Measuring the Stock of the Aggregate R&D capital in the Growth Accounting Framework

by Ning Huang

University of British Columbia

Paper prepared for the CEA Annual Meeting, June 1st- June 3rd, 2007

Abstract

The Canberra Group on Capital Measurement has recommended the capitalization of R&D expenditure for the next international version of the System of National Accounts (SNA). How to treat R&D expenditure in the standard Growth Accounting framework provides some new challenges for the productivity analysis. In this paper, we propose a new method of treating R&D expenditure in the Growth Accounting Framework and investigate how to construct the proper measures for the stock of the aggregate R&D capital. We divide the stock of R&D capital into two parts: the R&D capital stock for knowledge production and the R&D capital stock for non-knowledge production. These two parts play different roles in the production function. In addition, we derive a general formula for measuring the quantity of newly created R&D capital that depends not only on R&D expenditure but also on the progress of knowledge productivity. Based on our model, we show the difference of our treatment in productivity analysis from the usual treatment by using US manufacturing data as an example.

1. Introduction

The New Economy is characterized as the knowledge-based economy. Technical innovation that is mainly attributed to firms' R&D investments is regarded as the major source of productivity and long-run economic growth. Economists have been long interested in identifying the quantitative relationship between R&D and productivity growth. In order to better reflect the role of R&D playing in the productivity growth, the Canberra Group on Capital Measurement has recommended the capitalization of R&D expenditure for the next international version of the System of National Accounts (SNA). Treating R&D as capital formation has direct impacts on the productivity analysis. We are particular interested in two issues associated with productivity analysis, namely how to treat R&D expenditure in the growth accounting framework and how to construct an appropriate measure for the aggregate R&D capital stock used in this framework.

The capitalization of R&D expenditures provides some new challenges for the standard growth accounting methodology[1]. The "standard" methodology treats R&D capital in a similar manner as physical capital. R&D capital is treated as an additional input factor in the production function. The measure of the R&D capital stock is constructed by using the Perpetual Inventory Method (PIM) based on a plausible assumption about the depreciation rate for R&D investments. However, Pitzer (2004) and Diewert (2005b) suggested that the treatment of R&D assets is not quite so straightforward as the "standard" methodology suggests since these R&D assets do not behave in the same manner as ordinary reproducible capital inputs. Investments in the ordinary capital increase the capital inputs, such as machine or structures, which in turn will generally lead to a positive increment in production. R&D investments can be regarded as a knowledge production process, which creates new knowledge that can be employed for both normal production of goods and services and future round of knowledge production. R&D capital is the output of R&D investment. It is not like a "normal" reproducible capital asset that depreciates with use. However, its value declines with the creation of even newer technologies, diffusion of knowledge and changing tastes. In sum up, just as Diewert (2006) pointed out that "there is a great deal of theoretical work that remains to be done in adapting the standard growth accounting methodology to deal with the complexities that are inherent in the treatment of R&D investments".

In order to proceed with capitalization of R&D in the national accounts, we need to

[1] Refer to Diewert (2006) for more detail discussion.

construct an accurate measure of the R&D capital stock. Constructing the right measure of the R&D stock is the core part of implementing the idea of capitalizing the R&D expenditures. In addition, as a good proxy for knowledge capital stock, the stock of R&D capital plays an important role in conducting economic research on many interesting issues, such as the sources of economic growth, the contribution of knowledge capital to productivity growth, the rate of return to knowledge capital, and many other R&D related research. The credibility of the results derived from these various researches heavily relies on the proper measure of the R&D capital stock. If each economist constructs his "own" R&D stock in the research, it is obvious that the results stemmed from these "self-made" R&D stock are not suitable for conducting comparative studies. Therefore constructing a widely recognized and appropriate measure for the R&D capital stock is crucial for conducting reliable economic research and providing meaningful policy implications. Given the importance of the R&D stock, it is worthwhile for us to investigate how to construct a proper measure for it.

When we conduct productivity analysis, or estimate the rate of return to R&D capital, the corresponding concept of the R&D capital stock employed in these works should be the aggregated R&D capital stock. Therefore the aggregation process is essential for constructing an appropriate measure of the R&D stock. There are many interesting topics arising in the aggregation of the R&D stock. For example, at a certain point in time, a firm may utilize technologies that have different efficiency levels, perhaps associated with certain vintage of capital investments. How then can we aggregate over the old technology and the new technology, and construct the aggregate R&D stock for the firm? Different firms in the same industry may apply different types of technologies, how can we aggregate different firms' R&D stocks and build the aggregate R&D stock at industry level? In an industry, some firms may invest in R&D and create their own technologies, while other firms may just "purchase", "borrow" or even "steal" technologies from other sources. How should we deal with the knowledge diffusion and build industrial level R&D stock that reflects the aggregate technological level of the industry? Firms in the same industry may conduct double research and create similar technology, what would be the impact of the similar knowledge on the aggregation of R&D capital stock? Aggregating R&D capital stock is a complicated issue. In order to build a reliable measure of R&D capital stock, we should attempt to find answers to the above questions. In the literature, we lack papers that explicitly and systematically discuss the aggregation of knowledge capital. However, there are some papers talking about the aggregation of physical capital. Diewert (1980) discussed aggregation problems in the measurement of capital. Cooper and Haltiwanger (1993) explored the spillover effect of machine replacement on the other sectors of the economy. Harper (2004) and Diewert (2005) both discussed the aggregation of capital over vintages when technology improvement was embodied in the capital.

Conventionally, the R&D capital stock are constructed by accumulating R&D expenditures based on the perpetual inventory method and an arbitrarily chosen depreciation rate with the range from 10% to 15%. The R&D stock at time t can be formulated by the previous R&D investment and the depreciated past R&D capital stock. This PIM-based measure of R&D capital stocks is based on some restrictive assumptions, such as a constant productivity of R&D activity, a constant structure of R&D by type of activity and a constant lag structure within types of activity[2]. These assumptions are problematic. For example, there is plenty of evidence that the productivity of R&D has not remained constant over time. In the literature, there are some papers that discuss the construction of the "knowledge capital stock" based on the R&D expenditures. Griliches (1979) and Jones and Williams (1998) used a "knowledge production function" to link the current technological knowledge and all the current and past R&D expenditures. Alston, Craig and Pardey (1998) and Esposti and Pierani (2003) modified the "knowledge production function" model. The former group built the knowledge capital stock in terms of the sum of the depreciated old knowledge stock and the new knowledge. The latter just wrote the knowledge capital stock in terms of all the previous R&D investments. In short, all these measures depend on R&D inputs. There is uncertainty about the accuracy with which R&D expenditure represents delivery of R&D outputs. We need to be aware of the limitation of the input-based measure of R&D capital stock.

