] provides a review of this strand of monetary theory. Lagos [41] show how differences of search frictions widen the risk-based premium between two asset classes. Duffie, Gârleanu and Pedersen [22] show how search frictions may rationalize spreads between bid and ask prices proposed by market intermediaries.

cannot choose which bond to deliver to unwind its corresponding repo position; she must rather find and deliver the same security she has originally borrowed. Empirically, Jordan and Jordan [35] confirm the link between lower repo rates and liquidity premia of "special"[4]bonds. Cherian, Jacquier and Jarrow [13] document the same relationship around auction dates. Buraschi and Menini [10] find a negative link between repo rates and bond prices in the market for German government bonds. Longstaff [44] shows that off-the-run Treasury bonds also carry a liquidity premium relative to other government-issued securities (i.e. Refcorp's bonds).

Ericsson and Renault [24] find that a recently issued corporate bond has a lower credit spread over the corresponding government bond yield. Also, they find a positive correlation between the on-the-run premium and the corporate spread. An increase in the on-the-run premium decreases the valuation of corporate securities. Finally, Krishnamurthy and Vissing-Jorgensen [39] presents evidence of a negative correlation between the corporate spread and the US Debt/GDP ratio. Increasing the quantity of U.S. Treasury securities available for trading decrease their equilibrium prices. They interpret this finding as evidence for the presence of a convenience yield component in U.S Treasury prices and attribute the later to a liquidity and a safety motive.

A wider relationship between the state of aggregate liquidity and asset prices also finds support in the theoretical literature. Bansal and Coleman [7] introduces transaction costs in the market for consumption goods to generate a demand for money. Government-issued bonds can be used to back checkable accounts to complement money. This monetary demand for bonds introduces a premium between short and long term bonds as well as bonds and equity. Holmstrom and Tirole [33], [34] proposes a link between the liquidity demand of financially constrained firms and asset prices.

The theoretical literature offers alternative channels through which the state of liquidity may affect future returns. Acharya and Pedersen [1] proposes a liquidity-adjusted CAPM model where transaction costs are varying. Alternatively, Vayanos [54] takes transactions costs as fixed but introduces the risk of having to liquidate a portfolio. In both cases, liquidity affect prices either through its direct effect on expected cash flows or its indirect effect through risk premia.

Finally, the presence of a liquidity premium has important implications for our interpretation of observed term premia. Deviations from the expectations hypothesis may appear less significant once prices are adjusted for the effect of liquidity. Longstaff [43] studies the U.S. repo rate with maturities up to three months. He cannot reject the expectations hypothesis in this sample, which contrasts with previous results

[4]A bond is called special if it offers lower financing costs on the repo market. On-the-run bonds are generally special but other bonds may become special depending on market conditions.

obtained from T-bills by Fama [26][5]. These deviation from the expectation hypothesis are often presented as stylized facts that must be explained by no-arbitrage models.

The rest of the paper is organized as follows. Section 2 presents the model and its state-space representation. Section 3 describes the data and section 4 introduces the estimation method based on the UKF. We report estimation results for the benchmark model in section 5 together with results from predictability regression. Estimation results from the model with a Liquidity factor are reported in in section 6. We use predictive regressions to evaluate the empirical success of the model and highlight the predictive power of the liquidity factor. We discuss our interpretation of the results in section 7 and conclude in Section 8.

2 A model with liquidity

Accounting for differences in coupon rates or maturity requires a term structure model. We use the Extended Nelson-Siegel [ENS] model proposed in Diebold and Li [18] and Diebold et al. [19]. This approach is more parsimonious and imposes a smoother shape to the discount rate curve than the bootstrap method. The Nelson-Siegel representation combines robustness to over-fitting with a good empirical fit when estimating yield curves[6].

The ENS model delivers performance in line with, or better, than other methods when pricing out-of-sample bonds from the same cross-section. Its forecasting performance is also superior to most other benchmark models of the term structure, especially at longer horizon (Diebold and Li [18]). Note that imposing a smooth shape to the curve is also a way to better identify idiosyncracies among prices. This has long been used by practitioners as a way to identify dear or cheap securities. In our context, this will help us identify the liquidity effect.

The ENS model is based on the following representation of the zero-coupon yield curve:

$$y_t^{(m)} = F_{1,t}b_{1,t}(m) + F_{2,t}b_{2,t}(m) + F_{3,t}b_{3,t}(m), \tag{1}$$

[5]T-Bills yields and repo rates carry similar inflation and credit risks but T-bills can be used as collateral, while repurchase agreement cannot. T-bills may also offer lower transactions costs.

[6]See Bliss ([9] and Anderson et al. [4] for a discussion and evaluation of different methods for estimating yield curves.

where $m = 1, 2, ...$ is the number of months to maturity and $F_{i,t}$ are latent factors driving the evolution of discount rates. The loadings $b_{i,t}(m)$ are given by

$$
\begin{aligned}
b_{1,t}(m) &= 1, \\
b_{2,t}(m) &= \left(\frac{1 - \exp\left(-\lambda_t m\right)}{\lambda_t m} \right), \\
b_{3,t}(m) &= \left(\frac{1 - \exp\left(-\lambda_t m\right)}{\lambda_t m} - \exp\left(\lambda_t m\right) \right). \qquad (2) \\
& \qquad (3)
\end{aligned}
$$

These loadings are constructed so that factors have the usual interpretation of level, slope and curvature. Following Diebold and Li, we keep λ fixed and drop the time subscript from loadings.

The price $D_t(m)$ of a discount bond with m periods to maturity and the price $P_t(M)$ of a M-period coupon bond follow directly from this representation:

$$
\begin{aligned}
D(F_t, m) &= exp(-mb(m)'F_t) \qquad m = 1, 2, \cdots \\
P_t(M_n) &= \sum_{m=1}^{M_n} D(F_t, m) \times C_t(m, M_n) \qquad n = 1, \cdots, N \qquad (4)
\end{aligned}
$$

where we use vector notations for factors F_t and factor loadings $b(m)$.

We cast our model within a state-space representation. The observed prices are driven by the level, slope and curvature factors F_t and, possibly, by a liquidity factor L_t. Latent factors have the following transition equation:

$$
\begin{aligned}
(F_t - \bar{F}) &= \Phi(F_{t-1} - \bar{F}) + \Sigma\epsilon_t \qquad (5) \\
(L_t - \bar{L}) &= \phi^l(L_{t-1} - \bar{L}) + \sigma^l\epsilon_t^l
\end{aligned}
$$

where ϵ_t and ϵ_t^l are uncorrelated $(K \times 1)$ and scalar Gaussian white noises respectively.

Our pricing model is an extension of equation 2 where we also specify measurement errors and a liquidity premium. The premium is a function both of a systematic liquidity factor L_t and bond-specific data $Z_n, n = 1, \cdots, N$.

$$
P_t = C_t D(F_t) + \zeta(L_t, Z) + \nu_t \qquad (6)
$$

where C_t is the $(N \times M_N)$ payoffs matrix obtained from stacking the N row vector of individual bond payoffs. Shorter payoff vectors are completed with zeros. $D(F_t)$ is the vector $(M_N \times 1)$ vector of discount bond prices. The measurement error ν_t is a $(N \times 1)$ gaussian white noise with covariance matrix Ω uncorrelated with innovations in the state variables. We will specify the $(N \times 1)$ liquidity premium $\zeta(L_t, Z_n)$ later in this section.

In the following we further assume that the Φ matrix is diagonal. Results in Diebold, Rudebusch and Aruoba [19] show the transition matrix is close to diagonal[7]. We also give ν_t a diagonal covariance matrix. This is consistent with its interpretation as measurement error. Moreover, the square root of each of its diagonal element is modeled as an affine function of maturity,

$$\omega_i = \omega_0 + \omega_1 M_n/12,$$

where M_i is the maturity category of bond i. This assumption substantially reduces the dimension of the estimation problem. Interestingly, leaving the diagonal elements of Ω unrestricted does not affect our results.

The following observation may explain the relationship we observe between maturity and measurement errors. In statistical studies of yields, the level factor explain most of the variability. In turn, duration measures sensitivity of bond prices to parallel shifts in the yield curve. Therefore, duration is an important measure of a bond's variability. It is also generally a nonlinear and increasing function of maturity. Nonetheless, a linear approximation is sensible for maturities up to 10 years (see Figure 2). However, bid-ask spreads may also play a role as their width typically increases with maturity.

Recall that we left the liquidity factor unspecified. We construct this liquidity premium from a latent factor common to all bonds. However, its loadings will vary with maturity and age. The liquidity premia is given by

$$\zeta(L_t, Z_n) = L_t \times \beta(M_n) \exp\left(-age(n,t)/\kappa\right) \qquad n = 1, \cdots, N \tag{7}$$

where $age(n,t)$ is the age of bond with maturity M_n at time t. We now explain in detail the role of parameters $\beta(M_n)$ and κ. $\beta(M_n)$ is the scale parameter for the liquidity premium. For a given age, the average effect of the liquidity factor on prices is determined by β. We allow the average effect on prices to vary freely with maturity. Note that we fix $\beta(120) = 1$ to identify the level of the liquidity factor.

[7] Preliminary period-by-period estimation (not reported here) of the observation equation generated filtered factors whose VAR representation had a Φ matrix not statistically different from a diagonal matrix. The assumption of no correlation between elements of F_t and L_t is for simplicity. Note however, that correlations between filtered values are small.

The parameter κ controls the decay of the liquidity premium as a bond grows older. For instance, immediately following its issuance (i.e.: $age = 0$), the loading on the liquidity factor is $\beta(M_i)$. Varying the age of bond, but keeping the liquidity factor fixed, we can explore how the liquidity premium decays with age. Taking $\kappa = 0.5$, the effect on price decreases by half after a little more than 5 months following issuance.

This specification reflects our priors about the relationship between the liquidity effect, the time from issuance and the time to maturity. Nonetheless, the signs of these relationship is left unrestricted and our specification is flexible enough to accommodate constant or zero loadings, as well as a continuum of shapes for the decay of liquidity with age.

3 Data

We use monthly observations on prices of U.S Treasury bills, notes and bonds from the CRSP data set. We exclude callable bonds, flower bonds and other bonds with tax privileges, issues with no outstanding securities as well as bonds and bills with less than 2 months to maturity. We also excluded some suspicious quotes. For instance, quotes with yields to maturity equal to zero and quotes where yields to maturity differed markedly from nearby observations. Finally some observations had either bid or ask prices missing while one security was excluded on the basis of a contradictory issuance date.

We focus on the period from 1986 to 2004. This restricts the sample to the post-Volcker period and avoids potential structural breaks. It also reduces the impact of tax effects on prices. A tax premium, linked to differences in coupon rates, was prevalent in the earlier sample. Interest rates have been rising for most of the seventies until reaching a peak in the early eighties. Investors favored high coupon bonds for their tax treatments, which gave higher prices to recently issued bonds. Consequently, it is difficult to untangle tax and liquidity premia in this period. Green and Ødegaard [30] documents that this effect mostly disappeared when the asymmetric treatment of interest income and capital gains was eliminated in the 1986 tax reform.

From 1962 to October 16, 1996, bid and ask prices in the CRSP data set were supplied by the Federal Reserve Bank of New York[8]. These quotes reflected the information available to the bank. In particular, CRSP's documentation reports that bid prices "...were the most widely quoted price from the range of quotation." Ask prices were determined from typical spreads as perceived by the bank.

[8]See Elton and Green [23] and Piazzesi [47] for discussion of the CRSP data set.

Before 1996 quotes do not indicate potential transaction prices, but an average of prices reported by 5 dealers chosen at random. This may average out the noise caused by differences in dealers's knowledge and trading interest. It may also incorporate the ignorance of the less-informed traders. Starting in 1996, CRSP uses data provided by GovPX Inc. This electronic trading platform emerged following a dramatic increase in transaction volume for U.S Treasuries. Here, the spread reflects the best available bid and ask prices. We follow previous literature and use the mid-point of bid-ask spreads throughout the sample.

The liquidity premia, and other latent factors, are extracted from coupon bond prices using the model in 5 and 6. However, non-linearities in the measurement equation make the estimation computationally expensive. The challenge is compounded by the large number of observed bond prices in the CRSP data set.

Part of our strategy is to reduce the number of observations. At each point in time, we pick pairs of securities around key maturities. For each maturity category, we first pick a security with time to maturity closest to its category's reference. We then pick the on-the-run security in each category, if available. We focus on maturities 3, 6, 9, 12, 18, 24, 36, 48, 60, 84, 120 months.

On-the-run bonds are not directly identified in the CRSP database. Instead, we use time since issuance as a proxy and pick the most recently issued security in each maturity bin even if no on-the-run security is available. Rather than a limitation, this helps identify how the effect of the liquidity premium varies with age because the liquidity factor's loadings vary with age.

Other features of this pairing help us identify the liquidity premium. By construction, securities within each pair have the same credit quality, similar time to maturity and similar coupon rates[9]. Small coupon and maturity differences can be accounted for by the model. Any remaining price differences, associated to the relative ages of bonds and common across maturities will be attributed to the liquidity effect.

We now investigate some features of our sample. There are 228 monthly observations, running from January 1986 to December 2004. At each date, we focus on pairs of securities around 11 key maturities, which makes a total of 22 observations. Our results are robust to changes in the start year.

Table 1a presents means and standard deviations for the age of bonds within in each liquidity-maturity category. At each maturity, the reference security (see first two columns) has typically been circulating for more than a year. In contrast, at maturity of 6, 12 and 24 months, the most recently issued security is only only a

[9]Early in the sample, some older bonds will have high coupon. However, given the relative stability of interest rates since then, most pairs exhibit similar coupon rates.

few days old. This reflects the frequent issuance pattern around these maturities. In other categories, the most recent securities have been circulating for only a few months which reflects either a regular issuance patterns (e.g. 60 and 120 months) or a short distance from another maturity with regular issuance (e.g. 9 and 18 months). The relatively large standard deviation for recent bonds with 84 months to maturity reveals the effect of variations in the issuance patterns around this maturity through time.

Table 2a and Table 2b show means and standard deviations of bond durations. Duration is a weighted average of a security's times to payments, using payments as weights. It is a measure of maturity which account for differences in coupon rates. This is the relevant measure to compare securities with potentially different coupon rates.

As expected, duration is similar across pairs. Moreover, we can see that categories with regular issuance patterns exhibit pairs with even smaller durations differences on average. This was precisely one of our objectives. Difference in duration will not be able to account for differences in prices between bonds in these categories.

Consider now means and standard deviations of coupon rates, presented in Table 3a and 3b. First note that maturities of 12 months and less are not included as T-Bills make a large part of the sample around these maturities, but do not pay any coupon.

The term structure of coupons is generally upward sloping. Old bonds have lower coupon rates, on average, at maturity of 120 months than at maturities from 24 to 84 months. This is a consequence of some bonds being older on average and bearing high coupon rates associated with the Volcker's era. Standard deviations indicate important variations in coupon rates. This also reflects the general decline of interest rates through the sample. Notwithstanding the variation in coupon rates through the sample Table 3a indicates that differences in coupons are typically small. This was another of our objectives.

To summarize, we have constructed a sample which put more emphasis on differences in bond ages. Within each pair, differences in duration and coupon rates have been minimized in order to maximize out ability to capture the effects of an on-the-run premium, if any.

4 Estimation Methodology

The model in equations 5 and 6 can be summarized by

$$(X_t - \bar{X}) = \Phi_X(X_{t-1} - \bar{X}) + \Sigma_X \epsilon_t$$
$$P_t = \Psi(X_t, C_t, Z_t) + \nu_t$$

where $X_t = [F_t^T L_t]^T$ and Ψ is the mapping of cash flows C_t and current states X_t into prices.

Estimation of this system is challenging because the measurement equation is a non-linear. In fact, we do not know the joint density of latent factors and observed bond prices. Various strategies to deal with non-linear state-space systems have been proposed in the filtering literature: the Extended Kalman Filter (EKF), the Particle Filter (PF) and more recently the Unscented Kalman Filter (UKF) which is described below[10]. To set up notation, we state the standard Kalman filter algorithm as applied to our model. We then explain how the unscented approximation helps overcome the challenge posed by non-linearities.

