A solution to the problem of externalities when agents are well-informed: Another comment

In this *Review* Hal R. Varian (1994) introduced a so-called *compensation mechanism* for solving a very general class of externality problems when players know one another's pay-offs. In Theorem 1 of that paper Varian argues that every sub-game perfect equilibrium of the compensation mechanism is Pareto efficient provided a *local invertibility* assumption is satisfied. Local invertibility ensures that each agent can set a price for the other agent that will induce the other agent to make any desired choice. Local invertibility does not require differentiability and thus implies that the compensation mechanism works for discrete choice externality problems. The purpose of this note is to show that the compensation mechanism results in equilibrium prices that implement both efficient and inefficient outcomes for two discrete choice externality problems, namely common property resource problems and free rider problems associated with joint ventures.

In order to analyze both classes of externality problem let $x_i = 0$ and $x_i = 1$ refer to agent i emitting or not emitting an externality respectively. In the common resource problem an agent emits an externality by using the common property resource (e.g. public road) and thereby emitting either pollution and/or congestion externalities. In the joint venture setting an agent emits an externality by withholding effort and/or information and thereby reducing the revenue shared by the joint venture partners.

If there are two agents then the compensation mechanism involves the agents playing the following game

Announcement stage – Agent i names the Pigovian tax p_{ij}^i that agent j must pay if j emits an externality as well as the compensation p_{ji}^i that agent j receives if i emits an externality. Agent i pays a penalty given by $\left\| p_{ji}^i - p_{ji}^j \right\|$ if the compensation that i announces that j should *receive* if i emits the externality, differs from the Pigovian tax announced by j that i should *pay* if i emits an externality.

Choice stage – Agent i chooses x_i to maximize the following pay-off

$$\pi_i = u_i(x_1, x_2) - p_{ji}^j x_i + p_{ij}^j x_j + \left\| p_{ji}^i - p_{ji}^j \right\|$$

where $u_i(x_1, x_2)$ is agent i's utility function. In order to facilitate discussion of the results the following notation will be used.

$b_i(x_j)$ = Benefit to i if i emits an externality given j's choice (x_j)
$c_j(x_j)$ = Cost to j if i emits an externality given j's choice (x_j)

where benefits and costs are measured by the change in utility.

Example 1 : Joint venture

The utility functions for the joint venture example are given as follows

	$x_2 = 0$ (don't free-ride)	$x_2 = 1$ (free ride)
$x_1 = 0$ (don't free ride)	(20, 20)	(10, 21)
$x_1 = 1$ (free ride)	(21, 10)	(17, 17)

The entries in the pay-off matrix can be justified as follows. If agent 1 free-rides given than agent 2 does not free ride then both agents lose revenues of 10 and agent 1 incurs a cost saving equal to 11. As a result agent 1 gains $11 - 10 = 1$ (pay-off rises from 20 to 21) and agent 2 loses 10 (pay-off falls from 20 to 10). If agent 1 free-rides given that agent 2 also free-rides then both agents lose revenues of 4 and agent 1 again incurs a cost saving equal to 11. As a result agent 1 gains $11 - 4 = 7$ (pay-off rises from 10 to 17) and agent 2 loses 4 (pay-off falls from 21 to 17). The reason that the loss in revenue caused by agent 1's free-riding is lower when agent 2 free rides than when 2 does not free ride is due to the complementarity in effort. That is, agent 1's marginal product of effort is lower when agent 2 free rides than when agent 2 does not free ride.

We now assume transferable utility and thus obtain that the efficient outcome for the above pay-offs occurs when both agents don't free ride. Let me now show that the compensation mechanism implements the both don't free ride and the both free ride outcomes with the same Pigovian tax.

Equilibrium 1: Both free ride $(x_1 = x_2 = 1)$

If agent i free rides given that j free rides then agent i's utility rises by

(1) $$b_i(1) = 17 - 10 = 7, i = 1,2$$

and agent j's utility falls by

(2) $$c_j(1) = 21 - 17 = 4, j = 1,2$$

The both free ride outcome is achieved using the following transfers

(3) $$c_1(1) \leq p_{21}^1 = p_{21}^2 \leq b_2(1)$$
(3b) $$c_2(1) \leq p_{12}^1 = p_{12}^2 \leq b_1(1)$$

Substituting (1) and (2) yields

(3a)' $$4 \leq p_{21}^1 = p_{21}^2 \leq 7$$
(3b)' $$4 \leq p_{12}^1 = p_{12}^2 \leq 7$$

In order to understand the equilibrium we thus have to show why each agent free rides and why they allow the other individual to free ride. The argument is as follows. Agent i free rides because they pay a tax set by agent j which is no greater than agent i's benefit from free riding (i.e. $p_{ij}^{j} \leq b_i(1)$). Agent i is willing to let agent j free ride by setting $p_{ji}^{i} \leq b_j(1)$ because they receive compensation set by agent j which is greater than of equal to agent i's disutility cost of bearing j's free riding (i.e. $p_{ji}^{j} \geq c_i(1)$). Furthermore agent i sets the compensation received by j equal to the tax paid by i in the event that i free rides (i.e. agent i sets p_{ij}^{i} equal to p_{ij}^{j}) in order to avoid paying a penalty.