The main objective of this paper is to propose a new method of the R&D investment treatment in Growth Accounting Framework and investigate how to construct the proper measures for the stock of the aggregate R&D capital. The major issues we deal with are the construction of the measures for the R&D asset and aggregate R&D capital stock. Two different levels of the aggregation are discussed in the paper, namely the aggregation over different R&D vintages and the aggregation across knowledge stocks from different firms.

The following part of the paper is organized as this. Section 2 suggests a new way of incorporating R&D capital into the standard growth accounting framework. Section 3 discusses some conceptual issues related to R&D capital and the aggregate R&D capital stock. Section 4 focuses on the construction of the measure of the aggregate R&D capital stock under the assumption of no knowledge diffusion. Aggregation of R&D capital stocks over different vintages and across different types is discussed in this section. The differences of our treatment in productivity analysis from the conventional treatment are presented in the fifth section. Section 6 provides a brief conclusion.

[2] For more detail discussion, refer to Shanks and Zheng (2006).

2. Treatment of R&D Capital in the Growth Accounting Framework

The main objective of R&D adventure is to create new knowledge for future applications. Therefore, we can treat R&D investment as a process of knowledge production, which creates new knowledge to be used for the future knowledge creation as well as for the general production procedure. In another word, an R&D project produces two types of output: additions to the stock of knowledge and new technologies that can be product innovation or process innovation. In general, we can assume that each economic unit conducts two different activities: *knowledge production* and *non-knowledge production* (or *ordinary product production*). Although the available knowledge (pool of knowledge) is same for both activities, the effective knowledge is different for these two activities. *Knowledge production* works on the existing ideas and produces new ideas. The existing knowledge stock, accumulating as the creation of new knowledge, acts as an object being processed in the knowledge production. On the contrary, the *ordinary product production* transforms the intermediate inputs other than knowledge into a desired product by applying some technologies that are treated as the technological shifter. Therefore, we can see that knowledge stock plays different roles in the two different production activities. As long as firms invest in R&D activities, they will create a certain amount of new knowledge that can be used for the next round of knowledge production; but it may be incomplete and not ready for the ordinary product production. Generally speaking, the lag of knowledge stock serving for knowledge production is shorter than that of the knowledge stock used by ordinary production. To better incorporate R&D expenditures into growth accounting framework, we propose to distinguish between the knowledge stock, or the stock of R&D capital, for *knowledge production* and *non-knowledge production*. Let R_t^K and R_t^{NK} denote the stock of R&D capital for knowledge production and for non-knowledge production at the end of time t, respectively. Because knowledge production creates knowledge products for the whole economy, we can include the knowledge product into the aggregate output. If Y_t^K represents the aggregate output including knowledge product at the end of time t, C and L represents capital and labour inputs excluding R&D expenditures, $I_{R,t}$ represents the R&D investment in period t, then the aggregate production function can be written as:

(1) $Y_t^K = F(C, L, I_R; t, R_{t-1}^K, R_{t-1}^{NK})$

where t is the time variable that captures the general technical evolvement. The knowledge stock variables, R_{t-1}^K and R_{t-1}^{NK}, capture the change of technological efficiency due to the accumulation of knowledge capital. Both time variable and R&D capital stock variables represent the shift in the production function. We differentiate knowledge stock for knowledge production and for non-knowledge production in the aggregate production function, because these knowledge stocks play different roles in the production function. R&D investment during period t, $I_{R,t}$, is treated as one input factor used to produce knowledge product.

We are also interested in the production of the ordinary product. Let Y_t denote the aggregate output excluding knowledge products, thus, we can write the corresponding aggregate production function as follows:

(2) $Y_t = F(C, L; t, R_{t-1}^{NK})$

Note that the treatment of R&D capital determines the size of the R&D variable and how it enters the aggregate production function, and this would have impact on the productivity analysis. Totally differentiating equation (2) with respect to time t and dividing both sides by Y_t, we have the following equations after some rearrangements:

(3)
$$\begin{aligned}\frac{\dot{Y}}{Y} &= \frac{\partial F}{\partial C}\frac{C}{Y}\frac{\dot{C}}{C} + \frac{\partial F}{\partial L}\frac{L}{Y}\frac{\dot{L}}{L} + \frac{\partial F}{\partial R_{t-1}^{NK}}\frac{R_{t-1}^{NK}}{Y}\frac{\dot{R}_{t-1}^{NK}}{R_{t-1}^{NK}} + \frac{1}{Y}\frac{\partial F}{\partial t} \\ &= \eta_C \frac{\dot{C}}{C} + \eta_L \frac{\dot{L}}{L} + \eta_R \frac{\dot{R}_{t-1}^{NK}}{R_{t-1}^{NK}} + \frac{1}{Y}\frac{\partial F}{\partial t}\end{aligned}$$

where dots indicate time derivatives. η_X denotes the output elasticity with respect to input X. Equation (3) provides us an explanation about how different elements contribute to the change of the aggregate output growth. Based on this equation, we can also conduct productivity analysis. The residual resulted from regressing the output growth rate on the change rate of capital input, labour input, and R&D capital stock is used as the measure of the productivity in the growth accounting framework. From equation (3), we can see that the treatment of R&D capital stock would have some impact on the estimation of productivity. We will investigate this impact later in this paper.

The main difficulty of applying this growth accounting framework is constructing the proper measures of the stock of the aggregate R&D capital for both knowledge production and non-knowledge production, denoted by R_t^K and R_t^{NK}, respectively. In the following part of this paper, we focus on how to construct the appropriate measure of the aggregate R&D capital stock for both knowledge production and non-knowledge production. Two different aspects of the aggregation are discussed in the paper, namely the aggregation over different R&D vintages and the aggregation across knowledge stocks from different sectors.

3. Conceptual Issues

Before we go further into the basic methodology for constructing an appropriate measure of the aggregate R&D capital stock, we need to clarify a few conceptual issues.

3.1 R&D Capital and Stock of R&D Capital

R&D capital is defined as the knowledge asset created by R&D investment in each period. It is the incremental part of the knowledge stock. Thus, the stock of R&D capital is the accumulation of knowledge asset, which forms a good proxy for knowledge stock. The question is what should be included in the stock of knowledge. Should we include all the knowledge created in the history? To answer this question, we should base on the definition of R&D capital. Note that R&D capital is defined as one type of asset. To be qualified as an asset, R&D capital should be able to generate economic benefits for its owner. R&D investment has the potential of creating new technology. The new technology may be used to produce a new product or a quality-improved product that can be either a final good or an intermediate good; it can also be a cost-saving new process. With a successful R&D venture, the performer can benefit from newly created knowledge through the following channels: (a) commercializing the new technology through patenting. Thus, the inventor can benefit from selling or leasing the right of using new knowledge. This benefit is due to the exclusive right of the owner to new knowledge and the high efficiency associated with the new technology. Because of the difficulty of maintaining the exclusive right and the obsolescence of knowledge, the incremental income accrued from new knowledge will decrease as time goes by. (b) Employing new knowledge in the production process and grabbing the monopolistic profit from the newly emerging market stimulated by the new knowledge, or enjoying a higher profit margin because of the improvement of efficiency. Monopolistic profit may shrink over time as the diffusion of knowledge. However, as long as the firm still use the knowledge, it still benefit from the utilization of knowledge. Therefore, we propose that the stock of R&D capital should include all knowledge assets that are created by R&D investment and are still utilized by an economic unit. This means that R&D capital stock should include patent, business secrecy and freely used knowledge. The useful life of R&D capital depends on its efficiency and utilization level.