Consider the case where Ψ is linear in X, which implies that state variables and bond prices are jointly Gaussian. In this case, the Kalman filter provides optimal estimates of current state variables given past and current observations of prices. The recursion works off ex-post values from the previous step:

$$\hat{X}_{t+1|t} \equiv E\left[X_{t+1}|\Im_t\right] \tag{8}$$
$$Q_{t+1|t} \equiv E\left[(\hat{X}_{t+1|t} - X_{t+1})(\hat{X}_{t+1|t} - X_{t+1})^T\right] \tag{9}$$

where $\hat{X}_{t+1|t}$ is a time-t forecast of X_{t+1} and $Q_{t+1|t}$ is the associated mean squared errors (MSE). The associated forecast and MSE of the price vector are given by

$$\begin{aligned} \hat{P}_{t+1|t} &\equiv E\left[P_{t+1}|\Im_t\right] \\ &= \Psi(\hat{X}_{t+1|t}, C_{t+1}) \end{aligned} \tag{10}$$
$$\begin{aligned} R_{t+1|t} &\equiv E\left[(\hat{X}_{t+1|t} - X_{t+1})(\hat{X}_{t+1|t} - X_{t+1})^T\right] \\ &= \Psi(\hat{X}_{t+1|t}, C_{t+1})^T \hat{Q}_{t+1|t} \Psi(\hat{X}_{t+1|t}, C_{t+1}) + \Omega \end{aligned} \tag{11}$$

[10]See Leippold and Wu [42] and Bakshi, Carr and Wu [6] for application in finance, Julier et al. [37] and Jullier and Uhlmann [36] for details and Wan and van der Merwe [56] for textbook treatment.

which use the linearity of Ψ. The next step is to use current bond prices to update our estimate of state variables. The filtering updates are

$$\hat{X}_{t+1|t+1} = \hat{X}_{t+1|t} + K_{t+1}(P_{t+1} - P_{t+1|t}) \tag{12}$$
$$\hat{Q}_{t+1|t+1} = \hat{Q}_{t+1|t} + K_{t+1}^T(\hat{R}_{t+1|t})^{-1}K_{t+1} \tag{13}$$

where

$$\begin{aligned} K_{t+1} &\equiv E\left[(\hat{X}_{t+1|t} - X_{t+1})(\hat{P}_{t+1|t} - P_{t+1})^T\right] \\ &= \hat{Q}_{t+1|t}\Psi(\hat{X}_{t+1|t}, C_{t+1}), \end{aligned} \tag{14}$$

which correspond to the covariance between forecast of state variables and bond prices. Finally, the transition equation gives us a time-$t+1$ forecast of X_{t+1} required for the next recursion:

$$\hat{X}_{t+2|t+1} = \Phi_X \hat{X}_{t+1|t+1} \tag{15}$$
$$\hat{Q}_{t+2|t+1} = \Phi_X^T \hat{Q}_{t+1|t+1}\Phi_X + \Sigma_X \Sigma_X^T. \tag{16}$$

We treat $\hat{X}_{1|0}$ as parameter. Then the Kalman recursion outlined above delivers a series $\hat{P}_{t|t-1}$ and $R_{t|t-1}$ for $t = 1, \cdots, T$ and the sample log-likelihood is

$$L(\theta; \Im_{t-1}) = \sum_{t=1}^{T}\left[\log f(\hat{P}_{t+1|t}, R_{t+1|t})\right] \tag{17}$$

where $f(\cdot,\cdot)$ is the multivariate Gaussian density.

However, because $\Psi(X, C)$ is not linear, equations 10 and 11 do not correspond to the conditional mean and covariance of prices. Also, 14 does not correspond to the conditional covariance between state variables and prices. Note, however, that the updating equations 12 and 13 remain linear. These are justified as linear projections, which suggests that we may recover the Kalman recursion provided we obtain approximations for conditional moments.

The Unscented Kalman Filter, like the Particle Filter, produces approximations of conditional moments. The PF uses Monte-Carlo simulations of the relevant distributions to get estimates of moments. In contrast, the UKF uses a deterministic rule to select points in the distributions. This reduces the computational burden but maintain second-order accuracy[11]. The Extended Kalman Filter linearizes all functionals in the state-space system. While this approach maintains a first-order accuracy, it

[11]See apprendix A for details.

does so with no gain in computational cost relative to the UKF. The loss in accuracy not only affects estimation and inference, but may also cause the recursion to be numerically unstable when embedded in the estimation algorithm.

The UKF provides approximated conditional means and covariances required for the Kalman recursion. We can then still use the likelihood given in 4, but in a QML context. The non-linear measurement equation makes the distribution of bond prices non-Gaussian. In this case, we can still recover an asymptotic approximation for the distribution of estimators. We thus have $\hat{\theta} \approx N(\theta_0, \Omega)$ where the covariance matrix is

$$\Omega = E\left[T^{-1}\left(\zeta_H \zeta_{OP}^{-1} \zeta_H\right)^{-1}\right] \tag{18}$$

where ζ_H and ζ_{OP} are the alternative representations of the information matrix, in the Gaussian case. Formally, these can be consistently estimated via their sample counterpart. We have

$$\hat{\zeta_H} = -T^{-1}\left[\frac{\partial^2 L(\hat{\theta})}{\partial\theta\partial\theta'}\right] \tag{19}$$

and

$$\hat{\zeta_H} = -T^{-1}\sum_{t=1}^{T}\left[\left(\frac{\partial L(\hat{\theta})}{\partial\theta}\right)\left(\frac{\partial L(\hat{\theta})}{\partial\theta}\right)^T\right] \tag{20}$$

Finally, the specification proposed here implies some restrictions on parameters space. First, ϕ_l and the elements of Φ must lie in $(-1, 1)$. Also, κ and the diagonal elements of Σ must be strictly positive. Finally, we maintain the covariance contour of the state variables inside the parameter space associated with positive interest rates because the filtering algorithm may otherwise fail. This constraint is not binding around the estimated values for the parameters.

5 Benchmark Model

5.1 A Benchmark Model Without Liquidity

We first proceed to estimate a restricted version of our model, excluding the liquidity premium, with two goals in mind. First, we will be able to compare our results, using coupon bonds, to the existing literature, based on bootstrapped zero-coupon bonds. Second, the presence of liquidity premia should be apparent in the residuals from the benchmark model. This may provide a direct justification for the introduction of a liquidity factor.

Diebold et al. [19] estimate a model similar to our benchmark, but using prices of zero-coupon bonds. These prices are not observed but, rather, are constructed using the bootstrap method (see Fama and Bliss [28]). Using coupon bond data, as we do here, eliminates potential approximation error, or the loss of information, due to the preliminary estimation of the discount rate curve. However, our approach introduces approximation errors when dealing with nonlinearities. Pre-processing coupon bond prices to produce a discount rate curve smooths away evidence of the on-the-run premia. Therefore, our focus on liquidity imposes the use of coupon bonds. Interestingly, both the bootstrap model and our model provide a similar picture for the behavior of forward rates and returns on discount bonds.

We focus on pairs of securities around a set of fixed maturities. In contrast, the bootstrap method uses substantially more observable prices at any point in time. This difference is less pronounced than it may appear because previous studies, including Diebold et al. [19], typically restrict their attention to a similar subset of fixed maturities.

Finally, the benchmark model allows us to assess the potential of a liquidity factor. Indeed, in this model only differences in coupons and maturities can affect prices. Then, any remaining systematic differences between on-the-run and off-the-run bonds will be attributed to measurement errors. This provides a diagnostic. The case where on-the-run bonds are typically more expensive than off-the-run bonds corresponds to a case where residuals of on-the-run bonds are higher, on average, than residuals for the other bond in each pair. Thus, in this case, the difference between residuals within a pair will be positive on average.

5.2 Benchmark Results

Figure 3 shows filtered values for the latent factors. Estimation of the benchmark model yields $\hat{\lambda} = 0.0627$ (with std. dev.: 0.0020) which pins the maximum loading for the curvature factor at a maturity of 28.6 months. Diebold et al. [19] obtains a similar value for λ, which pins the maximum at 23.3 months. Estimates for the transition equation are given in Table 4a. These are similar to results obtained by Diebold et al. [19]

The estimates imply average short and long term discount rates of 2.8% and 4.7% respectively, which are reasonable values for this period. Looking at the dynamics, we see that the first factor is very persistent. This reflects the steady decline in the level of interest rates since the early 1980s. The second factor is slightly less persistent. Its usual interpretation as the *negative* of the slope is justified given its association with business cycle. It enters positive territory only before the recessions of 1990 and 2001. The curvature factor is closely related to the slope factor.

Standard deviations of measurement errors are given by

$$\sigma(M_i) = 0.0189 + 0.031112 \times M_i/12,$$

with standard deviation for the parameters of 0.0022 and 0.0019 respectively. This results in standard deviations of 0.05 and 0.33 dollars for the measurement errors on bonds with 1 and 10 years to maturity respectively.

Table 5a gives more information on the fit of the benchmark model. We use parameters estimated from the entire sample. Mean Absolute Pricing Errors (MAPE) and Root Mean Squared Pricing Errors (RMSPE) indicate that the dispersion of pricing errors increases with maturity. As discussed above, this may reflect higher sensitivity of longer maturity bonds to interest rates. It may due to higher uncertainty regarding the true price, as signaled by wider bid-ask spreads.

More importantly, Mean Pricing Errors (MPE) are systematically higher for on-the-run securities than for off-the-run securities. This is in line with our expectations. A systematic liquidity premium implies that observed prices are higher for on-the-run issues than for seasoned issues, even after adjusting for differences in coupon rates and maturities. In a model where we omit the liquidity factor, we should observe higher residuals for on-the-run securities.

It is unclear, however, how the liquidity premium affects different bonds as they age to become off-the-run. Panels a of Figure 4 to Figure 9 plot the difference over time between residuals of the recently issued security and residuals of the older security. Panels b of these figures exhibit years from issuance for both securities.

Figure 7a corresponds to the 60 months category. This category has experienced regular issuance over the entire sample. As expected, the difference between residuals is almost always positive. That is, the on-the-run bond appears overpriced compared to the off-the-run bond. At the other end of the spectrum, the category with 18 months to maturity did not experience regular issuance. Figure 5a reports the pattern of residuals in this category. There is no evidence of a role for the liquidity premium at this maturity.

The 84-month category provides an interesting example, as illustrated in Figure 8. At first, the liquidity premium seems positive but collapses towards zero near the middle of the sample. However, the lower panel reveals a lack of new issuance in this later period. This highlights the relationship between time from issuance and the liquidity premium.

Other categories also fit this pattern except, possibly, at 24 month to maturity. Residuals in this category reveal no evidence of a liquidity premium even though there has been regular monthly issuance in this category. Interestingly, Jordan and Jordan [35] could not find evidence of a liquidity or specialness effect on prices at that maturity[12].Moreover, Goldreich, Hanke and Nath [29] also find that the on-the-run premium on 2-year notes goes to zero, on average, within one month from issuance.

The absence of a persistent price premium for on-the-run 2-year notes is intriguing and we can only conjecture as to its causes. The on-the-run notes may not exhibit a liquidity premium if they are expected to offer lower repo rates and lower transaction cost only for a short time period. This may be the case if other bonds with similar maturity becomes easily available for trade on the market. Then, perhaps older securities undergo a change of habitat as their maturity crosses the key two years marks. This could be the case if holders of long-term bonds re-allocate funds from their holding of now-short maturity bonds into longer term securities. If this two-year mark serves as a focus point for buyers and sellers, this may cause a large supply of older securities with this key maturity. In turn, these become cheap substitutes to the on-the-run notes, while offering relatively low transaction costs.

Table 6 summarizes the observations above. It lists mean differences between residuals at each maturity. This also give an appreciation of the results for the model with liquidity. We can see what maturities are, on average, most affected by a liquidity premium.

The premium for liquidity increases with maturity. On average, the price of a 12-month T-Bills was 0.09$ than an older, but otherwise similar, security. With the one year interest rate level at five percent, this translates in a yield difference around 10 basis points between on-the-run and off-the-run securities. Similarly, the price difference at the 60 months maturity is 0.28$. Assuming five-year interest rate at five percent, this translates in a yield difference of 3.25 basis points on average. This is consistent with the results of Amihud and Mendelson [3] who found that the price premium increases with maturity but that the yield premium decreases with maturity[13].

Taken together, this evidence supports a role for a systematic liquidity factor for pricing U.S. Treasury security. Moreover, the evidence suggests the liquidity premium increases with time to maturities but decreases with time from issuance.

[12]See Jordan and Jordan [35] p. 2061: "With the exception of the 2-year notes [...], the average price differences in Table II are noticeably larger when the issued examined is on special."

[13]The price premium at 84 months appears to be lower than at 60 monts, but note that the average combines information about the average magnitude of the liquidity premium and the number of occurrences of a premium. A large liquidity effect combined with infrequent issuance will lead to a relatively small average price difference.

5.3 Predictability Results

We now establish, within our model, stylized facts related to bond returns predictability[14]. The ability to capture patterns of predictability observed in the data is an important measure of success for term structure models[15].

In the following, we choose Cochrane and Piazzesi [11] (CP) as reference. We show that our model reproduces their evidence of predictability and, hence, is a valuable representation of the data. This results is robust wether we use returns and forward rates computed from our model or computed from the FF data set. Moreover, this conclusion still hold if use the regression results of Campbell and Shiller [12].

CP compute annual excess returns on discount bonds with two to five years remaining until maturity. As predictor, they use annual forward rates up to five years ahead. They find that the same linear combination of forward rates forecasts excess returns with R^2 around 35%.

We compute excess returns and forward rates directly from the model and repeat their exercise. However, forward rates are nearly collinear. This was also noted by Bansal, Tauchen and Zhou [8] in a different framework. Of course, forward rates are highly correlated in the data, and this property is amplified by the smooth shape imposed by the Nelson-Siegel curve. But this high linear dependance also means that not much is lost by discarding some of the forward rates. Keeping with standards established in the literature, we use annual forward rates with horizon of at one, three and five years.

Table 7a presents results from predictive regressions using data computed from our model. As in CP, we recover a tent-shaped factor structure for the coefficients on forward rates. We discuss this in more details belwo. Repeating the exercise using returns computed from the Fama-Bliss data set yields similar results. This is a convenient way to highlight differences between both set of excess returns, at least in terms of predictability. Table 7b presents the results. Perhaps surprisingly, coefficients and explanatory power are almost the same wether we use returns from FF data or from the model. Overall, the model seems to capture the same features of excess returns than the FF procedure. Figure 10 illustrates the similarities between excess returns in these data sets.

[14]See Fama [25], [26], and[27], as well as Startz [53] for maturities below 1 year; Campbell and Schiller [12], Fama and Bliss [28] and Schiller [48] for multi-year maturities. See also Cochrane andPiazzesi [11].

[15]Fama [27] originally identified this modeling challenge. Backus et al. [5] and Dai and Singleton [16] evaluate the performance of models in the affine class. Bansal, Tauchen and Zhou [8] consider models with regime shifts.

But CP reported higher R^2 for similar, unrestricted regressions. Why would the information content of forward rates be much here? This could be a consequence of our estimation strategy or, alternatively, the relationship between forward rates and risk premia might have changed in our shorter sample.

In Table 7c, we report results of regressions using both returns and forward rates from FF data, exactly as in CP[16], but for our shorter sample. This establishes the reduced predictability of forward rates for excess returns in our shorter sample. This lower predictability does not, however, invalidate the main results of CP. The predictive power of forward rates is substantial, with R^2 varying between 11% and 15%. We also recover a tent-shaped factor structure for coefficients of the forward rates. In fact, coefficient of the forward factor are almost identical to results reported by Cochrane and Piazzesi [11]. Also, we cannot reject a joint restriction on the factor structure[17].

But comparing Table 7a and Table 7c reveals some differences between the two data sets. Using forward rates from the FF data set decreases standard errors and increases explanatory power, especially for excess returns at shorter maturities. This suggests smoothing the yield curve remove some of the information content of forward rates for bond returns. From a similar observation, Dai et al. [17] conjectured that microstructure and liquidity effects may thus be responsible for some of the predictability by forward rates. We add that the effect of smoothing seems to be more important for returns on bonds of lower maturity.

To conclude, our model captures the main features of returns predictability from forward rates. There are some differences, though, between results using FF data or using our model. In particular, the predictive power of forward rates decreases when we switch to model-based forward rates. Still, Figure 11 shows the forward rates series in both data sets. Once again differences are small[18].

[16]Note that we chose to include the same maturity as in previous regression for comparability. Including the full set of forward rates does not affect any of the conclusions.

[17]Estimated coefficients of the forward factor are 0.48, 0.88, 1.23, 1.41 in our sample and 0.47, 0.87, 1.24 and 1.43 in Cochrane and Piazzesi [11]. We also obtain p-value less than 0.005 for each regression for a test of the joint restriction imposed by the factor structure.

[18]This is not trivial. Forward rates are constructed from differentiation discount bonds adjacent on the term structure. However, return computations involve differentiation of the same discount bonds but 12 months apart through time. Somehow, returns are similar in both data set but not forward rates.

6 Term structure model with liquidity

We now introduce the liquidity premium as in equation 6. Compared to the benchmark model, this does not affect parameter estimates substantially. In fact, even the path of filtered latent factors is left almost unchanged. However, the liquidity factor decreases pricing errors substantially and shows a positive correlation with stock market peaks and troughs and a negative correlation with bond excess returns.

Figure 12 displays the filtered values of the level, slope and curvature factors. Clearly, adding a liquidity factor has little effect on their behavior. Estimation yields $\hat{\lambda} = 0.0628$ (with std. dev. 0.0021) which is almost the same value as in the benchmark model. This pins the maximum loading on the curvature factor at a maturity close to 28 months. Results for the transition equation are given in Table 4b. The estimates imply average short and long term discount rates of 2.9% and 4.7% respectively. Overall, parameter estimates are very close to the benchmark model.

The liquidity factor unconditional mean is 0.65, measured in dollars. This implies that a just-issued bond with 10 years to maturity has on average a price 0.65$ higher than a similar off-the-run bond. This corresponds to a yield difference of close to 4 basis points for two bonds with the same coupon and with the on-the-run bond selling at par. Figure 13 shows filtered values of the liquidity factor. A few observations are warranted here.

First, Figure 13 shows how the liquidity factor varied in the sample, but nonetheless remained positive at all dates. We also highlight observation dates corresponding to the 1987 equity market crash, the 1998 Russian default crisis and the peak of the stock bubble in 2000.

