Log versus level in VAR forecasting: 16 Million empirical answers - expect the unexpected

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Abstract

The use of log-transformed data has become standard in macroeconomic forecasting with VAR models. However, its appropriateness in the context of out-of-sample forecasts has not yet been exposed to a thorough empirical investigation. With the aim of filling this void, a broad sample of VAR models is employed in a multi-country set up and approximately 16 Mio. pseudo-out-of-sample forecasts of GDP are evaluated. The results show that, on average, the knee-jerk transformation of the data is at best harmless.

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1 Introduction

In the field of forecasting macroeconomic time series with VAR models, the use of log-transformed data has become standard or at least good practice. The rationale for using log-transformed data in time series regressions and macroeconomic forecasts is rooted in the normality assumption of classical econometric approaches. It aims at limiting the detrimental effects of heteroscedasticity and skewness in the level data on estimation and testing results. To choose between the use of data in levels and log-transformed data, several in-sample statistical tests based on the Box-Cox-transformation have been developed and applied (see for example Shin and Kang (2001)). However, macroeconomic forecasters are typically interested in the out-of-sample forecast accuracy of a model. So far, the appropriateness of the log-transformation in this context has not been exposed to a thorough empirical investigation. With the aim of filling this void, a broad sample of models is employed over different estimation periods and forecast horizons. It is thereby assumed that the researcher is interested in the forecasts of the levels of a time series. In order to re-transform the forecasts of the estimations in log-transformed data back to levels the adjusted re-transformation approach following the lines of Arino and Franses (2000) as well as the “naive” approach, i.e. simply using the exponential of the logarithmic forecast values, are considered and evaluated. In order to put the results of our study on a firm empirical footing, the analysis is implemented on a G4[1] data set. All reasonable combinations of the variable of interest, the real GDP, and an extended set of explanatory macroeconomic variables are estimated employing VAR models. Using a recursive window approach, for each model the 1 to 8 step ahead pseudo out-of-sample forecasts based on an estimation in log-transformed data and on levels of the time series are compared on the basis of the absolute forecast errors (AFE). The results show that according to the forecast performance the standard use of log-transformed data at least has to be questioned. This finding is independent of the use of the adjusted or the “naive” re-transformation approach.

Our paper is structured as follows. Section 2 reviews the relevant literature. Sections 3 to 5 lay out the set up for estimation and forecasting, the procedure of the analysis, and the statistical tests employed, respectively. Section 6 describes the data and section 7 presents the results. The last section concludes.

[1] The G4-countries comprise Germany, Great Britain, Japan, and the United States of America

2 Literature review

So far, no empirical analysis has been implemented comparing the accuracy of forecasts based on VAR models estimated in logarithms to those of VAR models estimated in level data. However, several studies have considered methods for transforming logarithmic forecasts to level forecasts, thus making both approaches comparable. Granger and Newbold (1976) examine methods for forecasting the levels of transformed series using the autocovariance structure of the forecast errors. Employing the Hermite polynomial expansion, they consider a very general class of instantaneous transformations including the quadratic and the exponential. Building on these findings, Arino and Franses (2000) present explicit expressions for the level-forecasts of a time series when such forecasts are derived from a VAR model in log-transformed data. An empirical analysis demonstrates that re-transforming logarithmic forecasts to levels taking the naive approach leads to inferior results. Van Garderen (2005) considers unbiased predictions of levels when the time series are modeled as a random walk with drift and other exogenous factors after taking logs. He thereby develops unbiased predictors for growth and its variance. The results employing a disaggregated eight sector model of UK industrial production demonstrate, that his approach yields more accurate forecasts than usual techniques. Lopes and Ehlers (1997) show, that from a Bayesian point of view the theoretical issues raised by Granger and Newbold (1976) do not exist. An application of Bayesian techniques to forecast the levels of vector autoregressive log-transformed time series is presented. Thereby they make use of a Monte Carlo simulation of the posterior distribution of the parameters of the VAR adjusted to the log-transformed data.