3.2 R&D Capital versus Knowledge Capital

As we have pointed out before, the stock of R&D capital can be regarded as the proxy of the stock of knowledge capital. However, R&D capital is not exactly equal to knowledge capital. Although R&D investment can create a lion's share of new knowledge, other sources, such as "learning by doing" or "education" process can generate knowledge too. Therefore, the stock of R&D capital can be a good representative for knowledge stock, whereas the scope of knowledge stock is wider than that of R&D stock. When we try to make some implications about knowledge capital from the analysis of R&D capital, we should be aware of its limitation. In this paper, we only consider knowledge capital created by R&D investment. Without further specification, knowledge capital actually means R&D capital in our paper.

3. 3 Aggregate R&D capital stock

In this paper, the stock of R&D capital is the accumulation of disembodied knowledge created by R&D investments. For general production, it can be regarded as a technological index that indicates the efficiency level of knowledge or the position of production frontier. Aggregate R&D stock, which can be the aggregation of same type of knowledge from different vintages or the aggregation across different types of knowledge, is designed to reflect the average efficiency level of the technology. When we talk about the aggregation of R&D capital stock, we do not concern about the value, or price concept of R&D capital, but focus on the ***real quantity*** of R&D capital.

R&D capital belongs to the family of intangible assets. Un-observability of this type of asset causes many difficulties in its measurement. To document the role of R&D within the economy, it would be better to measure the output of R&D— new knowledge it creates. However there are difficulties in measuring the created knowledge. First, we lack a direct measure of knowledge capital created by R&D investment. Patent data can provide a measure for some R&D outputs, but they have significant limitations. Shanks and Zheng (2006) pointed out that patent protection tended to be used more for product innovation than for process innovation, many new knowledge simply could not be patented, in addition some R&D-performing industries did not prefer to use patents. Second, the future expected yields accrued from newly created knowledge are generally not observable due to the lack of future markets for these knowledge assets. In addition, there is always technical progress embodied in the different vintages of knowledge assets. Thus it is impossible for us to observe the prices of the exactly same knowledge assets of different ages at the same time. Because of the limitation of the information and the special features of knowledge assets, we use the information about R&D expenditures to construct a so-called ***input-based measure*** of R&D capital stocks.

3.4 Problems in Measuring the Stock of the Aggregate R&D Capital

To construct a proper measure for the stock of aggregate R&D capital, we have to face the following problems.

First, there is a problem of applying the usual index number theory. Knowledge assets have different contents, different efficiency levels, and more importantly, they are unobservable. How can we measure the aggregate R&D capital stock? Usually, the aggregation procedure involves applications of the index number theory, which require positive price. If the prices of some items are zero, these items are not taken into account in the aggregation. In the case of aggregation of R&D stock, if we apply the usual index number theory directly, we fail to include knowledge that can be freely used by the economic agents. In order to construct the right measure of the aggregate R&D capital stock, we cannot directly apply the usual index number theory without making some modification.

Second, we have the problem of incorporating two opposite forces— *depreciation and up scaling,* in the aggregation procedure. As knowledge capital accumulates over time, old knowledge becomes obsolescent due to the emergence of new knowledge. Obsolescence of an outdated knowledge may cause the depreciation of knowledge capital, in the mean time, new knowledge increases the stock of R&D capital. Usually, the retirement of old knowledge and the introduction of new technology imply the improvement of the aggregate technological level. In other words, except for the depreciation of knowledge capital, there are quality adjustments associated with the newly updated knowledge. Thus, in the aggregation of knowledge capital stock, there are two forces playing opposite roles in constructing the aggregates of knowledge capital. Depreciation reduces knowledge stock and up-scaling increases the stock. In order to aggregate knowledge capital properly, we should find an appropriate way to incorporate these two forces.

Third, we have to deal with the problem associated with knowledge diffusion—*positive knowledge spillovers*. R&D investment is an important method for firms to achieve a higher level of technological efficiency, but firms not investing in R&D can still on the cutting edge of the technology. How can they do that? They can purchase, borrow or even steal the new technology. Spillover effect arises whenever the firms undertaking R&D investment are not able to fully appropriate all of the benefits from their research results. With the existence of the positive R&D externality, how should we aggregate firms' R&D stock into one aggregate knowledge stock that indicates the average technical efficiency level of the industry? Therefore, another big problem we have to face in the aggregation is to deal with knowledge spillover within industries.

Finally, there is a problem of *knowledge similarity*. In some circumstances, two firms in the same industry may create similar knowledge. If knowledge stock reflects the technological efficiency level of knowledge, how to aggregate similar knowledge into an aggregate knowledge capital stock is an interesting topic worth exploring.

As we have pointed out before, the aggregation of R&D capital stock includes the aggregation over different vintages and the aggregation across different types of knowledge. In this paper, we only discuss the R&D capital aggregation problems *without knowledge diffusion*[3].

4. Measuring the Stock of the Aggregate R&D Capital without Knowledge Diffusion

In this section, we discuss how to construct proper measures for the aggregate R&D capital stock under the assumption that there is **no knowledge diffusion**. This means that each firm has to rely on its own R&D activity to improve its technological efficiency and apply its own R&D capital in the production procedure. We still assume that each firm conducts two different activities, namely *knowledge production* and *non-knowledge production*. In order to construct the right measure for R&D capital stock, we start with measuring the knowledge product or R&D capital, which is the output of knowledge production and the basic building block of formulating the stock of R&D capital.