Even more surprising is how the liquidity factor rises steadily *before* the stock market peaks of 1987 and 2000. In other words, the relative scarcity of liquid bonds decreases as the stock market approaches its peak. Perhaps investors realize the heightened risk of a sharp decline in equity valuation and seek temporary refuge in bonds. Perhaps this increases the quantity of bonds held with short horizon and available for borrowing[19].

However, flights to liquidity do not affect only the valuation of equity. Investors' substitution from equity to bonds should be associated with higher bond prices today,

[19] A simple regression analysis of stock returns and the liquidity factor fail to uncover a systematic relationship in our sample. This may imply that the link between liquidity and the stock market is not linear, or that other variables are missing from the model. In particular, anticipations of monetary policy affect both stock valuation and the bond market. This inquiry is beyond the scope of this paper.

and lower returns in the future. Of course, an increase in the on-the-run premia correspond by definition to an increase of on-the-run bond prices. However, the next section shows that prices of *seasoned* bonds increase at the same time.

Consider now the properties of the liquidity term. Table 8 gives estimated values for β. The average liquidity premium increase with maturity, as expected. Nonetheless, the scale parameter has a value close to zero in categories with 18 and 24 months to maturity. This is consistent with observations made in a previous section where we found no evidence of a liquidity effect at these maturities.

Also, we get $\hat{\kappa} = 0.61$ which implies a reduction by half of the liquidity premium after a little more than 5 months. Recall that this estimate for the speed of decay is an average over the sample period, and across all maturities.

Standard deviations of measurement errors follow

$$\sigma(M_i) = 0.0313 + 0.0270 \times M_i/12$$

which is similar to the benchmark case. The standard deviation for these parameters are 0.0037 and 0.0021 respectively. Adding a liquidity factor improves the fit: RMSE and MAPE improves for almost all maturities (see Table refnewtbl:ENS:mpe:Model5).

From Table 6, we see that the model eliminates most of the systematic differences between on-the-run and off-the-run bonds. Also, Figures 14 to 19 repeat the analysis for differences in residuals within each category. There again, the model is doing a nice job removing any systematic differences in residuals. The liquidity factor captures important features of the on-the-run premia present in the data.

6.1 Predictability results with liquidity

Our model shows the existence of a common liquidity factor affecting prices of on-the-run bonds. It also throws lights on the cross-section and time series behavior of on-the-run bonds. But what about a relationship between this measure of liquidity and the valuation of other governments bonds? We show that all bonds are affected by shifts in liquidity demand, as measured by on-the-run premia.

We thus ask what is the relationship between on-the-run premia and prices of seasoned bonds. One approach is to measure the predictive power of the liquidity factor for returns on seasoned bonds. If a liquidity component is nonetheless present in current valuation of seasoned bonds, and if it is correlated with the on-the-run premium, our liquidity factor should predict excess returns of seasoned bonds. Note that our model delivers prices, and thus returns, of seasoned bonds if we restrict the liquidity component to zero. This correspond to a bond issued sufficiently far in the past.

Clearly, on-the-run premia do not affect off-the-run prices directly: liquidity loadings decreases sharply for older bonds. But the on-the-run factor may itself be driven by an underlying variation in demand for liquidity in the economy. Although less liquid than recently issued securities, seasoned bonds are also readily converted into cash via the repo market. As demand for liquidity increases, the prices of both types of bonds may be affected with the on-the-run premium exactly compensating for their differences in liquidity. According to the flight-to-liquidity hypothesis, coefficients on the liquidity factor should be negative.

Tables 9a reports results from predictability regressions using excess returns, forward rates and the liquidity factor computed from the model. The evidence supporting predictability from the liquidity factor is solid. Coefficients are statistically significant at the one-percent level and R^2s increase substantially, even doubling for the shorter maturities regressions.

Coefficients for the liquidity factors are negative. Indeed, this negative relationship is apparent in Figure 1 where we draw excess returns along with the liquidity factor. We also highlight the stock market crash of 1987 and the financial crisis of 1998. One key period where the relationship seems to be absent is the "surprise" monetary tightening of 1994.

Also, the effect on returns increases with maturity. The current state of liquidity has a bigger effect on prices of off-the-run bonds with longer maturity. This parallels the positive link between the on-the-run price premium and maturity and it is consistent with results by Amihud and Mendelson [3].

Coefficients on forward rates decrease in absolute value compared to regressions where we omit liquidity but the differences are not significant. Moreover, we still observe a factor structure and cannot reject the null of no predictability from forward rates. Finally, as a robustness check, we repeat the regression but with returns and forward rates from FF data. Table 9 presents the results. Again the predictive power of the liquidity factor is substantial, with R^2 increasing from the 10-15% range to 16-23%. Point estimates are negative, significant and follow the same pattern as we increase maturity. Note that the predictive power still increases when we move from the model-based data set to the FF data set. This shows that liquidity, or microstructure, effects others than the on-the-run premium drives the extra predictability of forward rates in the FF data set.

7 Discussion

Our measure of the state of liquidity captures a systematic effect, common to all on-the-run bonds. Moreover, looking at Figure 13, we see that large scale variations in the liquidity factor are related to large scale movements in the S&P 500 index. In particular, liquidity reaches its peak in period where the stock market reached historical peaks. Finally, predictive regressions show how its effect extends to off-the-run bonds. This support an important role for liquidity demand in asset pricing.

This complex pattern is consistent with popular accounts of a flight-to-liquidity behavior. There, the underlying assumption is that equity and bond market differ qualitatively in terms of their liquidity. Bonds are perceived as a safe haven and a vehicle of liquidity: they can readily be exchanged for cash at low cost. Equities, on the other hand, are affected by fear of large and sudden price movement, which may not allow for needed portfolio reallocation.

Thus, in flights-to-liquidity periods, investors move into bonds and away from other asset classes. This favors on-the-run bonds. However, seasoned bonds may also be perceived as close substitutes to cash and it suffices that liquidity demand focuses on these assets so that they become liquid. Then, all bonds see their prices increase. The simultaneous increase of on-the-run premia only requires that the most liquid bonds benefit most from the rising demand. In this scenario, an increase of on-the-run liquidity premia today is associated with higher prices for all bonds. Investors increase their valuation of assets more readily exchangeable for cash. Prices decrease when this transitory demand shift dissipates, which leads to lower observed financial returns.

A flight-to-liquidity interpretation is supported by theory. Building on Duffie [21] and Duffie et al. [22], Vayanos and Weil [55] rationalize liquidity premia of on-the-run premia through search frictions. Lagos [41] extends this argument to multiple assets and shows how search can introduce a premium between assets facing varying frictions. If bonds face lower search costs in a decentralized exchange, the risk premia on stocks will be amplified by a liquidity premium. What is still missing in this literature is a source of endogenous variations of search costs which, in turn, introduces variation in the relative liquidity value of different assets[20].

Thus bonds are valued for their monetary services. Also, the most liquid bonds are the most sensitive to variations in the demand for monetary services. This interpretation relates to a growing literature in monetary theory based on search frictions.

[20]However, Acharya and Pedersen [1] use variation in transaction costs to produce a conditional version of the CAPM where the state of liquidity in the market affects asset prices.

Shi [50] studies an economy with a decentralized market for goods and a centralized bond market. In this economy, search frictions rationalize money holdings. Moreover, an arbitrarily small legal restriction against bonds means that money drives bonds out as medium of exchange.

The empirical literature is also consistent with this interpretation. Longstaff [44] documents substantial price differences between U.S Treasury bonds and Refcorp bonds[21]. Longstaff reports that price differences between off-the-run Treasury bonds and Refcorp bonds can be attributed differences in liquidity. He argues that the discount on Refcorp bond is due to "varying preferences for the liquidity of Treasury bonds, especially in unsettled markets."[22]. Finally Ericsson and Renault [24] document that increases of the thirty-year on-the-run premium in the government bond market are associated with increases of corporate bonds spreads across all credit ratings. This provides further supports for important role for the aggregate liquidity demand in the determination of asset prices.

Alternative interpretation

Other mechanisms than the variation in the demand for monetary services may lie behind the empirical observations made in this paper. Here, we consider the following interpretations: the effect of volatility, variation in hedging demand, and frictions between the bond and the repo market. We argue that none of these channels offer a satisfactory account of the observed link between liquidity premia and excess returns.

First, consider explanations emphasizing the role of volatility. For example, in Vayanos [54] the risk of forced portfolio liquidation increases with volatility, leading investors to favor assets with lower volatilities. Under parameter restrictions considered by Vayanos in its paper, a systematic upward shift of asset volatilities decreases prices of all assets, but with lower severity if transaction costs are small.

There are two problems with this line of argument. First note that a flight to lower volatility is associated with increases in excess returns for all assets, contradicting our results. Furthermore, in this model the on-the-run premium is ultimately driven by differences in transaction costs. However, Amihud and Mendelson [3] and Goldreich

[21] Refcorp is an agency of the U.S. government. Its liabilities have both their principal and coupons explicitly guaranteed by the U.S. Treasury.

[22] He considers observed quotes for stripped zero-coupon Treasuries and Refcorp's bonds. He found that price differences cannot be related to differences in credit quality, bid-ask spreads, taxation or repo rates. Instead, price differences are associated with variation in consumer's confidence, Treasury buyback and fund held in money market fund

et al. [29] conclude that direct transaction costs account for only a small portion of the liquidity premia.

More generally, the same argument goes for explanations based on the direct effect of transaction costs. Also, they leave unanswered the question of what drives the variation in transaction costs in crisis period such as the 1987 crash, the 1998 Russian default or the bursting of the dot-com bubble. On the other hand, differences between asset classes in term of the ease with which they can be exchanged for cash may offer an answer. Consider only what are the search costs in these periods, and ultimately the execution risk, of finding a buyer to reallocate your wealth toward cash?

Consider now explanations based on variations in hedging demand. Models based on search frictions predict that a higher demand for short bonds will drive repo rates lower (i.e.: the short trader will offer better financing terms to attract bonds owners) and, ultimately, drive on-the-run bond prices higher compared to seasoned bonds.

But what lies behind this shift in hedging demand? An increase in the willingness to hedge against unexpected rises in interest rates would rise the risk premia and would be associated with higher excess returns. This leads to a positive relationship between returns on bonds and the liquidity premium.

Alternatively, speculative activities in other markets may also be driving the demand for short bonds. This would leave unchanged the anticipated distribution of interest rates but could affect bond prices through variation in the price of risk. Again, the difficulty here is to obtain the correct sign for the relationship. Presumably, the speculators' decisions to increase their risk exposure in other markets are associated with good states of nature, or with lower risk aversion, leading to a positive relation between liquidity and returns.

Another type of arguments would emphasize the presence of market imperfections. Consider the case where the bond market does not adjust immediately to information revealed in the repo market. Then anticipation of higher rates would increase the short interest rate, decrease repo rates and increase on-the-run premia but with no contemporaneous changes in bond prices.

As the bond market ultimately incorporates this information, prices fall decrease reflecting the anticipation of higher rates. This certainly produces the correct sign for the relationship between liquidity and bond returns. However, it is not clear how such an information lag would persist in these extremely liquid markets. Furthermore, these imperfections cannot account for the presence of the on-the-run premium.

8 Conclusion

We use the Extended Nelson-Siegel term structure model of Diebold et al. [19] and we allow for a latent liquidity factor driving the on-the-run premium. Estimation of the model proceed directly from coupon bond prices, based on recent advances in non-linear filtering. We find that the same liquidity factor drives the on-the-run premia at different maturities. Its effect increases with maturity and decreases with the age of a bond. Most interestingly, the liquidity factor predicts annual excess returns on *both* on-the-run and seasoned bonds.

We interpret our findings as evidence of flight-to-liquidity behavior. Perhaps the state of aggregate liquidity plays an important role in asset pricing. We argue that this interpretation is supported by both theory and recent empirical works. The literature studying variations in yield spreads of different bond types (e.g. corporate, agency) already contains evidence linking variation in spreads with liquidity on the market for U.S. Treasury bonds. The theoretical literature on search-based models also provides support for this interpretation: asset classes with lower search costs offer lower excess returns.

However, the literature still has to identify underlying factors driving the variations in the state of aggregate liquidity. What drives variations in the willingness of investors to hold assets with greater search costs? Or, alternatively, why do they sometime exhibit greater preference for a close substitute to cash? Another important aspect is the impact that the presence of a liquidity factor in bond prices should have on the prices of risk. To assess these effects, we need to set up a no-arbitrage model in the spirit of the recent literature linking bond yields and macroeconomic variables. We leave these questions for future research.

A Unscented Kalman Filter

The UKF is essentially an approximation of a non-linear transformation of probability distribution coupled with the Kalman filter. It has been introduced in the engineering literature by Julier et al. [37] and Jullier and Uhlmann [36]. (See also Wan and van der Merwe [56] for general introduction) and, to our knowledge, was first imported in finance by Leippold and Wu [42].

Given $\hat{X}_{t+1|t}$ a time-t forecast of state variable for period $t+1$, and its associated MSE $\hat{Q}_{t+1|t}$ the unscented filter selects a set of Sigma points in the distribution of $X_{t+1|t}$ such that

$$\begin{aligned} \bar{\mathbf{x}} &= \sum_i w^{(i)} x^{(i)} = \hat{X}_{t+1|t} \\ \mathbf{Q}_x &= \sum_i w^{(i)} (x^{(i)} - \bar{\mathbf{x}})(x^{(i)} - \bar{\mathbf{x}})' = \hat{Q}_{t+1|t} \end{aligned}$$

Jullier et al. [37] proposed the following set of Sigma points:

$$x^{(i)} = \begin{cases} \bar{\mathbf{x}} & i = 0 \\ \bar{\mathbf{x}} + \left(\sqrt{\frac{N_x}{1-w^{(0)}} \sum_x} \right)_{(i)} & i = 1, \ldots, K \\ \bar{\mathbf{x}} - \left(\sqrt{\frac{N_x}{1-w^{(0)}} \sum_x} \right)_{(i-K)} & i = K+1, \ldots, 2K \end{cases}$$

with weights

$$w^{(i)} = \begin{cases} w^{(0)} & i = 0 \\ \frac{1-w^{(0)}}{2K} & i = 1, \ldots, K \\ \frac{1-w^{(0)}}{2K} & i = K+1, \ldots, 2K \end{cases}$$

where $\left(\sqrt{\frac{N_x}{1-w^{(0)}} \sum_x} \right)_{(i)}$ is the ith row or column of the matrix square root.

Following Julier and Uhlmann [36], we can use a Taylor expansion to evaluate the approximation's accuracy. The Taylor expansion for the non-linear transformation $y = g(x)$ about $\hat{x}$ is

$$\begin{aligned} \bar{y} &= E\left[g(\bar{x} + \Delta x)\right] \\ &= g(\bar{x}) + E\left[D_{\Delta x}(g) + \frac{D^2_{\Delta x}(g)}{2!} + \frac{D^3_{\Delta x}(g)}{3!} + \cdots \right] \end{aligned}$$

where the $D^3_{\Delta x}(g)$ operator evaluates the total differential of $g(\cdot)$ when perturbed by Δx, and evaluated at $\bar{x}$. A useful representation of this operator in our context is

$$\frac{D^i_{\Delta x}(g)}{i!} = \frac{1}{i!}\left(\sum_{j=1}^{n} \Delta x_j \frac{\partial}{\partial x_j}\right)_i g(x)\Bigg|_{x=\bar{x}}$$

Different approximation strategies for $\bar{y}$ will differ by either the number of terms used in the expansion or the set of perturbations Δx, and its corresponding distribution. Note that the expansion can be further reduced in our case. All odd-ordered terms sums are zero if the distribution of Δx is symmetric. Moreover, we can re-write the second terms as a function of the covariance matrix P_{xx} of Δx. We get:

$$\bar{y} = g(\bar{x}) + \left(\nabla^T P_{xx} \nabla\right) g(\bar{x}) + E\left[\frac{D^4_{\Delta x}(g)}{4!} + \cdots\right]$$

Linearisation of $g(\cdot)$ will then lead to the approximation $\hat{\bar{y}}_{lin} = g(\bar{x})$, which agree with the expansion up to first-order terms. On the other hand, the unscented approximation will be exact up to third-order term. The σ-points χ have the correct covariance matrix by construction. However, the approximation could be arbitrarily worse than linearisation, depending on the the behavior of higher-order terms. This behavior is driven by differences in moments of the distribution of Δx and χ. In the gaussian case, Julier and Ulhmann [36] show further that same-variable fourth moments agree and that all other moments of χ are lower than the true moments of Δx. Since linearisation assume all these terms are zero, approximation errors are necessarily smaller in the case of the unscented transformation.

Using a similar argument, Julier and Uhlmann [36] show that linearisation and the unscented transformation agree with the Taylor expansion up to the second order term. However, the approximation errors in higher order terms are again smaller for the unscented approximation.

References

[1] Acharya, V. V. and Pedersen, L. H. (2004). "Asset pricing with liquidity risk." NBER, working paper.

[2] Amihud, Y. and Mendelson, H. (1986). "Asset pricing and the bid-ask spread." Journal of Financial Economics 17, 223-249.

[3] Amihud, Y. and Mendelson, H. (1991). "Liquidity, maturity and the yields on U.S. Treasuries securities." The Journal of Finance 46, 1411-1425.

[4] Anderson, N., Breedon, F., Deacon, M., Derry, A., Murphy, G. (1996). "Estimating and interpreting the yield curve." Chichester, England. John Wiley and Sons, Series in financial economics and quantitative analysis.