Let me now elaborate on the incentive for individual i to set the appropriate Pigovian tax (i.e. $p_{ji}^{i} \leq b_j(1)$ in this case) by indicating what happens to individual i's pay-offs if they either lower or raise the Pigovian tax. If agent i lowers the Pigovian tax then nothing happens to i's pay-off because j continues to free ride and thus i continues to receive the same compensation set by j (given by p_{ji}^{j}). If agent i raises the Pigovian tax above $b_j(1)$ and thus induces j to not free ride then i cannot be made better off because the loss of compensation set by j and given by p_{ji}^{j} is at least as great as i's disutility cost from having to put up with agent j's free riding (i.e. $p_{ji}^{j} \geq c_i(1)$).

Equilibrium 2: Neither free rides $(x_1 = x_2 = 0)$

If agent i free rides given that agent j does not free ride then agent i's utility rises by

(4) $$b_i(0) = 21 - 20 = 1, i = 1,2$$

and agent j's utility falls by

(5) $$c_j(0) = 20 - 10 = 10, j = 1,2$$

The neither free ride outcome is achieved using the following transfers

(6a) $$b_1(0) \leq p_{21}^{1} = p_{21}^{2} \leq c_2(0)$$
(6b) $$b_2(0) \leq p_{12}^{1} = p_{12}^{2} \leq c_1(0)$$

Substituting (4) and (5) yields

(6a)' $$1 \leq p_{21}^{1} = p_{21}^{2} \leq 10$$
(6b)' $$1 \leq p_{12}^{1} = p_{12}^{2} \leq 10$$

In order to understand the equilibrium we thus have to show why each agent does not free ride and why they induce the other agent not to free ride. The argument is as follows. Agent i does not free ride because they pay a tax set by agent j which is at least as great as i's benefit from free riding (i.e. $p_{ij}^{j} \geq b_i(0)$). Agent i induces j to not free ride

by setting $p_{ji}^i \geq b_j(0)$ because they receive compensation set by agent j which is less than or equal to i's cost of bearing j's free riding (i.e. $p_{ji}^j \leq c_i(0)$). Furthermore agent i sets the compensation received by j equal to the tax paid by i in the event that i free rides (i.e. agent i sets p_{ij}^i equal to p_{ij}^j) in order to avoid paying a penalty.

      Let me elaborate on the incentive for agent i to set the appropriate Pigovian tax (i.e. $p_{ji}^i \geq b_j(0)$ in this case) by indicating what happens to agent i's pay-offs if they either lower or raise the Pigovian tax. If individual i raises the Pigovian tax then nothing happens to i's pay-off because agent j continues to not free ride and thus agent i continues to receive no compensation and continues to incur no disutility cost of bearing agent j's free riding. If agent i lowers the Pigovian tax below $b_j(0)$ and thus induces agent j to free ride then agent i can not be made better off because the compensation that agent i would receive if agent j were to free ride (which is set by j and given by p_{ji}^j) is less than or equal to i's disutility cost of putting up with agent j's free riding (i.e. $p_{ji}^j \leq c_i(0)$).

Illustration of the equilibrium : $p_{21}^1 = p_{21}^2 = p_{12}^1 = p_{12}^2 = 5$. From the above analysis we obtain that any tax between 4 and 7 would implement either the both free ride or the both don't free ride outcome. In order to demonstrate this multiplicity of equilibria let us suppose that each agent choose a Pigovian tax equal to 5. In this case the pay-off matrix becomes

	$x_2 = 0$ (don't free-ride)	$x_2 = 1$ (free ride)
$x_1 = 0$ (don't free ride)	(20, 20)	(15, 16)
$x_1 = 1$ (free ride)	(16, 15)	(17, 17)

which is the same as the utility function matrix except that a transfer of 5 is made from the free rider to the non-free rider in the off diagonal elements of the matrix. The above matrix shows that both $x_1 = x_2 = 0$ and $x_1 = x_2 = 1$ are equilibrium outcomes. Clearly the efficient outcome ($x_1 = x_2 = 1$) dominates the inefficient one but nevertheless the fact remains that the compensation mechanism has confronted the agents with a co-ordination problem.