4.1 Measurement of Knowledge Product

R&D capital is the output of knowledge production or R&D investment. Unlike the ordinary physical capital, which is formulated just by purchasing and can be employed without further productive activity, R&D capital is created by R&D investment through a "production procedure". The underlying technology of creating new knowledge can be represented by a *knowledge production function*, which is a function of general input factors, such as labour input, capital input and intermediate input. In addition, knowledge production also relies on the stock of ideas—the existing knowledge stock, which determines "technological opportunities". The existing stock of knowledge capital plays a vital role in determining the productivity of the knowledge production. Let K_t denotes

[3] Aggregation problems with knowledge diffusion will be discussed in the future work..

new knowledge product created at the end of time t, then we can write the knowledge asset created by R&D investment during period t in the following form:

(6) $K_t = G(I_{R,t}, R_{t-1}^K, t)$

where G(.) is the knowledge production function that is determined by R&D investment in period t, $I_{R,t}$, effective R&D stock for knowledge production in the previous period, R_{t-1}^K, and time variable t. R&D investment can be decomposed into components, such as labour input, capital input and material input. We use input vector x_t to represent these input factor in the knowledge production function. Replacing $I_{R,t}$ by x_t, we rewrite the knowledge production function as follows:

(7) $K_t = G(x_t, R_{t-1}^K, t)$

Based on our understanding of knowledge creation process, we impose the following general assumptions on the knowledge production function:

- Knowledge production function increases in the inputs. For example, generally speaking, skilled labour accounts a large portion of R&D expenditure. Both quality and quantity of the skilled person are important elements determining the productivity of knowledge creation. The higher quality of the professional personnel (represented by a higher wage), the higher chance for the firm to create new knowledge. Similarly, a scientific project conducted by more personnel that are professional would be more likely to be successful. Therefore, we can say that increase in the inputs would increase the likelihood of creating new knowledge.
- Knowledge production function is assumed to increase in the existing knowledge stock. The existing knowledge is another important factor determining the productivity of the knowledge production; however, the role it plays in the knowledge creation is subtle. Because the current researchers "stand on the shoulders" of the previous researchers, usually we would expect to see that the more R&D stock is in position, the higher probability of creating new knowledge would be. However, in the meantime, the more R&D stock in position implies the chance of finding new technology is small. Knowledge capacity determines the relationship between the knowledge creation and existing knowledge. The newly created knowledge can be some important breakthrough or just small improvements. The possibility of a technological breakthrough will increase with the accumulation of knowledge capital, while the possibility of improvement-type knowledge will decline with the accumulation of knowledge. Therefore, it is difficult for us to fully incorporate the impacts of existing ideas on the knowledge creation into our knowledge production function. To avoid this complexity, we just

assume that knowledge creation is increasing in the existing knowledge stock[4].

Regarding to the choice of the function form of knowledge production, we think it is desirable to choose a flexible functional form. Although in the literature, the Cobb-Douglas knowledge function is a popular choice, we think flexible functional form is a better choice.

In order to work out an explicit index number formula for constructing the measure of the newly created knowledge product at each period, we impose some restrictive assumptions on the knowledge production function. Every R&D activity performer is assumed to minimize the cost of creating new knowledge with a given amount of output level. Then the total cost of knowledge production can be written as follows:

$$(8) \quad C(K_t, w_t) \equiv \min_x \{w_t \cdot x_t : G(x_t, R_{t-1}^K, t) = K_t\}$$

where w_t is a positive vector of input prices. If we assume that knowledge production function G(.) is a nondecreasing, linearly homogenous and concave function, given a positive input prices vector $w_t = (w_{t,1}, w_{t,2}, ..., w_{t,N})$, the solution to the cost minimization problem, total cost function $C(K_t, w_t)$, can be decomposed into a unit-cost function $c(w_t)$ times the output level, that is:

$$(9) \quad C(K_t, w_t) \equiv \min_x \{w_t \cdot x_t : G(x_t, R_{t-1}^K, t) = K_t\} = c(w_t)K_t$$

where the unit cost function can be defined as:

$$(10) \quad c(w_t) \equiv \min_x \{w_t \cdot x_t : G(x_t, R_{t-1}^K, t) = 1\}$$

Given the price and quantity data for two periods, w_0, w_1, x_0, x_1, we can define a price index and a quantity index as a function P and Q of prices and quantities, $P(w_0, w_1, x_0, x_1)$, and $Q(w_0, w_1, x_0, x_1)$, respectively. We generally assume that the price and quantity indexes satisfy the "product test"; that is, P and Q satisfy:

$$(11) \quad P(w_0, w_1; x_0, x_1)Q(w_0, w_1; x_0, x_1) = w_1 \cdot x_1 / w_0 \cdot x_0$$

[4] We believe that this assumption makes sense at most time.

Investigating the knowledge production function given by equation (7), we found out that the existing knowledge stock, R_{t-1}^{K}, has important impact on deriving the specific formula for the measure of new knowledge created at each period. In the following part of this section, we will try different assumptions about the role of the existing knowledge stock playing in the knowledge production function.

First, the existing idea can be treated as an additional input factor in the knowledge production function. As we have pointed out before, the main function of the knowledge production is transforming the existing idea into new knowledge. The existing knowledge is actually the "object" being processed in the knowledge production. Treating the existing idea as an input, we can write the knowledge production function as follows:

$$(12) \quad K_t = G(x_t, t)$$

where input vector x_t includes the existing knowledge as an additional input. If we use trans-log functional form as our knowledge production function, it can be written as:

$$(13) \quad \begin{aligned} \ln K_t &= \ln G(x_t, t) \\ &= \alpha_0 + \sum_{n=1}^{N} \alpha_n \ln x_{t,n} + (1/2)\sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_{ij} \ln x_{t,i} \ln x_{t,j} + \beta_0 t + \sum_{n=1}^{N} \beta_n t \ln x_{t,n} + \gamma t^2 \end{aligned}$$

where $K_t = G(x_t, t)$ is the new knowledge created during period t, and x_t = $(x_{t,1}, x_{t,2}, ... x_{t,N})$ is a vector of input used by the firm during period t. With the following restrictions on the parameters,

$$(14) \quad \begin{aligned} &\sum_{n=1}^{N} \alpha_n = 1,\ \alpha_{ij} = \alpha_{ji},\ \sum_{i=1}^{N} \alpha_{ij} = 0,\ \text{for j} = 1,2,..., \text{N and} \\ &\sum_{n=1}^{N} \beta_n = 0 \end{aligned}$$

G(.) defined by equation (13) is linearly homogeneous in x, and the resulting function can provide a second-order approximation to an arbitrary twice continuously differentiable function of (x, t) that is linearly homogeneous in x.

Equation (13) is also a quadratic form, thus we can apply Diewert Quadratic Identity and get the following equations:

$$
\begin{aligned}
&\ln G(x_1,t_1)-\ln G(x_0,t_0)\\
(15)\quad &=(1/2)[\hat{x}_1\nabla_x \ln G(x_1,t_1)+\hat{x}_0\nabla_x \ln G(x_0,t_0)]\cdot(\ln x_1-\ln x_0)\\
&+(1/2)\left[\frac{\partial \ln G(x_1,t_1)}{\partial t}+\frac{\partial \ln G(x^0,t^0)}{\partial t}\right]\cdot(t_1-t_0)
\end{aligned}
$$

where $\hat{x}_1 \equiv$ the vector x_1 diagonalized into a matrix, and $\hat{x}_0 \equiv$ the vector x_0 diagonalized into a matrix. If we assume that knowledge producer faces the input price vector $w_0 >> 0_N$, $w_1 >> 0_N$ during periods t_0 , t_1 and he competitively minimizes costs, then we can derive the following identity:

$$
(16)\quad \nabla_x \ln G(x_0,t_0)=w_0/w_0\cdot x_0;\ \ \nabla_x \ln G(x_1,t_1)=w_1/w_1\cdot x_1
$$