[5] Backus, D., Foresi, S., Mozumdar, A., Wu, L. (2001). "Predictable changes in yields and forward rates." Journal of Financial Economics 59, 281-311.

[6] Bakshi, G., Carr, P. and Wu, L. (2005). "Stochastic risk premiums, stochastic skewness in currency options, and stochastic discount factors in international economies." Journal of Financial Economics, forthcoming.

[7] Bansal, R., and Coleman, W.J. II (1996) "A monetary explanation of the term premium, equity premium and risk-free rate puzzle." The Journal of Political Economy, 104, 1135-1171.

[8] Bansal, R., Tauchen, G., and Zhou, H. (2004). "Regime shifts, risk premiums in the term structure, and the business cycle." Journal of Business and Economic Statistics 22, 396-409.

[9] Bliss, Robert R. (1997). "Testing Term Structure Estimation Methods" in Vol. 9 of Advances in Futures and Options Research, ed. P. Boyle, G. Pennacchi, and P. Ritchken, JAI Press, Greenwich, Connecticut, 197231.

[10] Buraschi, A. and Menini, D. (2001). "Liquidity risk and specialness." Journal of Financial Economics 64, 243-284.

[11] Cochrane, J.H. and Piazzesi, M. (2005a)."Bond risk premia." American Economic Review 95, 138-60.

[12] Campbell, J.Y. and Shiller, R.J. (1989). "Yield spreads and interest rate movements: A bird's eye view." The Review of Economic Studies 58, 495-514.

[13] Cherian, J. A., Jacquier, E. and Jarrow, R. A. (2004). "A model of the convenience yields in on-the-run treasuries." Review of Derivatives Research 7, 79-97.

[14] Constantinides, G.M. and Ingersoll, J.E. (1984)."Optimal bond trading with personal taxes." Journal of Financial Economics 13, 299-335.

[15] Dai, Q. and Singleton K.J. (2002). "Specification analysis of affine term structure models." The Journal of Finance, 55, 1943-1978.

[16] Dai, Q. and Singleton, K.J. (2002). "Expectation puzzles, time-varying risk premia and affine model of the term structure." Journal of Financial Economics 63, pp415-441.

[17] Dai, Q., Singleton, K.J. and Yang, W. (2004). "Predictability of bond risk premia and affine term structure models." Working Paper.

[18] Diebold, F.X. and Li, C. (2006). "Forecasting the term structure of govenment bond yields." Journal of Econometrics 130, 337-364.

[19] Diebold, F.X., Rudebusch, G.D. and Aruoba, S.B. (2006)."The macroeconomy and the yield curve: A dynamic latent factor approach." Journal of Econometrics 131, 309-338.

[20] Duan J-C. and Simonato, J-G. (1999). "Estimating and testing exponential-affine term structure models by kalman filter." Review of Quantitative Finance and Accounting, 13, 111-135.

[21] Duffie, D. (1996)."Special repo rates." The Journal of Finance 51, 493-526,

[22] Duffie, D., Gârleanu, N. and Pedersen,L.H. (2005)."Over-the-counter markets." Econometrica 73, 1815-1847.

[23] Elton, E.J. and Green, T.C. (1998). "Tax and liquidity effects in pricing governments bonds." The Journal of Finance 53, 1533-1562.

[24] Ericsson, J. and Renault, O. (2006). "Liquidity and credit risk." The Journal of Finance, 61, 2219-2250.

[25] Fama, E. (1976). "Forward rates as predictors of future spot rates" Journal of Financial Econometrics 3, 361-377.

[26] Fama, E. (1984a). "The information in the term structure." Journal of Financial Economics 13, 509-528.

[27] Fama, E. (1984b). "Term premium in bond returns." Journal of Financial Economics 13, 529-546.

[28] Fama, E.F. and Bliss, R.R. (1987)."The information in long-maturity forward rates." American Economic Review 77, 680-92.

[29] Goldreich, D., Hanke, B. and Nath, P. (2005)."The price of future liquidity: Time-varying liquidity in the US treasury market." Review of Finance 9, 1-32.

[30] Green, R.C. and Ødegaard, B.A. (1997)."Are there tax effects in the relative pricing of the U.S. governement bonds?" The Journal of Finance 52, 609-633.

[31] Glosten, L.R. and Milstom, P.R. (1984)."Bid, ask and transaction prices in a specialist market with heterogeneously informed traders." Journal of Financial Economics 14, 71-100.

[32] Hicks, J.R. (1948). "Value and Capital", Oxford, Clarendon Press.

[33] Holmström, B. & Tirole, J. (1998). "Private and public supply of liquidity." Journal of Political Economy, 106, 1-40.

[34] Holmström, B. & Tirole, J. (2001). "LAPM: A liquidity-based asset pricing model." Journal of Finance, 56, 1837-1867.

[35] Jordan, B.D. and Jordan S.D. (1997). "Special repo rates: An empirical analysis." The Journal of Finance, 52, 2051-2072.

[36] Julier, S.J. and Uhlmann, J.K. (1996). "A general method for approximating nonlinear transformations of probability distributions." Working Paper, Robotics Research Group, University of Oxford.

[37] Julier, S.J., Uhlmann, J.K. and Durrant-Whyte, H.F. (1995). "A new approach for filtering nonlinear systems." in The Proceedings of the American Control Conference, Seattle, 1628-1632.

[38] Krishnamurthy, A. (2002). "The bond/old-bond spread." The Journal of Financial Economics, 66, 463-506.

[39] Krishnamurthy, A. and Vissing-Jorgensen, J. (2007). "The demand for treasury debt." Working Paper 12881, NBER.

[40] Kiyotaki, N. and Wright, R. (1989). "On money as a medium of exchange." The Journal of Political Economy, 97, 927-954.

[41] Lagos, R. (2005). "Asset prices and liquidity in an exchange economy." Working Paper, New York University.

[42] Leippold, M. and Wu. L. (2003). "Design and estimation of multi-currency quadratic models." European Finance Review, 7, 47-73.

[43] Longstaff, F. A., (2000). "The term structure of very short-term rates: New evidence for the expectations hypothesis." Journal of Financial Econonomics, 58, 397-415.

[44] Longstaff, F. A., (2004). "The flight-to-liquidity premium in U.S. Treasury bond prices." Journal of Business, 77, 511-526.

[45] McCulloch, J.H. (1975). "The tax-adjusted yield curve.", Journal of Finance 30, 811-830.

[46] O'hara, M. (2003). "Presidential address: Liquidity and price discovery." Journal of Finance, 58, 1335-1354.

[47] Piazzesi, M. (2003). "Affine term structure model." Working Paper.

[48] Schiller, R.J. (1979). "The volatility of long-term interest rates and expectations models of the term structure." Journal of Political Economy 87, 1190-1219.

[49] Schiller, R.J. and McCulloch, J.H. (1987). "The term structure of interest rates." in Handbook of Monetary Economics.

[50] Shi, S. (2005). "Nominal bonds and interest rates." International Economic Review, 46, 579-612.

[51] Shi, S. (2005). "Liquidity, interest rates and output." working paper, University of Toronto.

[52] Shi, S. (2006). "Viewpoint: a microfoundation of monetary economics." Canadian Journal of Economics, 39, 643-688.

[53] Startz, R. (1982). "Do forecast errors of term premia really make a difference between long and short rates?" Journal of Financial Economics 10, 323-329.

[54] Vayanos, D. (2004). "Flight to quality, flight to liquidity, and the pricing of risk." Working Paper 10327, National Bureau of Economic Research.

[55] Vayanos, D. and Weill, P.-O. (2005). "A search-based theory of the on-the-run phenomenon." Working Paper, American Finance Association.

[56] Wan, E.A. and van der Merwe, R. (2001). "The unscented Kalman filter." in Simon Haykin eds., Kalman Filtering and Neural Network, Wiley and Sons Publishing, New York.

[57] Warga, A. (1992). "Bond returns, liquidity, and missing data." The Journal of Financial and Quantitative Analysis 27, 605-617.

	Age/Old	**Stdev/Old**	**Age/New**	**Stdev/New**
3	11.99	9.36	1.64	0.10
6	16.90	6.55	0.12	0.12
9	14.41	6.41	3.46	4.00
12	13.24	6.14	1.77	3.30
18	7.34	6.26	3.50	0.64
24	23.28	13.46	0.34	0.53
36	24.69	10.81	4.42	6.63
48	19.25	9.88	4.14	3.07
60	30.31	20.43	2.55	4.03
72	34.15	9.12	10.79	11.51
120	16.26	19.37	4.33	7.99

(a) Summary Statistics of Age (months) for both Liquidity types Types

	Age/Diff.	**Stdev/Diff**
3	10.34	9.38
6	16.77	6.56
9	10.95	7.04
12	11.47	6.89
18	3.85	6.25
24	22.94	13.41
36	20.27	12.40
48	15.11	11.67
60	27.76	20.77
72	23.36	13.28
120	11.92	18.10

(b) Summary Statistics of Age Differences (months)

Table 1: Panel 1a presents means and standard deviations of age for each reference maturity. New refers to the most recently issued security in each category; Old refers to the maturity with maturity closest to the reference. Panel 1b presents means and standard deviations, for each reference maturity, of age differences between Old and New security. Monthly data from CRSP (1984-2004), age and maturity are in months.

	Duration/Old	Stdev/Old	Duration/New	Stdev/New
3	3.01	0.03	4.37	0.10
6	5.99	0.10	5.89	0.12
9	8.88	0.12	10.02	0.38
12	11.75	0.24	11.94	0.92
18	17.33	0.42	19.35	0.81
24	22.49	0.56	22.61	0.72
36	32.33	1.35	32.55	2.50
48	41.59	2.19	43.45	4.17
60	49.93	2.95	50.87	2.85
72	64.89	4.54	67.06	7.56
120	83.10	7.94	83.96	8.50

(a) Summary Statistics of Duration for both Liquidity Types

	Duration/Diff.	Stdev/Diff
3	-1.36	0.10
6	0.10	0.17
9	-1.14	0.42
12	-0.18	0.88
18	-2.02	0.65
24	-0.12	0.66
36	-0.22	2.40
48	-1.86	2.96
60	-0.94	2.39
72	-2.18	5.01
120	-0.85	5.34

(b) Summary Statistics of Duration Differences

Table 2: Panel 2a presents means and standard deviations of durations for each reference maturity. New refers to the most recently issued security in each category; Old refers to the maturity with maturity closest to the reference. Panel 2b presents means and standard deviations, for each reference maturity, of duration differences between Old and New security. Monthly data from CRSP (1984-2004) and maturity is in months.

	Coupon/Old	**Stdev/Old**	**Coupon/New**	**Stdev/New**
18	7.18	3.05	7.19	3.10
24	7.54	2.71	7.14	3.09
36	7.98	2.79	7.47	2.79
48	7.81	2.95	7.67	2.77
60	8.19	2.63	7.54	2.82
72	8.20	2.37	7.98	2.50
120	7.51	2.13	7.85	2.61

(a) Summary Statistics of Coupon Rates for both Liquidity Types

	Coupon/Diff.	**Stdev/Diff**
18	-0.01	1.02
24	0.40	2.29
36	0.52	2.12
48	0.14	2.00
60	0.64	2.14
72	0.22	2.18
120	-0.34	1.25

(b) Summary Statistics of Coupon Rates Differences

Table 3: Panel 3a presents means and standard deviations of coupon rates for each reference maturity. New refers to the most recently issued security in each category; Old refers to the maturity with maturity closest to the reference. Panel 3b presents means and standard deviations, for each reference maturity, of coupon rate differences between Old and New security. Monthly data from CRSP (1984-2004) and maturity is in months.

	$\bar{F}$	Φ	Σ		
Level	0.0046	0.987	0.23		$(\times 10^{-3})$
	(0.0004)	(0.006)	(0.01)		
Slope	-0.0019	0.980	0.22	0.24	
	(0.0007)	(0.007)	(0.02)	(0.03)	
Curvature	-0.0009	0.937	0.20	0.06	0.52
	(0.0003)	(0.019)	(0.01)	(0.03)	(0.03)

(a) Benchmark Model

	$\bar{F}$	Φ	Σ		
Level	0.0047	0.985	0.23		$(\times 10^{-3})$
	(0.0013)	(0.006)	(0.02)		
Slope	-0.0019	0.982	0.23	0.21	
	(0.0016)	(0.007)	(0.02)	(0.05)	
Curvature	-0.0011	0.930	0.20	0.05	0.54
	(0.0005)	(0.022)	(0.01)	(0.05)	(0.04)
Liquidity	0.649	0.950		0.13	
	(0.114)	(0.012)		(0.02)	

(b) Model with liquidity

Table 4: Dynamic Nelson-Siegel term structure model: estimation results for the transition. Panel 4a: results for the benchmark model. Panel 4b: results for the model with liquidity. Coupon bonds data from CRSP 01/1986-12/2003.

	MPE	MAPE	RMSPE	MPE	MAPE	RMSPE
3	0.005	0.049	0.066	0.019	0.061	0.086
6	-0.011	0.099	0.128	0.017	0.089	0.120
9	-0.035	0.164	0.213	0.032	0.180	0.230
12	-0.039	0.238	0.304	0.052	0.227	0.293
18	-0.055	0.368	0.463	-0.049	0.431	0.537
24	-0.022	0.500	0.620	0.013	0.497	0.616
36	0.012	0.740	0.910	0.089	0.749	0.929
48	-0.015	0.949	1.193	0.094	0.998	1.244
60	-0.021	1.157	1.455	0.262	1.143	1.419
84	-0.237	1.511	1.913	-0.043	1.495	1.886
120	-0.192	1.729	2.231	0.137	1.782	2.270
All	-0.055	0.682	1.119	0.057	0.696	1.127

(a) Benchmark Model

	MPE	MAPE	RMSPE	MPE	MAPE	RMSPE
3	0.010	0.047	0.064	0.004	0.060	0.083
6	0.017	0.097	0.126	0.002	0.087	0.117
9	-0.012	0.158	0.206	0.007	0.177	0.226
12	-0.020	0.234	0.299	0.014	0.225	0.288
18	-0.013	0.365	0.458	-0.004	0.428	0.534
24	-0.003	0.498	0.620	-0.012	0.496	0.616
36	0.036	0.740	0.910	-0.004	0.742	0.925
48	0.017	0.950	1.188	0.015	0.993	1.242
60	0.045	1.153	1.441	0.008	1.120	1.397
84	0.016	1.477	1.868	-0.007	1.464	1.868
120	-0.119	1.712	2.210	-0.002	1.768	2.251
All	-0.002	0.675	1.105	0.002	0.687	1.117

(b) Model with liquidity

Table 5: Mean pricing errors (MPE), absolute pricing errors (MAPE) and squared pricing errors (RMSPE) within each maturity category for off-the-run (first three columns) and on-the-run (last three columns) securities. Dynamic Nelson-Siegel term structure model, Panel 5a: benchmark model and Panel 4b with no liquidity factor. Coupon bonds data from CRSP 01/1986-12/2003.

	Benchmark	**Liquidity**
3	0.014	-0.006
6	0.028	-0.015
9	0.067	0.019
12	0.092	0.034
18	0.006	0.009
24	0.035	-0.009
36	0.077	-0.039
48	0.112	-0.002
60	0.287	-0.037
84	0.193	-0.023
120	0.330	0.116
all	0.113	0.004

Table 6: Mean residual differences within each maturity category for the benchmark model (left column) and the liquidity model (right colum). Dynamic Nelson-Siegel term structure model. Coupon bonds data from CRSP 01/1986-12/2003.

	cst	$f_t^{(1)}$	$f_t^{(3)}$	$f_t^{(5)}$	R^2
$xr_{t+1}^{(2)}$	-0.001	-0.330	0.802	-0.361	0.046
	(1.25)	(0.41)	(0.99)	(0.75)	
$xr_{t+1}^{(3)}$	0.016	-1.095	2.526	-1.280	0.075
	(2.31)	(0.77)	(1.86)	(1.41)	
$xr_{t+1}^{(4)}$	-0.098	-1.962	4.467	-2.332	0.105
	(3.14)	(1.03)	(2.53)	(1.93)	
$xr_{t+1}^{(5)}$	-0.340	-2.829	6.408	-3.385	0.132
	(3.78)	(1.23)	(3.04)	(2.33)	

(a) Excess returns and forward rates from model

	cst	$f_t^{(1)}$	$f_t^{(3)}$	$f_t^{(5)}$	R^2
$xr_{t+1}^{(2)}$	0.066	-0.397	0.991	-0.494	0.048
	(1.30)	(0.43)	(1.03)	(0.77)	
$xr_{t+1}^{(3)}$	0.310	-1.135	2.724	-1.476	0.066
	(2.43)	(0.82)	(1.97)	(1.49)	
$xr_{t+1}^{(4)}$	-0.114	-2.058	4.671	-2.440	0.106
	(3.25)	(1.08)	(2.64)	(2.01)	
$xr_{t+1}^{(5)}$	-0.277	-3.003	6.844	-3.684	0.135
	(3.90)	(1.27)	(3.12)	(2.39)	

(b) Excess returns from FF and forward rates from model

	cst	$f_t^{(1)}$	$f_t^{(3)}$	$f_t^{(5)}$	R^2
$xr_{t+1}^{(2)}$	-0.318	-0.556	1.419	-0.734	0.109
	(1.08)	(0.24)	(0.49)	(0.41)	
$xr_{t+1}^{(3)}$	-0.868	-1.261	2.988	-1.493	0.129
	(1.97)	(0.47)	(0.99)	(0.80)	
$xr_{t+1}^{(4)}$	-2.065	-1.959	4.237	-1.877	0.155
	(2.59)	(0.64)	(1.38)	(1.10)	
$xr_{t+1}^{(5)}$	-3.360	-2.382	4.737	-1.801	0.155
	(3.11)	(0.79)	(1.72)	(1.35)	

(c) Excess returns and forward rates from FF

Table 7: Predictability regression of bond excess returns on forward rates. Newey-West S.E. in parentheses with 18 lags. FF is the Fama-Bliss dicount bond data set from CRSP. Model is a dynamic Nelson-Siegel term structure model with liquidity factor. Coupon bonds data from CRSP 01/1986-12/2003.