3 Estimation and Forecasting

Following Granger and Newbold (1976) we first consider the case of forecasting a univariate time series x_t and then extend the results to the more general case of a multivariate VAR(p) model according to Arino and Franses (2000). The series y_t corresponds to the natural logarithmic transformation of x_t, so that $y_t = log(x_t)$. The log-transformed time series y_t can be written as $y_t = m_t + \varepsilon_t$ with m_t being the conditional expectation of y_t, given the information set at time t, and with ε_t being a standard white noise process. The exponential of the forecast of y_{t+k} is referred to as the "naive" forecast of x_{t+k}:

$$\widehat{x}^*_{t+k} = exp(\widehat{m}_{t+k}). \tag{1}$$

As shown by Granger and Newbold (1976), this "naive" forecast does not equal the expected value of the time series x_{t+k} in levels at time t. It is biased since the expected value of the exponential of the forecast error η_{t+k} unequals one[2]. The unbiased forecast $\widehat{x}_{t+k}$ of the variable x_{t+k} in levels can be written as:

$$\widehat{x}_{t+k} = E_t[exp(m_{t+k} + \eta_{t+k})] = \widehat{x}^*_{t+k} E_t[exp(\eta_{t+k})]. \tag{2}$$

where E_t is the expectation operator at time t. Granger and Newbold (1976) showed that the required correction factor $E_t[exp(\eta_{t+k})]$ equals $exp(\sigma_k^2/2)$, where σ_k^2 is the variance of the k-step ahead forecast error of variable y. In the more general case of a VAR(p) model, the required covariance matrices of the k-step ahead forecast errors of the endogenous variables can be derived starting from the specified equation in logarithms[3]:

$$Y(t) = B_0 + \sum_{r=1}^{p} B_r Y(t-r) + \varepsilon(t) \tag{3}$$

$Y'(t) = (y_1(t), \ldots, y_m(t))$, $B_0' = (b_1, \ldots, b_m)$ and $B_r = (b_{ijr})_{i,j=1}^m$ are m-dimensional vectors and matrices with constant parameters, and $\varepsilon'(t) = (\varepsilon_1(t), \ldots, \varepsilon_m(t))$ is a vector of m normally identically and independently distributed random variables with mean zero and covariance matrix Σ, that is $E[\varepsilon_t] = 0, E[\varepsilon_t \varepsilon_t'] = \Sigma_\varepsilon$ and $E[\varepsilon_t \varepsilon_s'] = 0$ for $s \neq t$. The instantaneous covariance matrix Σ_ε is assumed to be nonsingular. Recursively inserting the observed and/or forecasted values of Y in equation 3 yields the k-step ahead forecasts of the endogenous variables. For a VAR(*1*) model the one-step ahead forecast of the vector of the endogenous expressed as a function of observed values of Y is given by:

$$Y(t+1) = B_0 + B_1 Y(t) + \varepsilon(t+1) \tag{4}$$

Making use of equation 4 the two-step ahead forecast $Y(t+2)$ is then given by:

$$Y(t+2) = B_0 + B_1 B_0 + B_1^2 Y(t) + B(1)\varepsilon(t+1) + \varepsilon(t+2) \tag{5}$$

The k-step ahead forecast $Y(t+k)$ of a general VAR(p) model where the coefficients in front of $Y(k)$, $\varepsilon(t+k)$ and the constant terms are written more compactly as $C_r(k)$, $D(i)$ and $C_0(k)$ respectively gives:

$$Y(t+k) = C_0(k) + \sum_{r=1}^{p} C_r(k) Y(t-r+1) + \sum_{i=1}^{k} D(i)\varepsilon(t+i) \tag{6}$$

[2]With $k > 1$ the variance of η, the forecast error, and the variance of ε are different in the VAR context.

[3]The estimation and forecasting in levels is analogous.

where

$$
\begin{aligned}
C_0(k) &= B_0 + \sum_{r=1}^{p} B_r C_0(k-r) \\
C_l(k) &= \sum_{r=1}^{p} B_r C_l(k-r) \\
D(k) &= \sum_{r=1}^{p} B_r D(k-r)
\end{aligned}
$$

with the initial conditions

$$C_0(j) = 0 \quad \text{for j=0,-1,\ldots,-p+1.}$$

$$C_l(j) = \begin{cases} I_m, & \text{if j} = 1-l, \text{ for } 1 \leq l \leq p, \\ 0, & \text{otherwise.} \end{cases}$$

$$D(j) = \begin{cases} 0, & \text{for } j = 0, -1, \ldots, -p+1, \\ I_m, & \text{otherwise.} \end{cases}$$

I_m denotes a m-dimensional Identity Matrix. It becomes clear, that the k-step ahead forecast error

$$\eta_{t+k} = \sum_{i=1}^{k} D(i)\varepsilon(t+i) \tag{6}$$

is a linear combination of normally distributed variables. The covariance matrix of this error term can be recursively computed as:

$$\Sigma_Y(k) = \Sigma_Y(k-1) + D(k)\Sigma_\varepsilon D(k)' \tag{7}$$

where $\Sigma_Y(1) = \Sigma_\varepsilon$. The sum of the i's row of $\Sigma_Y(k)$ contains the variance $\sigma_{i,k}$ of the k-step ahead forecast error of variable y_i. Accounting for this correction factor, equation 2 can be used to obtain the unbiased forecast of $x_{i,t+k}$. This is, the "naive" forecast has to be multiplied with $exp(\sigma_{i,k}^2/2)$.