Substituting (16) into (15) yields the following equation:

$$
\begin{aligned}
&\ln K_{t_1}-\ln K_{t_0}=\sum_{n=1}^{N}[s_{1,n}+s_{0,n}]\ln(x_{1,n}/x_{0,n})\\
(17)\quad &+(1/2)\left[\frac{\partial \ln G(x_1,t_1)}{\partial t}+\frac{\partial \ln G(x_0,t_0)}{\partial t}\right]\cdot(t_1-t_0)
\end{aligned}
$$

where $K_{t_r}=G(x_r,t_r)$ and $s_{r,n}=w_{r,n}x_{r,n}/w_r\cdot x_r$ for r = 0,1 and n = 1, 2,…,N. Rearranging and exponentiating equation (17), we can obtain the following relationship:

$$
(18)\quad \frac{K_{t_1}}{K_{t_0}}=Q_T(w_0,w_1,x_0,x_1)\exp\left\{(1/2)[\frac{\partial \ln G(x_1,t_1)}{\partial t}+\frac{\partial \ln G(x_0,t_0)}{\partial t}](t_1-t_0)\right\}
$$

where $Q_T(w_0,w_1,x_0,x_1)$ is the Törnqvist quantity index in inputs. The exponential part of the right-hand side of equation (18) represents a theoretical expression for the cumulative effects of technical progress on knowledge production. Equation (18) gives us a theoretical expression for constructing a measure of the knowledge capital. The difficulties of applying this equation include: (1) Determine the quantity of existing knowledge and its corresponding price; (2) Determine the effects of technical progress.

Second, instead of treating existing ideas as one additional input factor, we assume that knowledge production is separable, that is our knowledge production can be written in the following form:

$$
(19)\quad K_t=G\left(x_t,R_{t-1}^K,t\right)=g(x_t,t)h(R_{t-1}^K)
$$

where $g(x_t, t)$ is the knowledge production function in terms of variable inputs, while h(.) is a function of existing ideas; it reflects the effect of existing knowledge on knowledge creation. Again we choose trans-log functional form for $g(x_t, t)$. Applying Diewert Quadratic Identity, we can derive the following relationship:

$$(20)\quad \begin{aligned} &\ln g(x_1, t_1) - \ln g(x_0, t_0) \\ &= \ln Q_T(w_0, w_1, x_0, x_1) + (1/2)\left[\frac{\partial \ln g(x_1, t_1)}{\partial t} + \frac{\partial \ln g(x_0, t_0)}{\partial t}\right]\cdot(t_1 - t_0) \end{aligned}$$

Substitute equation (19) into equation (20) and exponentiate both sides of equation (20), we have:

$$(21)\quad \frac{K_{t_1} / h(R^K_{t_1-1})}{K_{t_0} / h(R^K_{t_0-1})} = Q_T(w_0, w_1, x_0, x_1)\exp\left\{(1/2)[\frac{\partial \ln g(x_1, t_1)}{\partial t} + \frac{\partial \ln g(x_0, t_0)}{\partial t}](t_1 - t_0)\right\}$$

Then we can derive a general formula for constructing the measure of newly created R&D asset:

$$(22)\quad \frac{K_{t_1}}{K_{t_0}} = Q_T(w_0, w_1, x_0, x_1)\exp\left\{(1/2)[\frac{\partial \ln g(x_1, t_1)}{\partial t} + \frac{\partial \ln g(x_0, t_0)}{\partial t}](t_1 - t_0)\right\}\left(\frac{h(R^K_{t_1-1})}{h(R^K_{t_0-1})}\right)$$

From equation (22), we can see that the ratio of new knowledge created during two different periods is determined by three parts: (1) Törnqvist quantity index in inputs, which can be calculated using observable price and quantity data obtained from the R&D expenditure information; (2) Effect of general technical progress on the productivity of knowledge production; (3) Effect of existing knowledge stock on knowledge creation. In the existing literature, only the first part is taken into consideration. Usually, the effects of technical progress and existing ideas are ignored or assumed to be constant. This treatment can simplify the problem, but may cause measurement errors. Even though the general measure of R&D capital defined by equation (22) reflects knowledge creation more closely, we have difficulties in applying it due to the data limitation. Difficulties associated with applying equation (22) include: (1) Estimating the effect of general technical progress; (2) Determining the effect of existing knowledge or the functional form of the *h* function. We cannot evaluate these effects by estimating the knowledge production function due to the un-observability of knowledge stock. An alternative way of solving this problem is to find some approximates for these effects through other channels.

In the following part, we discuss how to deal with the difficulties associated with applying equation (22).

If the time interval between t_1 and t_0 is short, we may not expect to see significant changes in the technology of producing new knowledge, thus we may assume that:

(23) $$\frac{\partial \ln g(x^1, t^1)}{\partial t} \approx 0, \text{ and } \frac{\partial \ln g(x^0, t^0)}{\partial t} \approx 0$$

(24) $$\frac{h(R_{t_1-1}^K)}{h(R_{t_0-1}^K)} \approx 1$$

Thus, for two close periods we can have:

(25) $$\frac{K_{t_1}}{K_{t_0}} \approx Q_T(w_0, w_1, x_0, x_1)$$

This implies that if we use chained quantity index, our measure of R&D capital may has relatively small measurement error. That is we can construct a measure of newly created R&D capital at each period using the following equations:

$$\begin{aligned}
&K_0 = 1 \\
&K_1 = Q_T(w_0, w_1, x_0, x_1) \\
(25)\quad &K_2 = Q_T(w_1, w_2, x_1, x_2)K_1 = Q_T(w_1, w_2, x_1, x_2)Q_T(w_0, w_1, x_0, x_1) \\
&K_3 = Q_T(w_2, w_3, x_2, x_3)K_2 = Q_T(w_2, w_3, x_2, x_3)Q_T(w_1, w_2, x_1, x_2)Q_T(w_0, w_1, x_0, x_1) \\
&\ldots
\end{aligned}$$

Another possible way of finding an approximate for the general technology changes in producing knowledge is to estimate the knowledge production function using patent data. Although patents are only a part of the new knowledge created in each year, it may still provide us some rough picture about the general tendency of technology changes in the knowledge production. The information required for the estimation includes:

(1) Number of new patents in each year;
(2) R&D expenditures of the corresponding industries (or firms) in each year;
(3) Other possible factors affecting the increase of new patents in each year (time trend etc)

Note that the factors affecting the number of patents are not limited to the changes in productivity of creating new knowledge. There are many other factors, such as the change of patent policy, playing similar rules in the application of patent. Therefore, we should be wary of the limitation of using patent data.