	β	**Std. Dev.**
3	0.073	0.006
6	0.066	0.006
9	0.120	0.009
12	0.120	0.009
18	-0.039	0.011
24	0.070	0.009
36	0.283	0.021
48	0.381	0.026
60	0.617	0.033
84	0.696	0.036
120	1.000	0.000

Table 8: Level parameters β of liquidity loadings for maturity 3 to 120 months. Dynamic Nelson-Siegel term structure model with a liquidity factor. Coupon bonds data from CRSP 01/1986-12/2003.

	cst	$f_t^{(1)}$	$f_t^{(3)}$	$f_t^{(5)}$	L_t	R^2
$xr_{t+1}^{(2)}$	0.185	-0.208	0.529	-0.162	-0.608	0.081
	(1.19)	(0.41)	(1.00)	(0.75)	(0.32)	
$xr_{t+1}^{(3)}$	0.432	-0.820	1.916	-0.833	-1.360	0.123
	(2.19)	(0.77)	(1.86)	(1.40)	(0.62)	
$xr_{t+1}^{(4)}$	0.552	-1.532	3.515	-1.634	-2.124	0.163
	(2.96)	(1.03)	(2.50)	(1.90)	(0.89)	
$xr_{t+1}^{(5)}$	0.539	-2.249	5.123	-2.444	-2.867	0.199
	(3.55)	(1.22)	(2.97)	(2.27)	(1.12)	

(a) Excess returns and forward rates from model

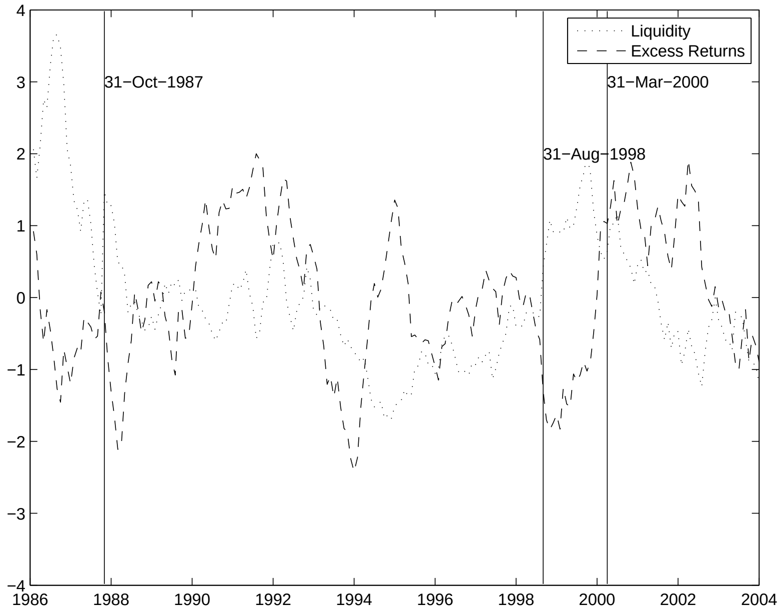
	cst	$f_t^{(1)}$	$f_t^{(3)}$	$f_t^{(5)}$	L_t	R^2
$xr_{t+1}^{(2)}$	0.263	-0.267	0.703	-0.283	-0.643	0.085
	(1.24)	(0.43)	(1.04)	(0.79)	(0.33)	
$xr_{t+1}^{(3)}$	0.701	-0.876	2.151	-1.056	-1.278	0.105
	(2.32)	(0.82)	(1.98)	(1.48)	(0.65)	
$xr_{t+1}^{(4)}$	0.478	-1.668	3.806	-1.806	-1.930	0.150
	(3.10)	(1.08)	(2.62)	(1.99)	(0.91)	
$xr_{t+1}^{(5)}$	0.498	-2.492	5.711	-2.854	-2.528	0.184
	(3.70)	(1.26)	(3.08)	(2.35)	(1.15)	

(b) Excess returns from FF and forward rates from model

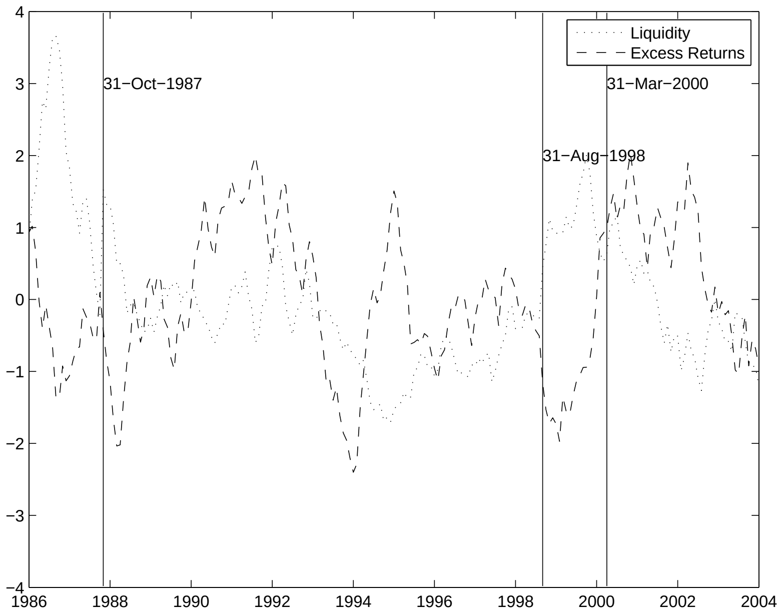
	cst	$f_t^{(1)}$	$f_t^{(3)}$	$f_t^{(5)}$	L_t	R^2
$xr_{t+1}^{(2)}$	0.008	-0.541	1.456	-0.746	-0.737	0.159
	(1.01)	(0.23)	(0.46)	(0.39)	(0.34)	
$xr_{t+1}^{(3)}$	-0.187	-1.231	3.066	-1.517	-1.540	0.187
	(1.85)	(0.46)	(0.93)	(0.76)	(0.67)	
$xr_{t+1}^{(4)}$	-1.010	-1.913	4.357	-1.914	-2.386	0.225
	(2.43)	(0.62)	(1.28)	(1.02)	(0.95)	
$xr_{t+1}^{(5)}$	-1.955	-2.320	4.897	-1.850	-3.178	0.236
	(2.91)	(0.76)	(1.59)	(1.25)	(1.21)	

(c) Excess returns and forward rates from FF

Table 9: Predictability regression of bond excess returns on forward rates. Newey-West S.E. in parentheses with 18 lags. Model is a dynamic Nelson-Siegel term structure model with liquidity factor. FF is the Fama-Bliss dicount bond data from CRSP. Coupon bonds data from CRSP 01/1986-12/2003.



(a) Excess returns from FF



(b) Excess returns from Model

Figure 1: Comparison of excess returns and liquidity factor. Model is a dynamic Nelson-Siegel term structure model with a liquidity factor. FF is the Fama-Bliss discount bond data set from CRSP. Coupon bonds data from CRSP 01/1986-12/2003.

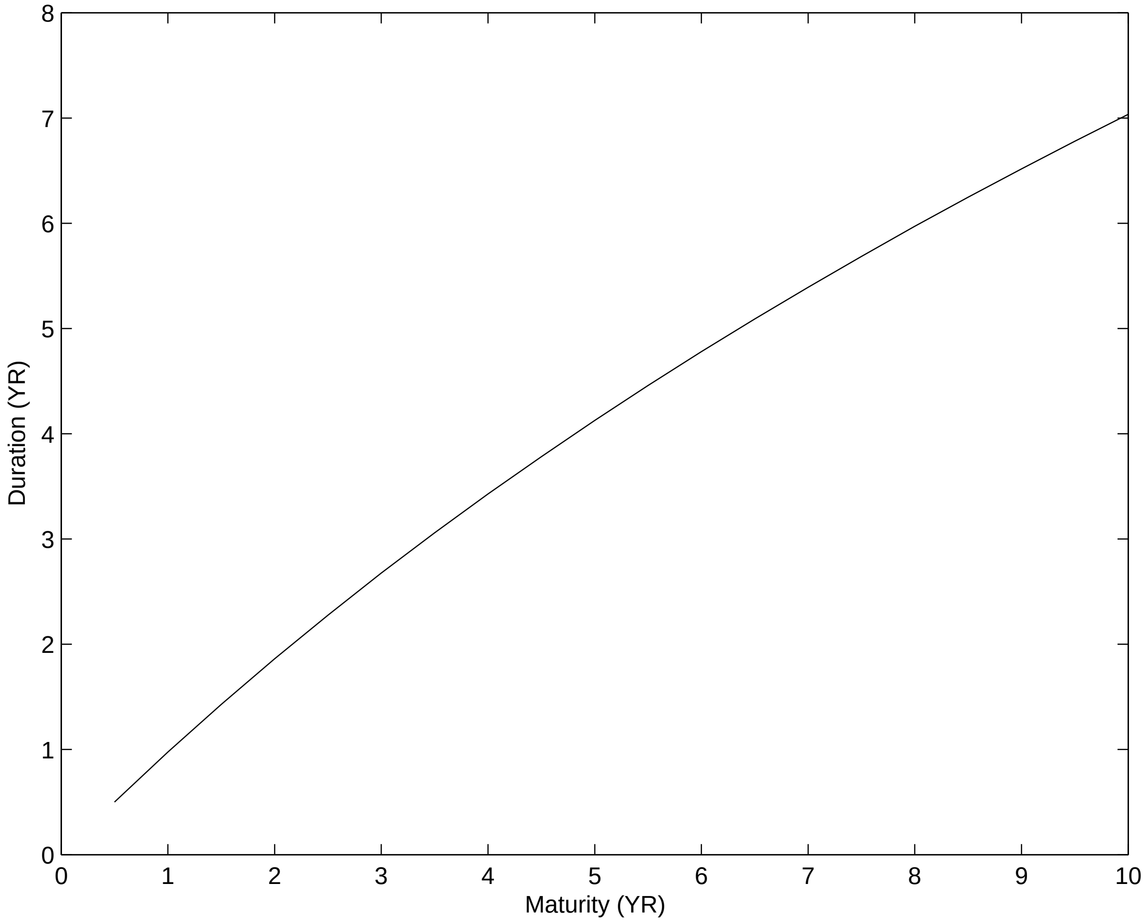


Figure 2: Duration, in years, for coupon bonds of different maturity. Yields to maturity and coupon are 5%.

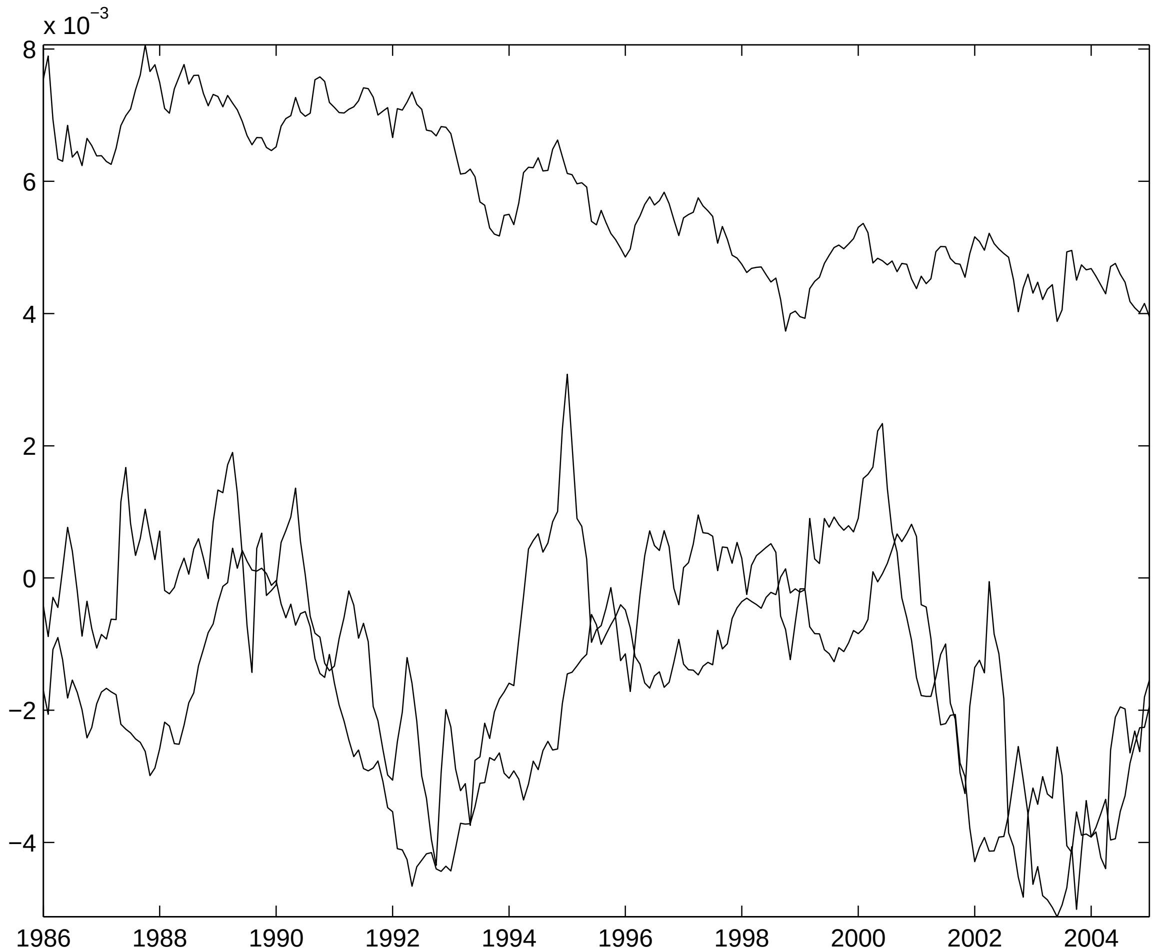
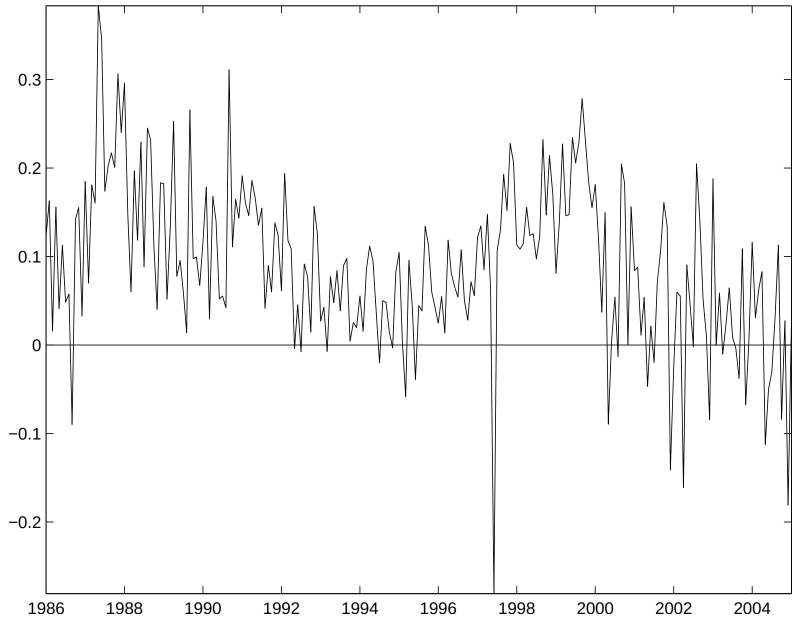
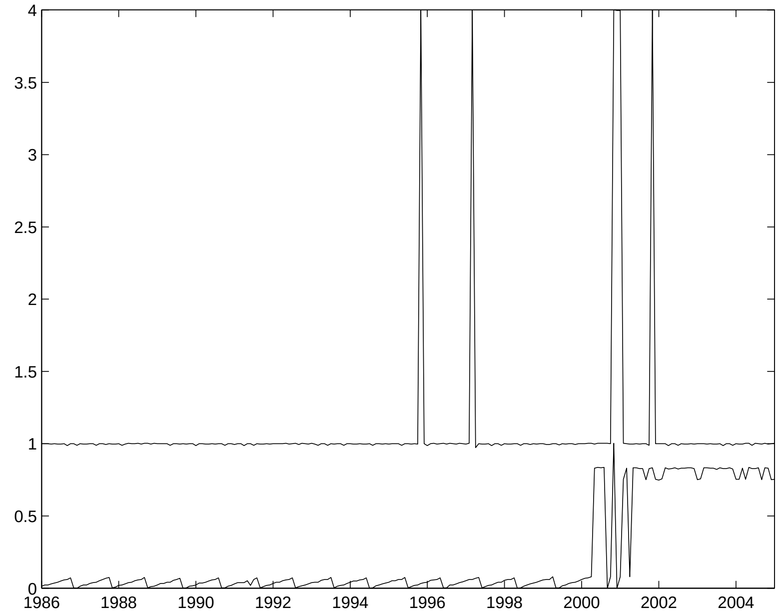


Figure 3: Latent factors from Dynamic Nelson-Siegel term structure model. Coupon bonds data from CRSP 01/1986-12/2003.