4 Procedure of the Analysis

For reasons of robustness the analysis of the forecast performance of the different data transformation approaches for VAR models comprises data of 4 countries, namely those of the G4 group. For each country and for each data transformation, a total number of 6885 VAR models is specified. The different models are build permutating a total set of 16 candidate endogenous variables plus the GDP as the variable of interest, which necessarily forms part of all models. Given the limited number of observations in the time dimension a maximum of up to 5 variables per single model is imposed. The resulting VAR models are then estimated using a recursive estimation scheme. The full sample, the end of the sample used in the first recursion as well as the number of recursions for each country is given in Table 2 in the appendix. Each iteration the estimation sample is expanded one quarter. The lag length for each VAR model at each estimation step and for each forecast horizon is dynamically optimized using the AIC information criteria. Building on the estimated models, the 1 to 8 step ahead pseudo out-of-sample forecasts are calculated at each period of the recursive estimation scheme. The forecasts of the log-transformed models and the models estimated in levels are then compared on basis of their AFEs using simple ratios as well as two-sided and one-sided statistical tests. The tests are implemented for the individual countries as well as for the total group. Within both subsets the average over all eight horizons as well as the different horizons separately are analyzed. In order to compare their forecast performance with the models estimated in levels, the forecasts of the log-transformed models are re-transformed to levels following the above described ways.

5 Tests

In order to test whether two forecasts differ significantly, the Wilcoxon signed-rank test and the sign test, two distribution-free, non-parametric procedures are applied. They test for the null hypothesis, that an independently distributed series δ_t $(t = 1, ..., T)$ has a zero median. In the current context, δ_t is defined as $\delta_t = abs(\hat{\epsilon_t^i}) - abs(\hat{\epsilon_t^j})$, .i.e. the difference of the AFEs of the two approaches. The sign test assumes, that under the null hypothesis the series is independently but not necessarily identically binomially distributed. Its test statistic is

$$S = \sum_{t=1}^{T} I_+(\delta_t) \tag{8}$$

where

$$I_+ = \begin{cases} 1, & \text{for } \delta_t > 0, \\ 0, & \text{otherwise.} \end{cases}$$

In large samples the studentized version is asymptotical normal:

$$\frac{S - \frac{T}{2}}{\sqrt{\frac{T}{4}}} \overset{a}{\sim} N(0,1) \tag{9}$$

Thus the critical values of the binomial or the normal distribution may be used. Under the assumption of distributional symmetry, the signed-rank test proposed by Wilcoxon (1945) is a more powerful distribution-free alternative to the sign test. Distributional symmetry implies, that the median equals the mean. Under the same null hypothesis as the sign-test, the test statistic of the signed-rank test is computed as the sum of the ranks of the absolute values of the positive observations:

$$W = \sum_{t=1}^{T} I_+(\delta_t) rank\left[\delta_t\right] \tag{10}$$

where the ranking is such that the largest absolute observation is given rank T and the other ranks are assigned correspondingly. The test is build on the idea, that if the underlying distribution is symmetric about zero, a "very large" (or "very small") sum of the ranks of the absolute values of the positive observations is "very unlikely." The exact finite sample distribution of W is free from nuisance parameters and independent of the true underlying distribution. In large samples, the studentized version is standard normal.

$$\frac{W - \frac{T(T+1)}{4}}{\sqrt{\frac{T(T+1)(2T+1)}{24}}} \overset{a}{\sim} N(0,1) \tag{11}$$