The determination of h function is another problem we need to solve for applying equation (22). h function reflects how exactly the existing ideas (stock of the current knowledge) affect the knowledge creation. The problem is what kind functional form we should choose for this h function. To answer this question, let us consider the basic properties that h function should have, based on the process of creating new knowledge:

1) Because the existing knowledge stock is constructed based on its relevance to the new knowledge, it is reasonable to assume that $h'(\cdot) > 0$, that is the more relevant of the existing idea, the higher productivity of creating new knowledge is.
2) h function acts as an efficiency index that indicates the productivity of creating new knowledge. Being an index, the value of h function should indicate the *relative efficiency level* of producing new knowledge. Therefore, it should be non-negative, that means $h(\cdot) \geq 0$ for any given level of knowledge stock.
3) In knowledge production function, h function adjusts the quantity of newly created knowledge product. It scales R&D capital up at different degrees based on its level. Because the current R&D investment plays the major role in creating new knowledge product, we do not want a huge adjustment to the quantity of the new knowledge product due to the scale of the existing knowledge stock. Therefore, the value of h function should not be a very large number, although the stock of existing knowledge is of a large scale. This implies that we need some functional form that can transform a large number into a small positive number.

Based on the above discussion, one possible candidate for h function is:

(26) $h(R_{t_1-1}^K) = (R_{t_1-1}^K)^{1/\alpha}$

where α should be greater than 1.

In this paper, we come up with a new idea that the knowledge stock serving in knowledge creation and knowledge stock serving in general product production are different. In the following part of this section, we focus on how to construct the separate measure of these two types of knowledge stock.

4.2 Aggregate R&D Capital Stock over Vintages

R&D capital of different vintages reflects the same type of knowledge created at different periods. There are always changes in technological efficiency embodied in the new knowledge asset. Thus, the aggregation of R&D capital over vintages is to aggregate same type of knowledge with different efficiency levels over time. To simplify our question, we assume that each firm creates *same* type knowledge over time. Therefore, at a certain point in time, it employs different knowledge created at different time. In order to construct the measure of R&D capital stock, which is the accumulation of the knowledge assets created by R&D investment, we need to aggregate these knowledge assets of different vintages into one number.

Again, we assume no knowledge spillover effect in the aggregation. Let K_t denote the knowledge product created during period t, thus the available knowledge can be used by each firm at the end of time t (or at the beginning of time t+1) can be written as {K_t, K_{t-1}, K_{t-2},…K_1, and R_0}, where R_0 denotes the initial R&D stock available to the firm. Let R_t^K and R_t^{NK} denote the R&D stock for knowledge production and ordinary product production at the *end of time t*, respectively. The superscript to the right indicates whether the R&D stock is for knowledge production or for normal production. We can construct R_t^K in the following way:

$$(27)\quad R_t^K = \mu_0^t K_t + \mu_1^t K_{t-1} + \mu_2^t K_{t-2} + \mu_3^t K_{t-3} + ... + \mu_{t-1}^t K_1 + \mu_t^t R_0$$

where μ_i^t is the relevancy index of i year old knowledge at period t. Relevancy index reflects to what extent the existing knowledge is related to the creation of new knowledge. Usually, we assume that the more updated the knowledge is, the more relevant it will be to the new knowledge creation. That is:

$$(28)\quad \mu_0^t \geq \mu_1^t \geq \mu_2^t \geq \mu_3^t \geq ...\mu_{t-1}^t \geq \mu_t^t$$

The effective knowledge stock for non-knowledge production, R_t^{NK}, which reflects the average efficiency level of a firm's technology, can be constructed in the following way:

$$(29)\quad R_t^{NK} = u_0^t e_0^t K_t + u_1^t e_1^t K_{t-1} + u_2^t e_2^t K_{t-2} + ... + u_{t-1}^t e_{t-1}^t K_1 + u_t^t e_t^t R_0$$

Where e_i^t denotes the time t efficiency index of the knowledge with age i. The efficiency indexes in equation (29) transform all the knowledge created at different time into the knowledge with same efficiency level. We may name these transformed knowledge as *efficiency adjusted knowledge*. u_i^t denotes the utilization index for the knowledge asset of i year age at time t. Utilization index reflects the utilization level of the knowledge.

Theoretically, the aggregate stock of R&D capital over vintages can be constructed based on equation (27) and equation (29). However, the problem is that the important factors determining the stock of knowledge capital, such as relevancy indexes, efficiency indexes and utilization indexes, are not observable. In the following part of this section, we discuss how to determine the relevancy indexes, efficiency indexes and utilization indexes.

4.2.1 Relevancy Index

According to equation (27), the sequence of aggregate R&D capital stock for knowledge production can be written as follows:

$$
(30)\quad
\begin{aligned}
&R_t^K = \mu_0^t K_t + \mu_1^t K_{t-1} + \mu_2^t K_{t-2} + \mu_3^t K_{t-3} + ... + \mu_{t-1}^t K_1 + \mu_t^t R(0) \\
&R_{t-1}^K = \mu_0^{t-1} K_{t-1} + \mu_1^{t-1} K_{t-2} + \mu_2^{t-1} K_{t-3} + ... + \mu_{t-2}^{t-1} K_1 + \mu_{t-1}^{t-1} R(0) \\
&R_{t-2}^K = \mu_0^{t-2} K_{t-2} + \mu_1^{t-2} K_{t-3} + \mu_2^{t-2} K_{t-4} ... + \mu_{t-3}^{t-2} K_1 + \mu_{t-2}^{t-2} R(0) \\
&...
\end{aligned}
$$

The superscript of the relevancy index, t-i, i = 0, 1,2,…, indicates that the corresponding indexes are valid at period t-i. The subscript of the index indicates the age of the specific knowledge at period t-i. Note that the relevancy index is designed to reflect the relevancy of a specific type of knowledge to the knowledge that will be created during the next period. Then the age of the knowledge pulsing one actually reflects the time lag between the new knowledge and the existing old knowledge. Usually the relevancy index is knowledge specific, this characteristic might complicate the determination of the relevancy index. To simplify our problem, we may assume that the relevancy index is independent of what would be created in the future, that is the relevancy index would depend only on the time lag between the new knowledge and the existing old knowledge, or the age of the knowledge at a particular time. Based on this assumption, the following condition should hold:

$$(31)\quad \mu_l^{t^i} = \mu_l^{t^j} = \mu_l \quad \text{for } \forall t^i \text{ and } t^j$$

where t^i and t^j denote two different points in time. Equation (31) tells us that the relevancy index would be same as long as the time lag between the new knowledge and the existing knowledge is same. Applying equation (31) to equation (30) and dropping the superscript of the relevancy index, the sequence of the aggregate R&D capital stock can be written as follow:

$$(32)\quad \begin{aligned} R_t^K &= \mu_0 K_t + \mu_1 K_{t-1} + \mu_2 K_{t-2} + \mu_3 K_{t-3} + \ldots + \mu_{t-1} K_1 + \mu_t R(0) \\ R_{t-1}^K &= \mu_0 K_{t-1} + \mu_1 K_{t-2} + \mu_2 K_{t-3} + \ldots + \mu_{t-2} K_1 + \mu_{t-1} R(0) \\ R_{t-2}^K &= \mu_0 K_{t-2} + \mu_1 K_{t-3} + \mu_2 K_{t-4} \ldots + \mu_{t-3} K_1 + \mu_{t-2} R(0) \\ &\ldots \end{aligned}$$