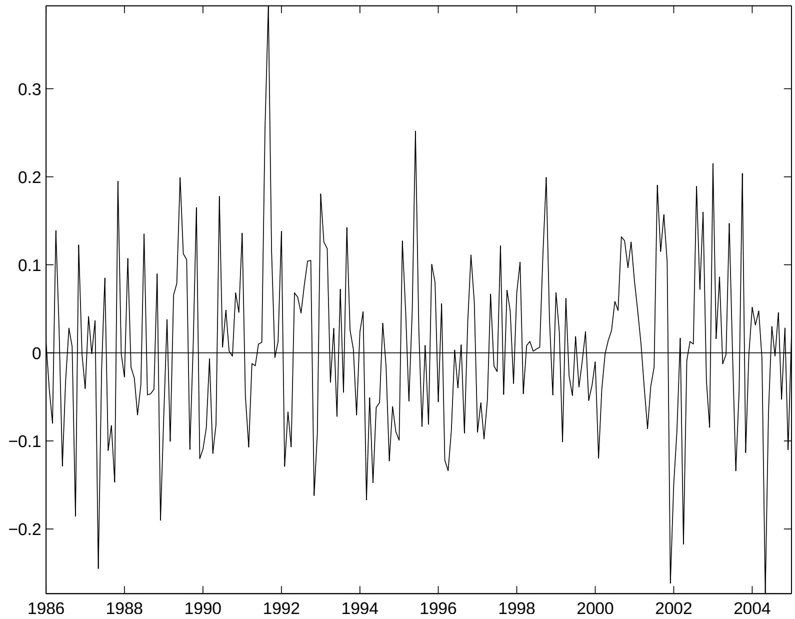


(a) Residual Differences

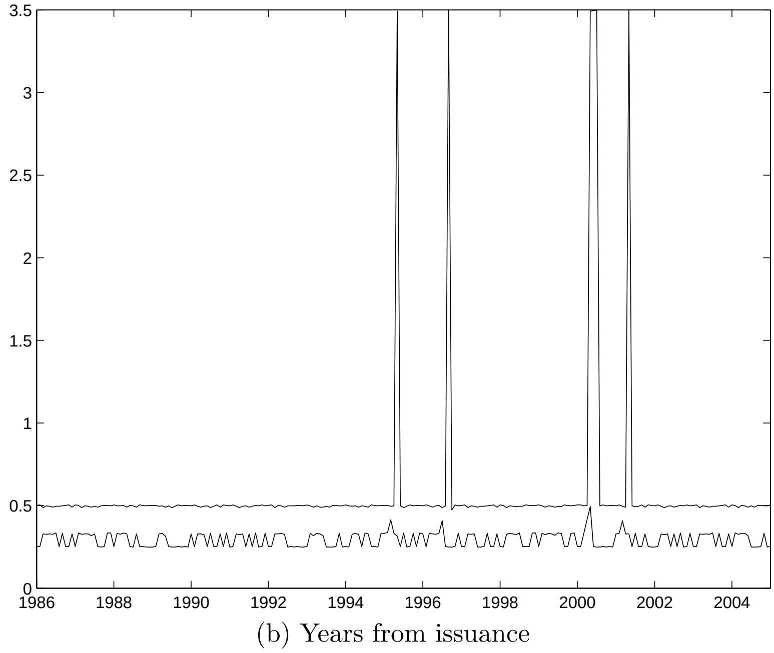


(b) Years from issuance

Figure 4: Years from issuance and dollar differences between residuals at 12 months to maturity. Dynamic Nelson-Siegel term structure model with no liquidity factor. Coupon bonds data from CRSP 01/1986-12/2003.

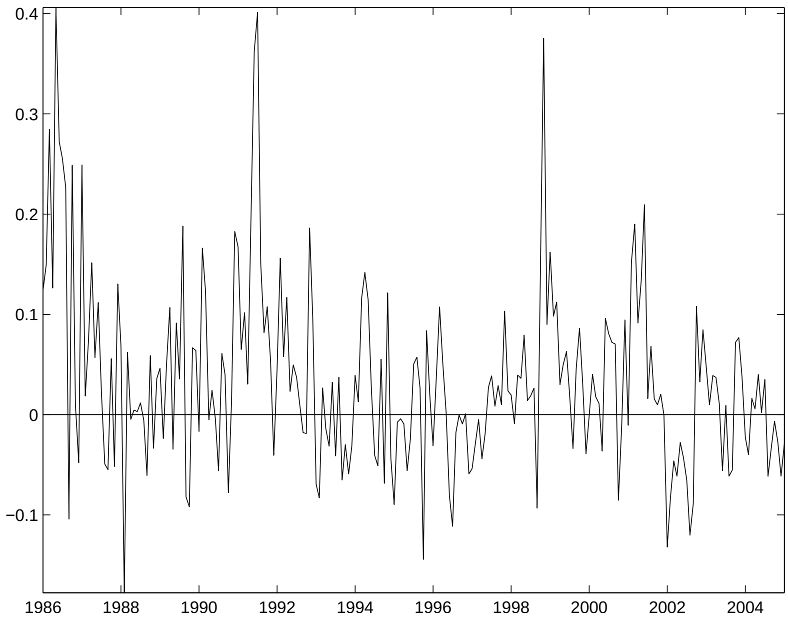


(a) Residual Differences

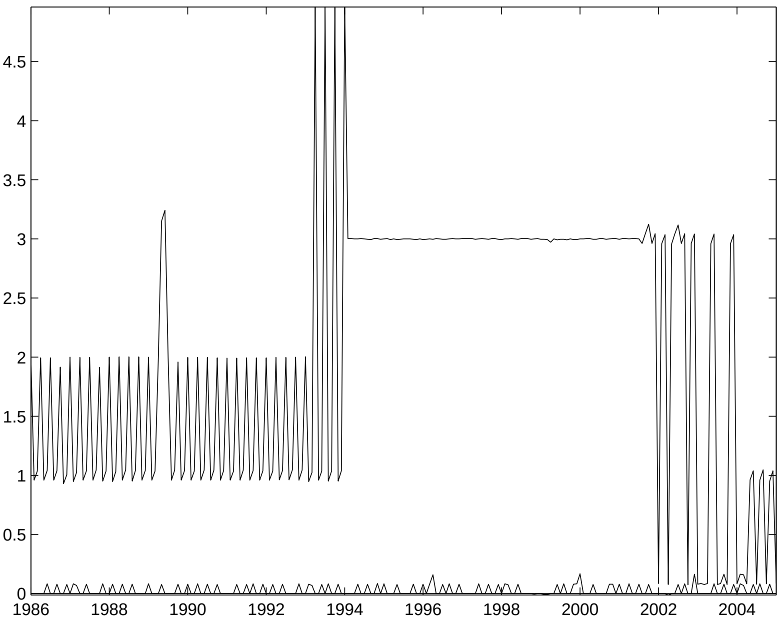


(b) Years from issuance

Figure 5: Years from issuance and dollar differences between residuals at 18 months to maturity. Dynamic Nelson-Siegel term structure model with no liquidity factor. Coupon bonds data from CRSP 01/1986-12/2003.

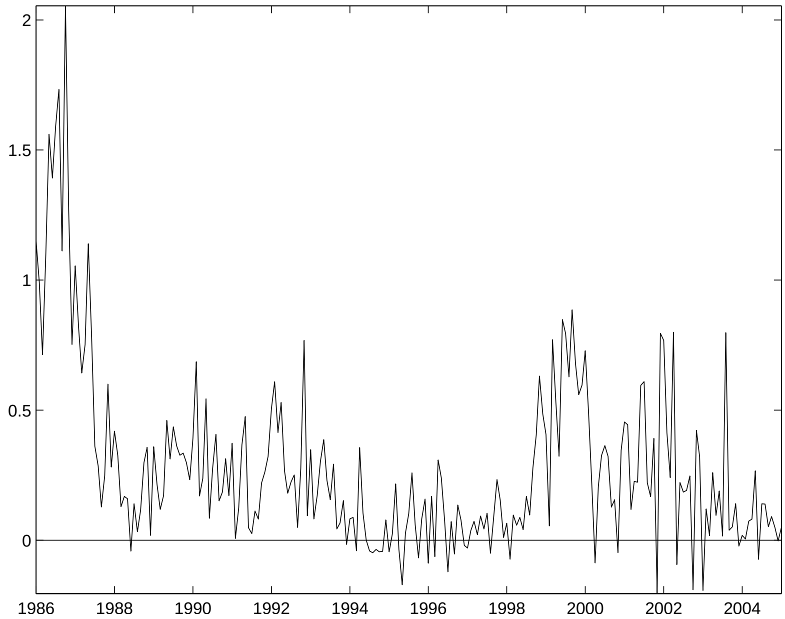


(a) Residual Differences

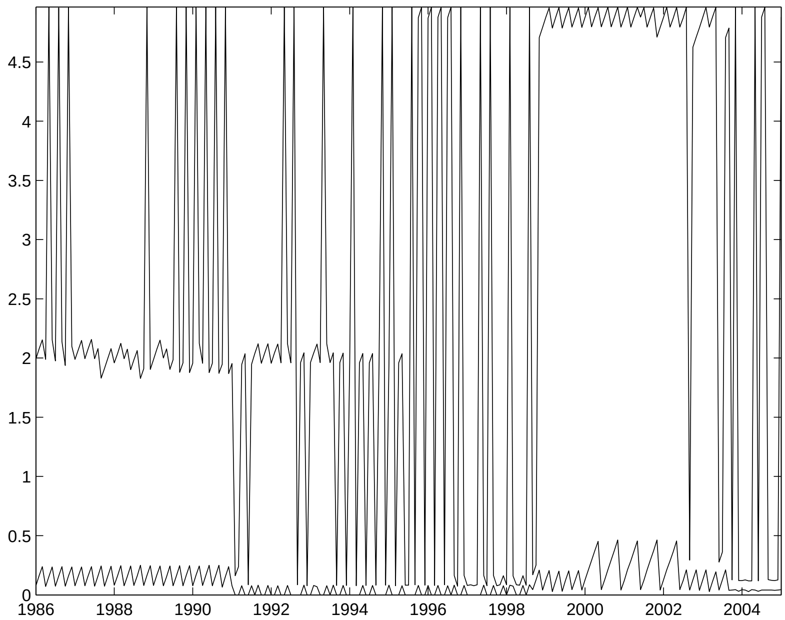


(b) Years from issuance

Figure 6: Years from issuance and dollar differences between residuals at 24 months to maturity. Dynamic Nelson-Siegel term structure model with no liquidity factor. Coupon bonds data from CRSP 01/1986-12/2003.

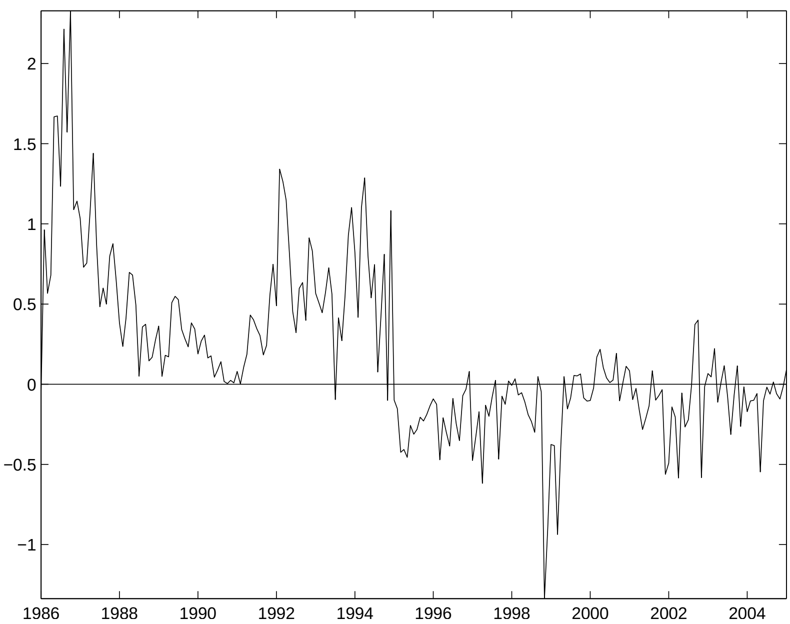


(a) Residual Differences

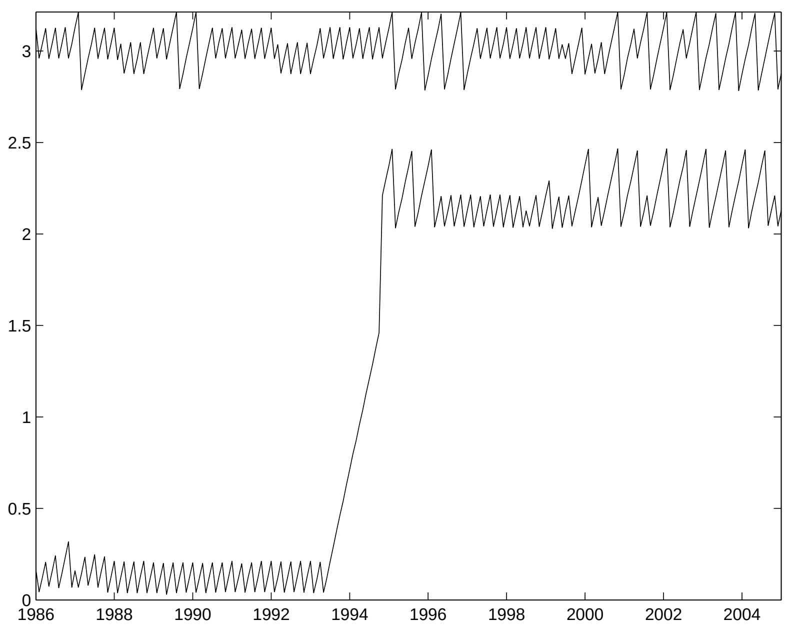


(b) Years from issuance

Figure 7: Years from issuance and dollar differences between residuals at 60 months to maturity. Dynamic Nelson-Siegel term structure model with no liquidity factor. Coupon bonds data from CRSP 01/1986-12/2003.

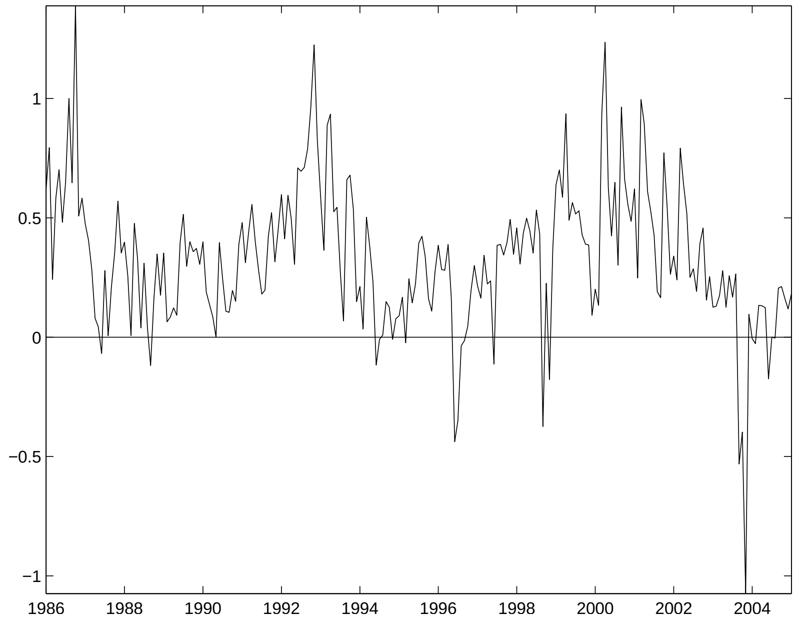


(a) Residual Differences

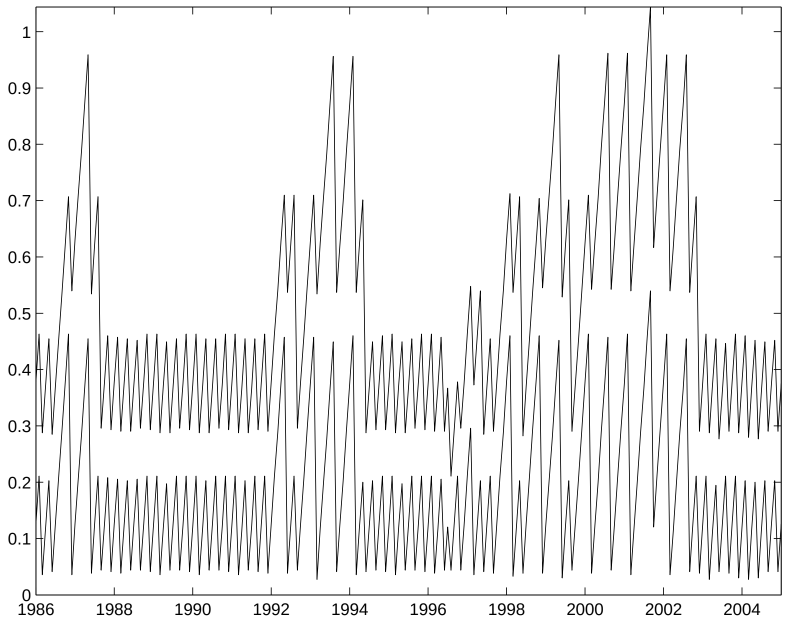


(b) Years from issuance

Figure 8: Years from issuance and dollar differences between residuals at 84 months to maturity. Dynamic Nelson-Siegel term structure model with no liquidity factor. Coupon bonds data from CRSP 01/1986-12/2003.