Extension : Different degrees of free-riding. Let me now extend the above example to show that the co-ordination problem would worsen if each agent had more than two possible discrete actions. For example if agents could choose the degree of free-riding (a lot or a little) and if the pattern of pay-offs described above were to continue then a possible utility function could be

	$x_2 = 0$ (don't free-ride)	$x_2 = 1$ (free ride a little)	$x_2 = 1$ (free ride a lot)
$x_1 = 0$ (don't free ride)	(20, 20)	(10, 21)	(-0.5, 21.5
$x_1 = 1$ (free ride a little)	(21, 10)	(17, 17)	(7, 18)
$x_1 = 1$ (free ride a lot)	(21.5, -0.5)	(18, 7)	(14, 14)

The pattern of pay-offs in the above matrix follows the pattern that agent i's benefit of free riding diminishes the more agent i free rides (e.g. $21.5 – 21 < 21 – 20$) and increases the more agent j free rides (e.g. $14 – 7 > 18 – 17 > 21.5 – 21$). In addition the costs that agent i's free riding imposes on agent j rises the more agent i free rides (e.g. $17 – 7 > 21 – 17$) and falls the more agent j free rides (e.g. $21.5 – 18 < 21 – 17 < 20 – 10$). This pattern of pay-offs can be justified by diminishing marginal product of effort and the complementarity of work effort between agents.

The other property of the pay-off matrix that is important to notice is that the entries along the main diagonal of the matrix represent the maximum joint pay-off in the column or row. For example $17 + 17 = 34$ is the highest pay-off in the second row or column. It is this property that will result in all elements along the main diagonal being possible equilibria using the compensation mechanism. In order to show this we can use arguments similar to the ones used above to show that taxes in the 1.5 to 10.5 range, 1 to 10 range and 4 to 7 range will bring about the both free ride, both free ride a little and both free ride a lot outcomes. This then implies that a tax in the 4 to 7 will bring about any of the three outcomes. If a tax of 5 is chosen then the pay-off matrix becomes

	$x_2 = 0$ (don't free-ride)	$x_2 = 1$ (free ride a little)	$x_2 = 2$ (free ride a lot)
$x_1 = 0$ (don't free ride)	(20, 20)	(15, 16)	(9.5, 11.5
$x_1 = 1$ (free ride a little)	(16, 15)	(17, 17)	(12, 13)
$x_1 = 2$ (free ride a lot)	(11.5, 9.5)	(13, 12)	(14, 14)

Which verifies that all three outcomes along the main diagonal are possible equilibria thereby demonstrating that the multiplicity of equilibria problem can increase with the number of possible discrete actions.

Comparison with the example chosen by Varian. The above examples demonstrate that the compensation mechanism can implement any outcome that maximizes the joint pay-off in both its row and column. In the examples that I have chosen there is more than one outcome that maximizes the pay-off in its row and column and thus inefficient equilibria are possible. In the discrete example chosen by Varian on p.1291, which is replicated below, the compensation mechanism implements a unique efficient outcome precisely

because this outcome (i.e. the both co-operate outcome) is the only outcome in the whole pay-off matrix that maximizes the joint pay-off in its row or column.

	$x_2 = 0$ (co-operate)	$x_2 = 1$ (defect)
$x_1 = 0$ (co-operate)	(5, 5)	(2, 6)
$x_1 = 1$ (defect)	(7, 1)	(3, 3)

Example 2 : Common property resources

Suppose that common property usage refers to the use of a public road for commuting purposes and that agents make the choice of whether or not to commute via their choice of where to work and live. Utility functions for this case can be given by

	$x_2 = 0$ (don't commute)	$x_2 = 1$ (commute)
$x_1 = 0$ (don't commute)	(20, 20)	(16, 28)
$x_1 = 1$ (commute)	(30, 16)	(19, 17)

The justification for the utility functions is as follows. If agent i commutes given that agent j does not commute then agent i receives the benefit of uncongested commuting and imposes an air/noise pollution externality equal to $20 - 16 = 4$ on agent j. The benefit of uncongested commuting is equal to $30 - 20 = 10$ for agent 1 and by $28 - 20 = 8$ for agent 2. If agent commutes given that agent j also commutes then agent i receives the benefits of congested commuting and imposes both an air/noise externality plus a congestion externality on agent j thereby reducing agent j's pay-off by $30 - 19 = 28 - 17 = 11$. The benefit of congested commuting is equal to $19 - 16 = 3$ for agent 1 and by $17 - 16 = 1$ for agent 2.