How can we obtain the estimates of these relevancy indexes? One possible way is to use the citation information. With a well-established citation network, citations provide us relatively well information for tracing the relationship between the new knowledge and the old knowledge. The following information might be useful for us to derive the relevancy index:

- Total citations made by a patent in a certain year
- The distribution of citations according to the age of the cited knowledge

If we use N_t to denote the total citations made at time t, and $N_{t,i}$ to denote the number of citations made at time t and the citing knowledge assets that are i years old, then the relevancy index, μ_i^t, can be obtained by the following formula:

$$(33)\quad \mu_i^t = \frac{N_{t,i}}{N_t}$$

The above index is a time-specific relevancy index of the knowledge asset that is i years old. The general time independent relevancy index can be obtained by taking weighted average over these time specific indexes, that is:

$$(34)\quad \mu_i = \sum_t \mu_i^t \frac{N_{t,i}}{\sum_t N_{t,i}}$$

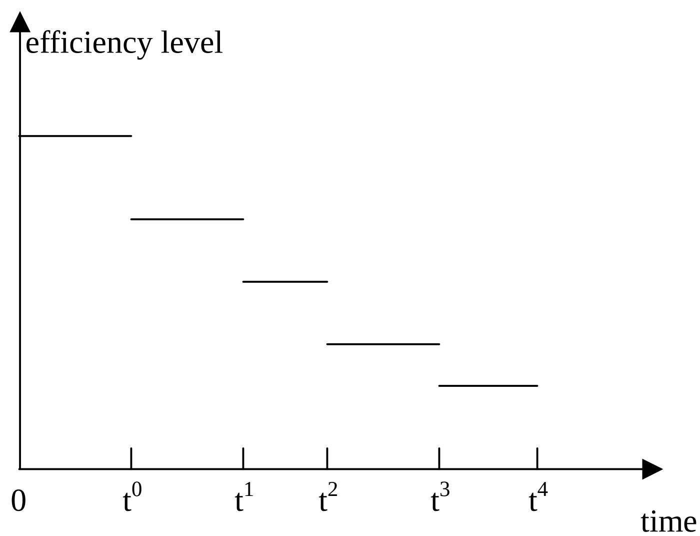
Note that it is difficult to get the accurate relevancy index, however, we may obtain close approximates with the improvement of citation information.

4.2.2 Efficiency Index and Utilization Index

The aggregate R&D stock for ordinary production depends on the utilization index and efficiency index. Generally speaking, we cannot observe the utilization level and efficiency level of a specific knowledge directly. Then, how should we construct these indexes? In the following part of this section, we attempt to explore possible ways to solve this problem.

A widely used assumption about productive efficiency of the physical capital is the geometric age-efficiency profile postulating that the efficiency of an asset declines at a constant rate. Is this a reasonable assumption for the efficiency change of R&D capital? At first glance, the answer to this question will be "no". A big difference between physical capital and R&D capital is that physical capital would wear or tear in its usage, therefore its efficiency would decline as time marches on; while the efficiency of the R&D capital would not change during its life of service. However, what really matters in aggregating R&D capital is the *relative efficiency level* of the R&D capital, because a firm's R&D capital stock for production is designed for representing the average efficiency level of its technology. The relative efficiency level of the R&D capital remains at its initial level until the emergence of a new R&D capital asset. The following figure shows how the relative efficiency level changes with time.

Figure 1: Changes in the relative efficiency level



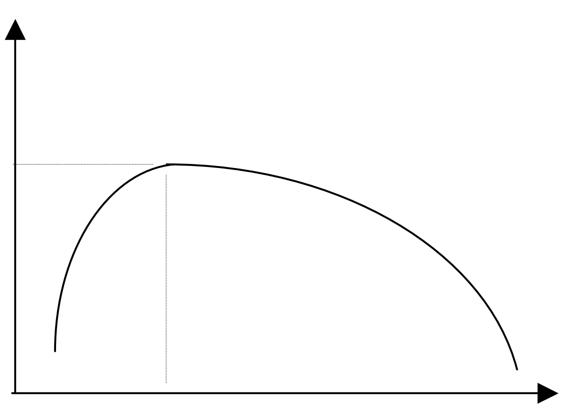
Where i^t denotes the time when the new knowledge is employed in the production. The interval between the two points in time, such as t^1-t^0, reflects the time lag between the two vintages of knowledge. The time lag actually shows the frequency of the evolvement of knowledge. When this frequency is high enough, geometric age-efficiency assumption can be applied in the case of R&D capital. Nevertheless, we need to remember that if the knowledge updating is slow or the time lag is long, using geometric age-efficiency assumption might cause measurement errors. In short, the main features associated with the changes in the efficiency level of the R&D capital include: (1) the absolute efficiency level of the R&D capital maintains same for its service life; (2) only the relative efficiency level of R&D capital maters in the aggregation of R&D capital; (3) the relative efficiency level of the existing R&D capital declines whenever there are new knowledge in position; (4) the efficiency level of the R&D capital is determined by the frequency of knowledge evolvement; (5) in the case of a high-frequency knowledge updating, geometric age-efficiency assumption can be applied to determine the efficiency level of knowledge capital.

Another index determining the weights on the different vintages of the knowledge assets is the utilization index. In order to determine the utilization level of the R&D capital, we need to make some assumptions about the utilization path or life cycle of a particular knowledge asset. Before we specify these assumptions, we list some observable facts about the application of knowledge:

- Usually, at the introductory stage of a new knowledge, the features of the new knowledge are not fully mastered by the owner, then the utilization level of a certain knowledge starts at an initial level and increases gradually later on;
- As time goes on, the properties of the knowledge are gradually mastered by its owner, thus the utilization level of the knowledge increases until reaching its peak point;
- The utilization level of a knowledge will remain at a high level for some time until another new knowledge becomes available;
- Firms have a tendency of employing more advanced technology in its production, thus the old technology would be partially replaced by a newly created technology, this implies that the utilization of the old technology would be reduced by a certain portion due to the emergence of the new technology.

Based on the above facts, we may draw the following diagram to illustrate the utilization path of a specific technology:

Figure 2: Utilization path



Usually there might be a flat part on the utilization path, to simplify the problem, we ignore this part. Thus, we assume that the utilization level of a newly created knowledge will increase at the early stage and it would start to decrease after reaching its peak point. This means that the utilization path exhibits an un-balanced bell shape. In summary, we need the following information to determine the whole path of the utilization:

(1) The initial utilization level of the R&D capital created at time t, u_0^t.