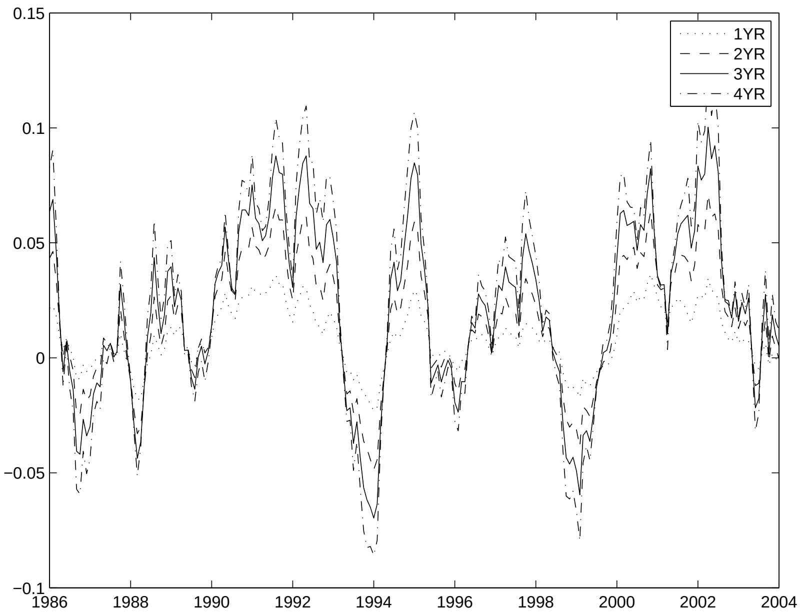


(a) Residual Differences

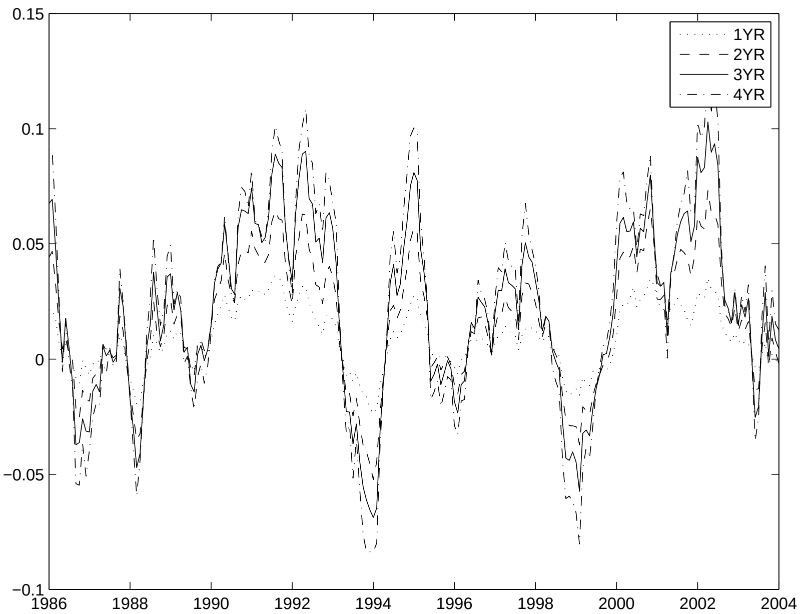


(b) Years from issuance

Figure 9: Years from issuance and dollar differences between residuals at 120 months to maturity. Dynamic Nelson-Siegel term structure model with no liquidity factor. Coupon bonds data from CRSP 01/1986-12/2003.

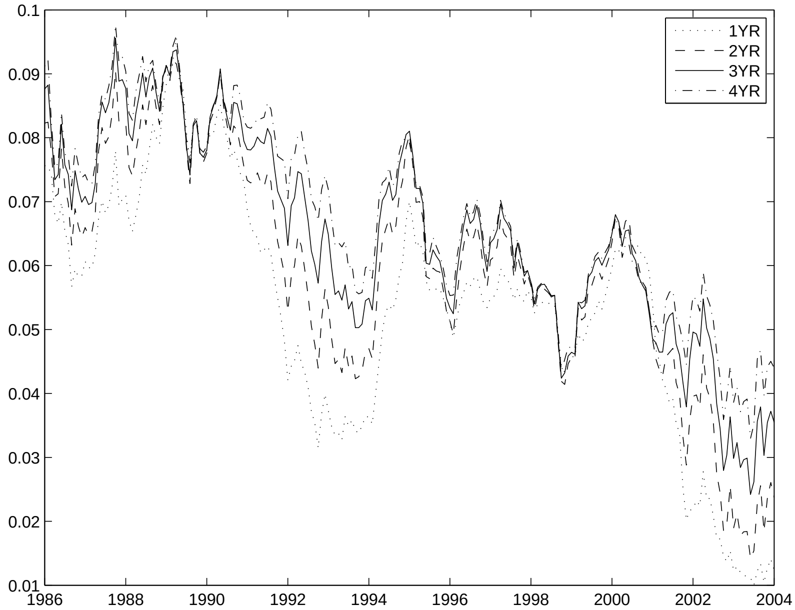


(a) Excess returns from FF

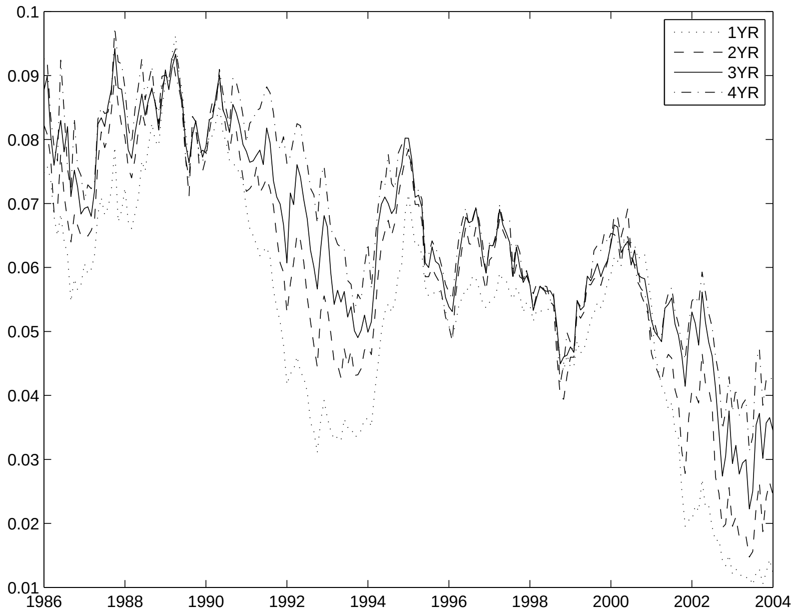


(b) Excess Returns from Model

Figure 10: Comparison of excess returns. Model is a dynamic Nelson-Siegel term structure model with a liquidity factor. FF is the Fama-Bliss discount bond data set from CRSP. Coupon bonds data from CRSP 01/1986-12/2003.



(a) Forward rates from FF



(b) Forward rates from Model

Figure 11: Comparison of forward rates. Model is a dynamic Nelson-Siegel term structure model with a liquidity factor. FF is the Fama-Bliss discount bond data set from CRSP. Coupon bonds data from CRSP 01/1986-12/2003.

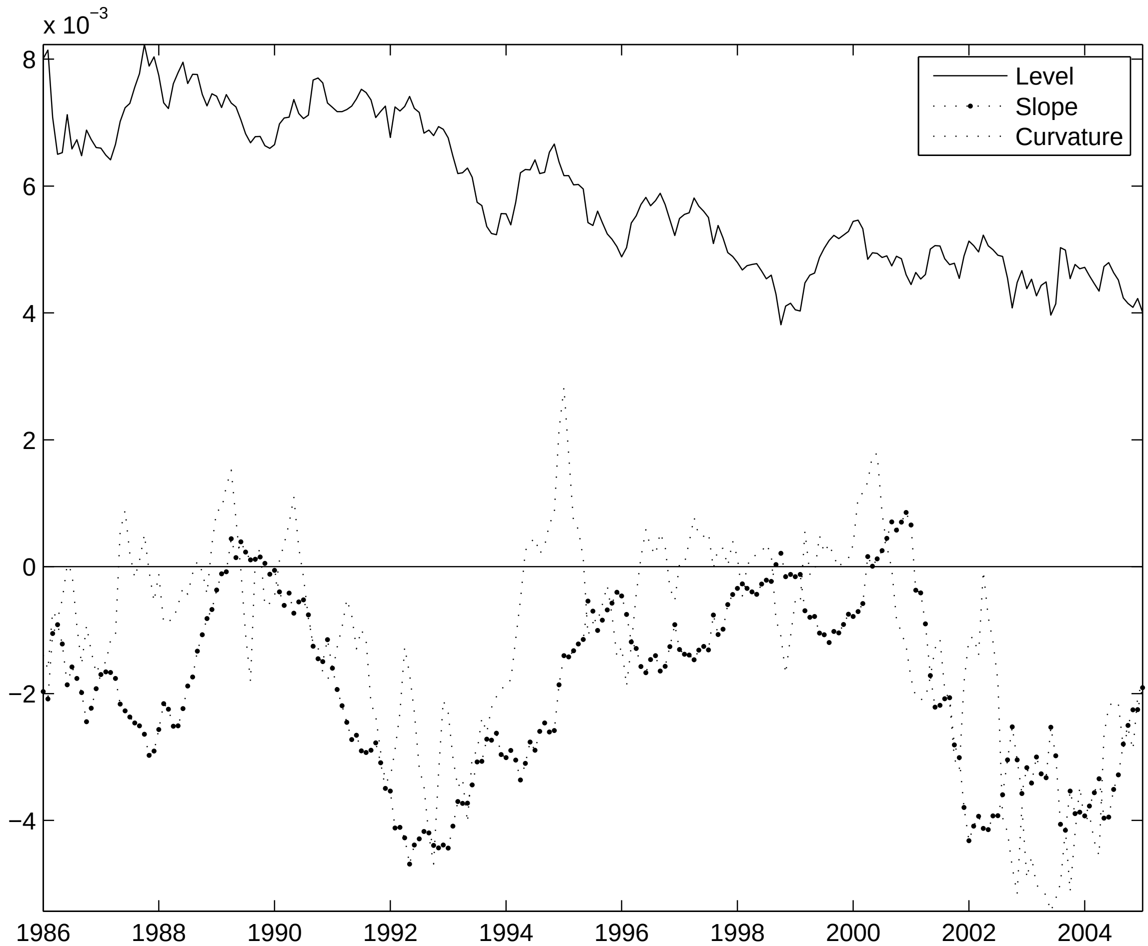


Figure 12: Latent factors from dynamic Nelson-Siegel term structure model with liquidity factor. Coupon bonds data from CRSP 01/1986-12/2003.

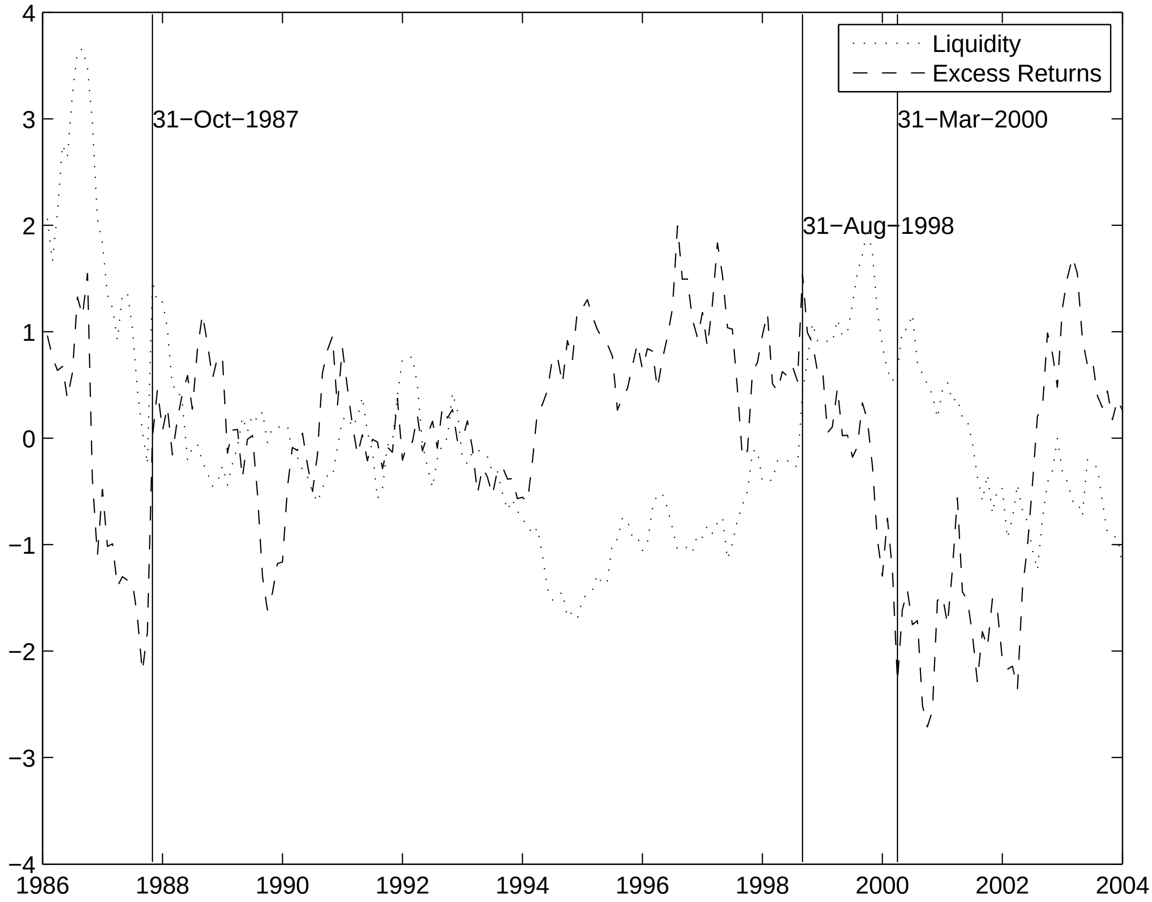
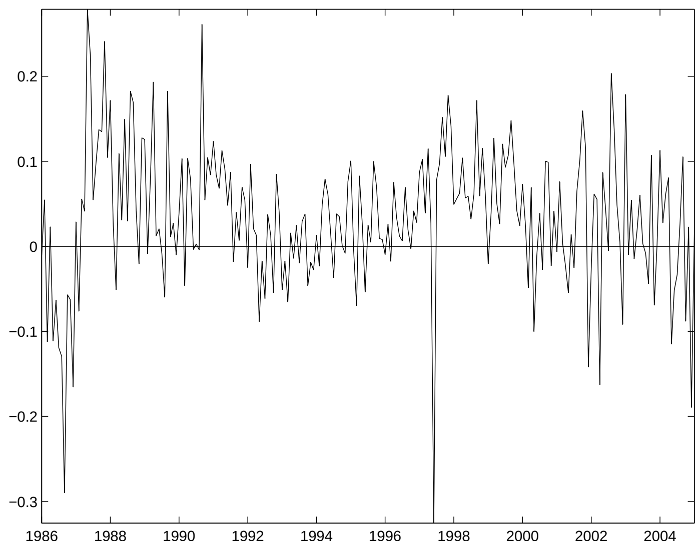
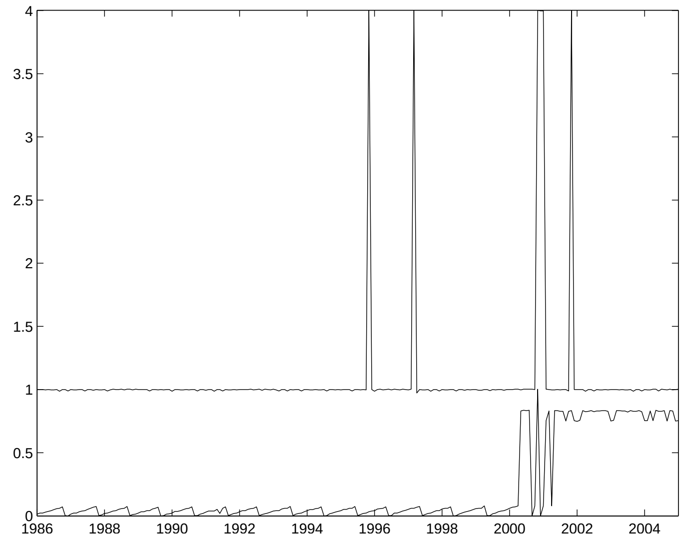


Figure 13: Annual excess returns on SP500 index and liquidity factor from dynamic Nelson-Siegel term structure model. Coupon bonds and SP500 data from CRSP 01/1986-12/2003.