To test whether or not two forecasts generated by different models are significantly different in one direction, i.e. whether one of them is significantly superior to the other several procedures have been developed. Diebold and Mariano (1995) proposed a test (DM test) of the null hypothesis of no difference in the accuracy of two competing forecasts that is widely applicable. Their test allows for a wide class of measures of forecast accuracy and is not restricted to a single loss function, e.g. the AFEs. Following Diebold and Mariano the tests considers the null hypothesis $H_0 : E[\delta_t] = 0$ and is based on the observed sample mean

$$\bar{d} = \frac{1}{T^*} \sum_{t=T_0}^{T} d_t \tag{12}$$

with $T^* = T - T_0 + 1$. The sequence of the forecast errors follows a moving average process of order $q = (k-1)$. If the autocorrelations of order k and higher are zero, the variance of the loss differential can be heteroscedastic and autocorrelation consistently (HAC) estimated as

$$\bar{V} = \frac{1}{T^*}(\hat{\gamma}_0 + 2\sum_{j=1}^{k-1}\hat{\gamma}_j) \tag{13}$$

where $\hat{\gamma}_j$ is the estimated $j-th$ autocovariance of the loss differential δ_t. Under the null hypothesis of equal forecast accuracy the DM test statistic can be computed as:

$$DM = \frac{\bar{\delta}}{\sqrt{\bar{V}}} \sim N(0,1) \tag{14}$$

To test if a model i is not dominated by a model j in terms of forecast accuracy a one-sided DM test has to be conducted. The modified null hypothesis is than given by $H_0 : E[\delta_t] \leq 0$. If the null is rejected one thus concludes that model j dominates model i. In order to reduce size distortions that might be significant in small samples Harvey, Leybourne, and Newbold (1997) suggest a corrected DM statistic:

$$HLN = DM[\frac{T^* + 1 - 2k + k(k-1)/T^*}{T^*}]^{1/2} \tag{15}$$

The modified statistic is compared to a Student's t-distribution with $T^* - 1$ degrees of freedom.

6 The Data

The data entering the VAR-models are standard macroeconomic time-series most commonly used for forecasting purposes. They have been obtained from the International Monetary Fund (IMF), Eurostat and/or national statistic agencies. Additionally to the standard VAR setup of the variable of interest, the Gross Domestic Product (GDP), the Policy Interest Rate and a measure for the Consumer Prices, several other aggregates and indicators that might explain real GDP are included. The Real Effective Exchange Rate, the Imports and Exports series as well as the Commodity Prices cover the external influences. The major Stock-Price Indices, the Unemployment Rate, Industrial Production, Consumption, Investment, Hours Worked and several early indicators such as Industrial Orders, Consumer and Producer Sentiments complete the set. The data, especially in the case of the early indicators, differ between the countries. A complete list of the variables included, as well as some basic descriptive statistics are given in the appendix (see Table

4 respectively Table 5 to Table 8). The number of observations range from 61 quarters in the case of Germany to 145 in the case of the United States (see Table 2). The relatively small number of observerations for Germany is due to the structural break stemming from German reunification in 1990. Given the variable of interest being GDP, the analysis is restricted to quarterly data.

7 Results

Table 1 compares the accuracy of forecasts based on models estimated in level data to those based on models estimated in log-transformed data. The columns give the percentage of the single test statistics for pairwise comparisons of the AFEs in the country-subsets and the total. The figures report the average values over all forecast horizons.[4] The log-forecasts have been re-transformed following Arino and Franses (2000). However, as the authors point out, it is not clear from the outset, whether the adjustments implemented for the optimal re-transformation yield more accurate results then the "naive" re-transformation approach. Therefore, we have repeated the tests for the "naive" re-transformation approach, as well. These results are given in Table 3 in the appendix. However, the main results of the analysis remain unchanged. The first row in both tables shows the results for

Table 1: Optimally retransformed log vs. level forecasts

	USA	Japan	Germany	UK	Total
Sign	0.32	0.35	0.13	0.35	0.33
Wilcoxon	0.37	0.46	0.18	0.43	0.41
Relative AFEs	0.56	0.49	0.53	0.56	0.54
DM	0.11	0.17	0.12	0.16	0.14
HNL	0.16	0.22	0.11	0.21	0.19
$\text{DM}^{reversed}$	0.01	0.09	0.05	0.02	0.04
$\text{HNL}^{reversed}$	0.01	0.09	0.04	0.03	0.04

the simple non-parametric sign test and the second row the ones for the Wilcoxon signed-rank test. As mentioned before, both statistics are two-sided procedures, testing the null-hypothesis of both competing model types yielding equally accurate forecasts. The values in the tables report the percentage of pairwise comparisons where the median difference between the

[4]The results do not vary significantly between the different forecast horizons and are not reported.