Note that the joint pay-offs are maximized when only agent 1 commutes. This asymmetric outcome arises because the agents impose the same externalities on one another but agent 1 gets a greater benefit from commuting than agent 2.

Equilibrium 1: Only agent 1 commutes ($x_1 = 1$ and $x_2 = 0$)

Agent 1's benefit of commuting given that 2 doesn't commute equals

(7a) $$b_1(0) = 30 - 20 = 10$$

The disutility cost incurred by 2 if 1 commutes given that 2 does not commute is given by

(7b) $$c_2(0) = 20 - 16 = 4$$

Agent 2's benefit of commuting given that 1 commutes equals

(8a) $$b_2(1) = 17 - 16 = 1$$

The disutility cost incurred by 1 if 2 commutes given that 1 commutes is given by

(8b) $$c_1(1) = 30 - 17 = 13$$

The outcome which involves 1 commuting and 2 not commuting can be achieved using the following transfers

(9a) $$c_2(0) \leq p_{12}^1 = p_{12}^2 \leq b_1(0)$$

(9b) $$b_2(1) \leq p_{21}^1 = p_{21}^2 \leq c_1(1)$$

Substituting (7) and (8) into (9) yields

(9a)' $$4 \leq p_{12}^1 = p_{12}^2 \leq 10$$

(9b)' $$1 \leq p_{21}^1 = p_{21}^2 \leq 13$$

The equilibrium logic is as follows. Agent 1 is willing to commute because they pay a tax set by agent 2 no greater than agent 1's benefit from commuting (i.e. $p_{12}^2 \leq b_1(0)$). Agent 1 is willing to induce agent 2 to not commute by setting $p_{21}^1 \geq b_2(1)$ because they receive compensation set by agent 2 which is no greater than agent 1's cost of bearing agent 2's commuting externalities (i.e. $p_{21}^2 \leq c_1(1)$).

Agent 2 is not willing to commute because they pay a tax set by agent 1 which is at least as great as their benefit from commuting (i.e. $p_{21}^1 \geq b_2(1)$). Agent 2 is willing to induce agent 1 to commute by setting $p_{12}^2 \leq b_1(0)$ because they receive compensation set by agent 1 which is at least as great as agent 2's cost of bearing agent 1's commuting externalities (i.e. $p_{12}^1 \geq c_2(0)$).

Equilibrium 2: Only agent 2 commutes ($x_1 = 0$ and $x_2 = 1$)

Agent 2's benefit of commuting given that agent 1 doesn't commute equals

(10a) $$b_2(0) = 28 - 20 = 8$$

The disutility cost incurred by 1 if 2 commutes given that 1 does not commute is given by

(10b) $$c_1(0) = 20 - 16 = 4$$

Agent 1's benefit of commuting given that 2 commutes equals

(11a) $$b_1(1) = 17 - 16 = 1$$

The disutility cost incurred by 2 if 1 commutes given that 2 commutes is given by

(11b) $$c_2(1) = 28 - 17 = 11$$

The outcome in which only agent 2 commutes can be achieved with the following transfers

(12a) $\qquad c_1(0) \leq p_{21}^1 = p_{21}^2 \leq b_2(0)$

(12b) $\qquad b_1(1) \leq p_{12}^1 = p_{12}^2 \leq c_2(1)$

Substituting (10) and (11) into (12) yields

(12a)' $\qquad 4 \leq p_{21}^1 = p_{21}^2 \leq 8$

(12b)' $\qquad 1 \leq p_{12}^1 = p_{12}^2 \leq 11$

Individual 2 is willing to pollute because they pay a tax set by individual 1 which is less than or equal to 2's benefit from pollution ($p_{21}^1 \leq b_2(0)$). Individual 2 is willing to induce 1 to not pollute by setting $p_{12}^2 \geq b_1(1)$ because they receive compensation set by individual 1 which is less than or equal to 2's cost of bearing 1's pollution (i.e. $p_{12}^1 \leq c_2(1)$).

Individual 1 is not willing to pollute because they pay a tax set by individual 2 which is greater than or equal to 1's benefit from pollution ($p_{12}^2 \geq b_1(1)$). Individual 1 is willing to induce 2 to pollute by setting $p_{21}^1 \leq b_2(0)$ because they receive compensation set by individual 2 which is greater than or equal to 1's cost of bearing 2's pollution (i.e. $p_{21}^2 \geq c_1(0)$).