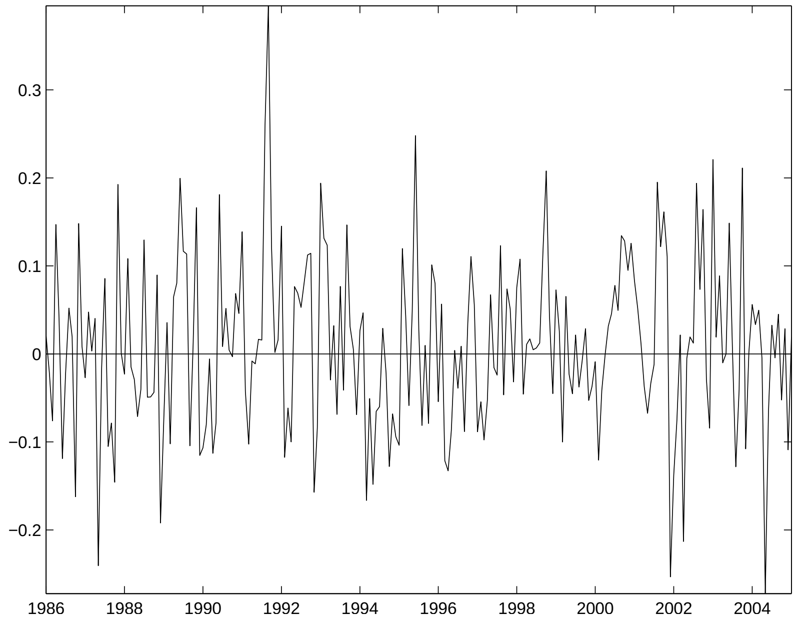


(a) Residual Differences

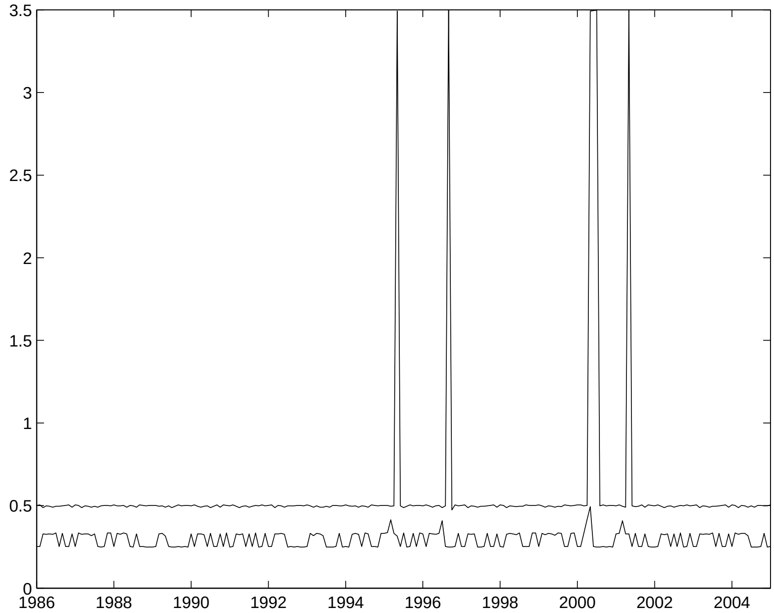


(b) Years from issuance

Figure 14: Years from issuance and dollar differences between residuals at 12 months to maturity. Dynamic Nelson-Siegel term structure model with liquidity factor. Coupon bonds data from CRSP 01/1986-12/2003.

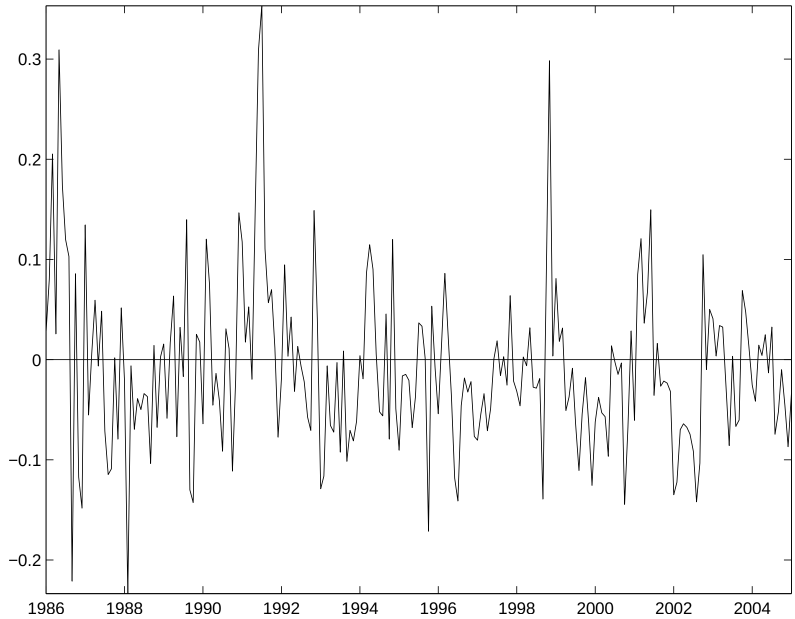


(a) Residual Differences

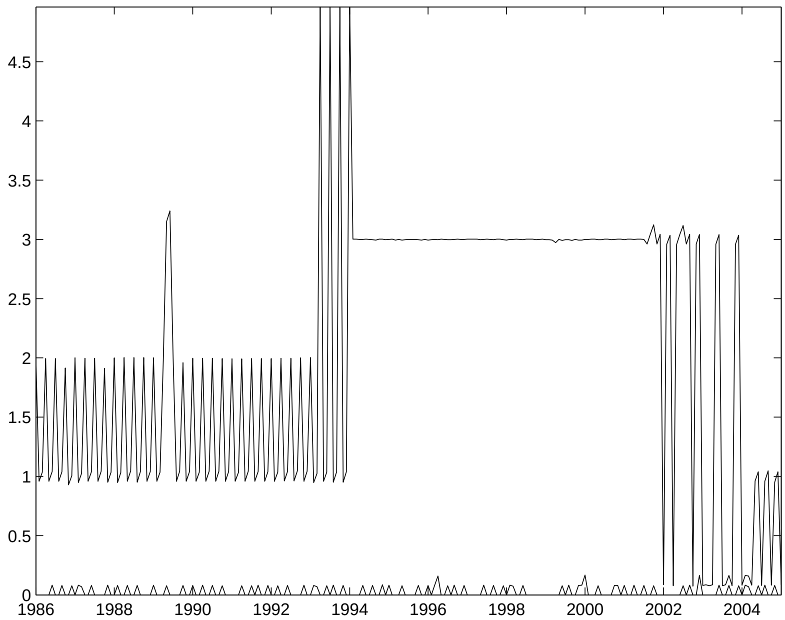


(b) Years from issuance

Figure 15: Years from issuance and dollar differences between residuals at 18 months to maturity. Dynamic Nelson-Siegel term structure model with liquidity factor. Coupon bonds data from CRSP 01/1986-12/2003.

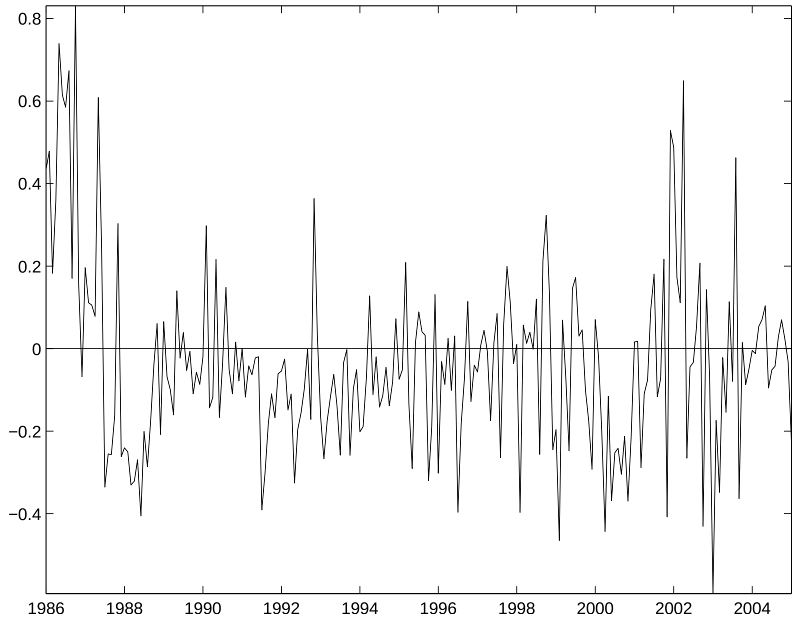


(a) Residual Differences

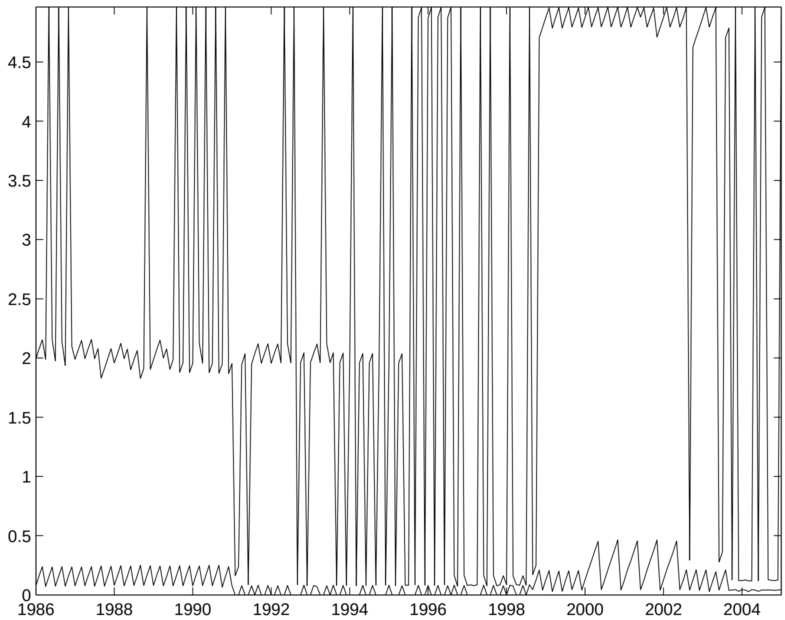


(b) Years from issuance

Figure 16: Years from issuance and dollar differences between residuals at 24 months to maturity. Dynamic Nelson-Siegel term structure model with no liquidity factor. Coupon bonds data from CRSP 01/1986-12/2003.

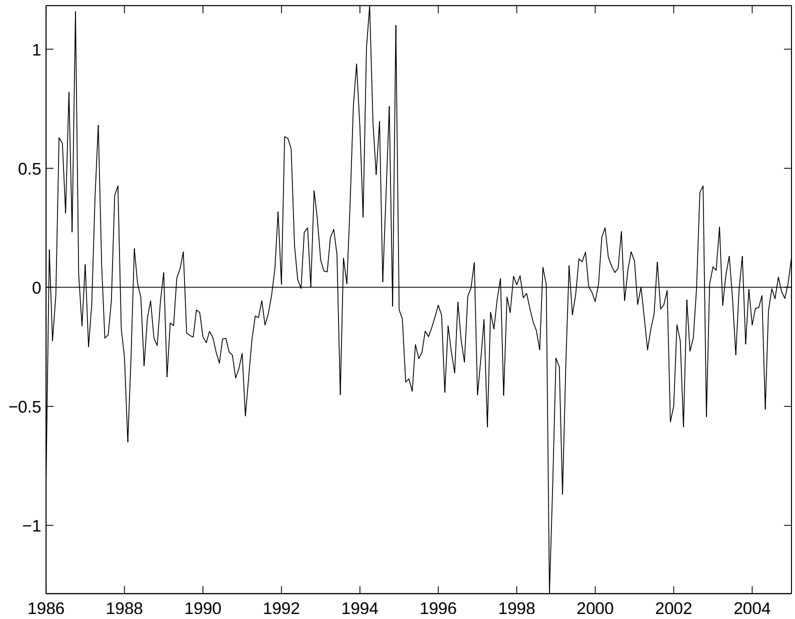


(a) Residual Differences

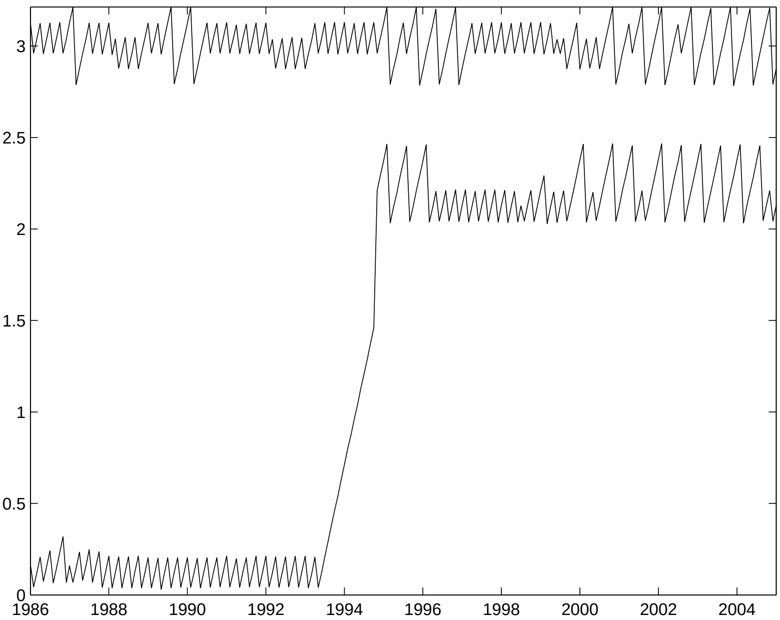


(b) Years from issuance

Figure 17: Years from issuance and dollar differences between residuals at 60 months to maturity. Dynamic Nelson-Siegel term structure model with liquidity factor. Coupon bonds data from CRSP 01/1986-12/2003.

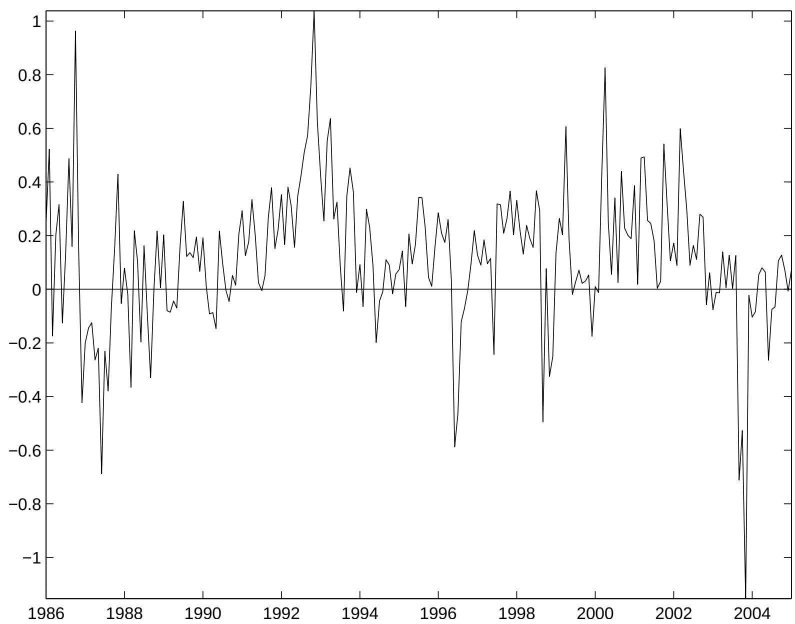


(a) Residual Differences

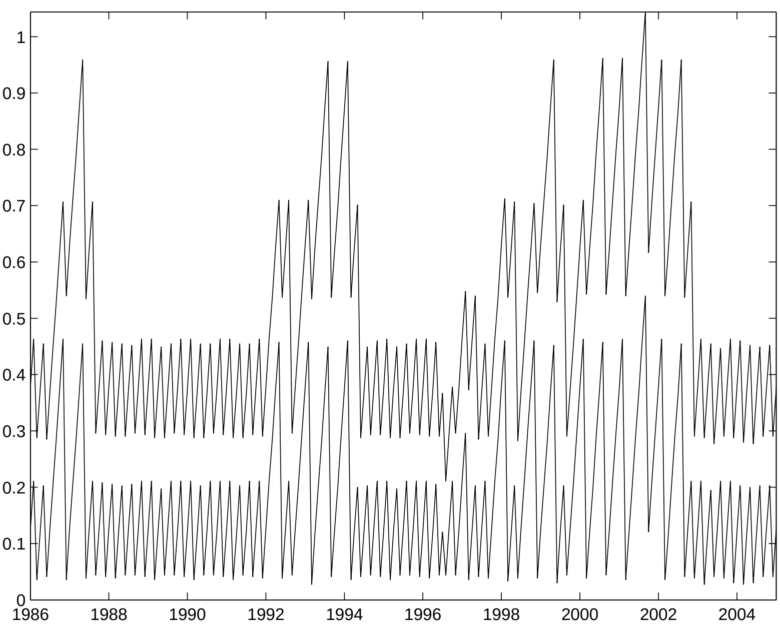


(b) Years from issuance

Figure 18: Years from issuance and dollar differences between residuals at 84 months to maturity. Dynamic Nelson-Siegel term structure model with liquidity factor. Coupon bonds data from CRSP 01/1986-12/2003.



(a) Residual Differences



(b) Years from issuance

Figure 19: Years from issuance and dollar differences between residuals at 120 months to maturity. Dynamic Nelson-Siegel term structure model with liquidity factor. Coupon bonds data from CRSP 01/1986-12/2003.