two alternatives is significantly different from zero to the 5 percent level. For the selected countries between $30 - 40$ percent of the pairwise comparisons show significant differences with respect to forecasting accuracy. Only for Germany, the percentage is considerably lower. This might be due to the relatively short sample for this country. The following rows give statistics that indicate, which of the two approaches is superior to the other. To start with, the third row simply gives a descriptive ratio. It shows the number of times in percent, the models in level data yield lower AFEs than their log-transformed counterparts. Forecasts based on level models outperform models in log-transformed data slightly more often. Only for Japan, when using the unbiased approach of re-transformation, the figures recommend an estimation and forecasting in logarithmic data. However, in 49 percent of the cases the level models still yield better results. The rows four and five, respectively six and seven, give the results of the DM test and the HNL test. Both statistics are specified as one-sided tests, exploring, whether one approach significantly outperforms the alternative. The null hypothesis of both statistics in row four and five is that the forecasts based on the models in level data are not significantly more accurate than the ones estimated in log-transformed time series. Rows six and seven report the results for the test with the null being that the forecasts based on log-transformed data are not significantly superior. The figures give the percentage of pairwise comparisons where the respective null hypothesis can be rejected at a 5 percent level of significance. For the DM (HNL) test on average, 14 (19) percent of the pairwise comparisons report a statistically higher forecasting accuracy for models in level data. In contrast, reverting the alternative H_1 hypothesis shows that the percentage of comparisons where the forecasts in levels are dominated by their logarithmic counterpart only reaches 4 for both tests. Combining the results of the different test statistics together with the ratios of the AFEs indicates a slight predominance of models in level data when forecasting out-of sample values for the GDP as variable of interest with VAR models.

8 Conclusion

VAR models are an essential tool for practitioners when it comes to forecasting macroeconomic time series. Log-transforming the data is common practice. The rationale behind this is that the log-transformation helps to overcome the detrimental effects of heteroscedasticity and skewness in the level data on estimation and testing. Our goal is to empirically evaluate the appropriateness of this approach in the context of forecasting with VAR

models.
In a recursive scheme VAR models for data of the G4 countries are estimated in logarithms as well as in levels and used to forecast 1 to 8 quarters ahead. The resulting logarithmic forecasts are transformed to levels and the forecast errors are compared to the ones of the respective level models. The re-transformation of the logarithmic forecasts to levels simply taking the exponentials leads to biased forecasts. Therefore, additionally, the optimal re-transformation proposed by Arino and Franses (2000) is applied, as well. However, the two approches basically yield the same results. One sided DM and HNL tests report on average a predominance of 14, respectively 19, percent of level forecasts over log-based forecasts, whereas only 4 percent of the pairwise comparisons give a predominance of the log-based forecasts. Overall, our analysis based on broad empirical evidence demonstrates that the automatical transformation of the data is at best harmless.

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Table 2: # of obs per country

Country	sample	1^{st} iteration	# iterations
Germany	1991:1 - 2006:1	1999:4	18
Japan	1970:1 - 2006:1	1980:1	97
UK	1972:1 - 2006:1	1982:1	89
USA	1970:1 - 2006:1	1980:1	97

Table 3: "Naive" log vs. level forecasts

	USA	Japan	Germany	UK	Total
Sign	0.30	0.36	0.11	0.38	0.33
Wilcoxon	0.36	0.46	0.16	0.49	0.42
Relative AFEs	0.56	0.50	0.52	0.59	0.55
DM	0.09	0.19	0.10	0.17	0.15
HNL	0.14	0.24	0.09	0.23	0.20
$\text{DM}^{reversed}$	0.01	0.08	0.06	0.00	0.03
$\text{HNL}^{reversed}$	0.01	0.08	0.04	0.00	0.03

Variable	Abbreviation	seasonal adj.	real
Business Confidence	BC		
Private Consumption	C	x	x
Consumer Confidence	CC		
Commodity Price Index	COM		
Consumer Price Index	CPI		
Government Spending	G	x	x
Gross Domestic Product	GDP	x	x
Housing Index	HI		
Hours Worked	HW		
Industrial Production	IP	x	x
Industrial Sales	IS	x	x
Investments	INV	x	x
Imports	M	x	x
Manufacturing Orders	MO		
Money	MON		
Policy Rates	R		
Government Benchmarks, long term	RL		
Retail Sales	RS	x	x
Real Effektive Exchange rate	RER		
Stock Index	STX		
Unemployment rate	U	x	
Hourly Earnings	W		
Exports	X	x	x

Table 4: Variables and abbreviations

Variable	Mean	Max	Min	Std. Dev.
BC*	52.55	70.43	32.23	6.76
CC*	97.18	118.63	64.97	12.05
COM*	101.10	164.30	36.63	32.90
C**	4651.68	8032.00	2434.40	1592.42
CPI*	116.34	199.30	38.10	48.91
G**	1394.54	2013.30	970.50	313.12
GDP**	6867.64	11381.40	3759.80	2191.83
HW				
I**	988.33	2013.40	411.90	439.021
IP‡	70.35	110.78	41.09	20.27
IS				
M**	708.78	1931.00	209.70	490.77
MO				
MON				
R	1.07	1.18	1.01	0.03
RER*	98.09	126.27	81.43	11.36
RL†	1.08	1.15	1.04	0.02
RS				
STX*	466.32	1475.51	69.42	430.29
U†	6.20	10.67	3.90	1.38
W‡	9.46	16.46	3.33	3.79
X	559.15	1252.80	156.10	336.74

Note: * marks variables that represent indices, ** are level data given in Billion USD, † are percentage variables, and ‡ indicate variables in USD.

Table 5: USA

Variable	Mean	Max	Min	Std. Dev.
BC				
CC				
COM*	98.04	115.77	56.09	15.91
C**	199270.00	291965.00	36508.40	84632.37
CPI*	81.09	101.61	31.24	21.32
G**	50274.04	90706.20	5036.80	28467.57
GDP**	346051.4	514023.0	69527.00	151179.50
HW				
I**	97228.42	151349.00	24695.60	38201.38
IP*	79.29	104.31	43.32	19.15
IS				
M**	34155.95	73516.90	6466.60	14515.32
MO**	1567.98	2576.00	478.40	640.18
MON*	134.24	394.27	17.54	102.25
R†	3.56	9.00	0.10	2.65
RER*	73.87	117.01	39.85	18.34
RL†	5.28	9.53	0.66	2.68
RS				
STX*	13098.44	37244	2030.33	8145.68
U†	2.83	5.43	1.07	1.20
W*	73.23	100.19	16.33	25.77
X**	39462.50	79025.10	7461.90	16556.18

Note: * marks variables that represent indices, ** are level data given in Billion YEN, and † are percentage variables.

Table 6: Japan

Variable	Mean	Max	Min	Std. Dev.
BC*	95.24	106.1	86.20	4.76
CC				
COM*	100.23	114.80	94.00	4.66
C**	281.32	302.86	245.8	17.12
CPI*	97.56	110.07	80.13	7.51
G				
GDP*	95.24	105.26	84.91	6.32
HW‡	12193.84	13202.10	11645.60	384.03
I**	100.43	111.65	92.36	4.74
IP*	95.23	107.93	85.77	5.50
IS				
M*	147.00	222.04	99.11	36.29
MO*	90.24	116.76	73.00	11.50
MON*				
R†	3.38	8.58	1.00	2.24
RER*	107.53	119.73	98.01	5.46
RL†	5.51	8.51	3.17	1.47
RS				
STX*	3719.63	7359.85	1494.99	1693.72
U†	8.10	10.00	5.53	1.20
W*	209.68	229.89	167.70	17.15
X*	153.23	249.59	95.89	46.29

Note: * marks variables that represent indices, ** are level data given in Billion EUR, † are percentage variables, and ‡ indicate variables in hours.

Table 7: Germany

Variable	Mean	Max	Min	Std. Dev.
BC				
CC				
COM*	86.79	116.91	18.49	24.41
C**	117.76	184.73	74.67	33.21
CPI*	111.30	194.2	21.10	52.98
G**	46.87	62.16	35.78	6.27
GDP**	200.35	296.51	134.24	46.74
HW				
INV**	16.67	29.55	8.75	6.73
IP*	86.53	104.10	63.23	11.85
IS*	79.77	104.17	56.89	11.54
M**	41.22	97.00	17.40	21.86
MO*	71.29	103.64	47.10	12.05
MON				
R†	8.53	16.06	3.41	3.37
RER*	86.92	104.53	65.75	9.73
RL†	9.39	16.54	3.97	3.36
RS†	76.16	127.80	48.90	22.78
STX*	1597.49	4009.38	180.17	1071.57
U†	7.32	11.87	3.43	2.54
W				
X**	41.62	87.01	17.03	18.51

Note: * marks variables that represent indices, ** are level data given in Billion GBP, and † are percentage variables.

Table 8: United Kingdom