(2) The increasing rate of the utilization level at the introductory stage of the knowledge.

(3) The position of the peak point. In order to locate the peak point, we assume that the utilization of one technology will reach its maximum point when another new technology is available for use. This means that the frequency of creating new knowledge plays an important role in determine the peak point.

(4) The decreasing rate of the utilization level after it reaches the peak point;

(5) The length of the useful service life of the R&D capital, L, and the utilization level just before the old technology is fully displaced.

Due to the data limitation, we do not have all the required information to determine the utilization path. In the following part, we would like to make some simple assumptions to show how the utilization level can be determined if we have the required information. The assumptions required for the calculation include:

(1) The initial utilization index is u_0; the maximum utilization level is 1; and the utilization level just before the knowledge is fully displaced by the new knowledge is u_L;

(2) The average time lag between the old technology and the new technology is γ;

(3) The useful life of the R&D capital is L;

(4) The utilization level increases or decreases at a constant geometric rate, denoted by α and β, respectively;

With the above information, the geometric increase rate can be calculated by the following formula:

(35) $\alpha = [1/u_0]^{(1/\gamma)} - 1$

Moreover, the decrease rate can be calculated as follows:

(36) $\beta = 1 - [u_L]^{[1/(L-\gamma)]}$

Then we can determine the utilization level for the knowledge of different vintages. Let u_t denote the utilization level of the knowledge asset that is t year old, and then we can derive the utilization level by using the following formulas:

(37) $u_t = u_0(1+\alpha)^t = u_0(1/u_0)^{t/\gamma}$ for t ⩽ γ

$u_t = (1+\beta)^{t-\gamma} = (u_L)^{(t-\gamma)/(L-\gamma)}$ for γ < t ⩽ L

$u_t = 0$ For t > L

With all the required information, theoretically we could construct the separate measures of the aggregate R&D capital stock for knowledge production and for general product production according to equation (27) and (29), respectively. Because the main difference between the different vintages of knowledge assets is their efficiency level, our work focus on how to transform the knowledge with different efficiency level into those with same efficiency level[5], that is into the *efficiency adjusted knowledge.* After we construct the measure of R&D capital stock at firm level, we need to aggregate these R&D stocks across firms to construct a measure of R&D capital stock at industry level. From the next subsection, we deal with the problems associated with aggregating over different types of knowledge.

4.3 Aggregation of R&D Capital across Sectors

Based on the availability of the data, there are two possible methods for aggregating R&D capital stock across sectors[6]:

- With **aggregate data**[7] about R&D expenditures and other input-output data at industry level, we can obtain the aggregate R&D capital stock by using these aggregate data. We can just treat the industry as one firm that produces an aggregate output by applying the aggregated technology. Then the approach proposed in the above section can be applied to construct the aggregate R&D capital stock.

---

[5] We want to point out that the purpose of this paper is to establish a scientific method of constructing an appropriate measure of R&D capital; we hope to provide some suggestions for the future data collection. Then when the data is ready, we can construct the proper measure. Right now, we lack information about the R&D lag, the frequency of knowledge evolvement, the useful life of the R&D capital. Because this information is crucial for constructing the proper measure of R&D capital, we strongly recommend collecting this information if we have enough resource for this complicated work. We believe that the quality of measuring R&D capital will increase with the improvement in techniques of collecting data.

[6] Aggregating across sectors is similar to aggregating across different types of knowledge, because different firms usually use differentiated knowledge to produce their products.

[7] Usually, the aggregate data is obtained using Törnqvist index number formula.

- With **disaggregate data** of individual firms in an industry, we can obtain the aggregate R&D capital stock by aggregating disaggregate R&D capital stock across firms, or we can construct the aggregate data at industry level and then obtain the aggregate R&D capital stock using these aggregate data.

In this section, we focus on the aggregation across firms using the disaggregate data of R&D capital stock.

Assuming that there are N firms in industry J, each firm applies one type of knowledge, and its R&D capital stock at period t is given, denoted by $R^J_{n,t}$, for n = 1, 2,N, then the aggregate R&D capital stock for the industry can be constructed by aggregating over these disaggregate R&D capital stocks. Note that we do not distinguish R&D stock for knowledge production from that for ordinary product production in this section, because the aggregating processes are same for both of them. To aggregate R&D stock across firms or different types of knowledge, we need to apply index number theory that requires price and quantity information. The stock of R&D capital offers us quantity information, whereas the price of the R&D capital stock is not observable. If there were complete markets for R&D capital, purchase price or rental price for R&D capital service could be directly observed. However, this is not the case for R&D capital. As we have mentioned before, R&D capital is defined as the knowledge asset created by R&D investment, it can be in form of patent, business secrecy or free knowledge. We can observe the price for a patent, whereas we cannot have information about the price of business secrecy and the price of the free knowledge is zero. In order to apply the index number theory, we need to find a replacement of the price. Borrowing the idea of aggregating ordinary physical capital, we may construct the user cost of R&D capital service to replace the price in the index number formula. That is we can use "firm-specific user cost shares" as weights to aggregate across R&D capital stock from different firms[8]. If we use $v_{n,t}$ to denote the user cost of R&D capital for firm n, then we can use the following formula to aggregate R&D capital across sectors in industry J:

$$(38)\quad \ln(R^J_t / R^J_{t-1}) = \sum_{n=1}^{N} (1/2)(s^J_{n,t} + s^J_{n,t-1}) \ln(R^J_{n,t} / R^J_{n,t-1})$$

where $s^J_{n,t}$ and $s^J_{n,t-1}$ are the firm-specific user cost share of firm n's R&D capital stock and can be defined as:

[8] This approach is used in the case of physical capital to aggregate across services from the different types of assets. For more information, please refer to Paul Schreyer (2003).

(39) $s^J_{n,t} = \frac{v_{n,t} \times R^J_{n,t}}{\sum_{n=1}^{N} v_{n,t} \times R^J_{n,t}}$ and $s^J_{n,t-1} = \frac{v_{n,t-1} \times R^J_{n,t-1}}{\sum_{n=1}^{N} v_{n,t-1} \times R^J_{n,t-1}}$

The crucial element for aggregating across sectors is to find the user cost of R&D capital stock for different firms. In order to derive the formula for calculating the user cost, we need to understand what exactly the user cost represents. In the setting of ordinary physical capital, user cost of the capital is the price paid for the service generated from the physical capital. Sometimes it is used as a synonymous terminology as the "rental price" of an asset. Usually the user cost of the physical capital good is determined by the rate of return to the asset, the depreciation rate of the asset and the price change of the asset. Because we cannot directly observe the price for R&D capital at different periods, we attempt to derive user cost of R&D capital from the available information.

In order to derive the user cost of R&D capital, we make some assumptions to simplify our problem. Free entry condition is assumed for firms conducting R&D activities. Then each firm earns zero benefit from creating new knowledge. This means that the cost for producing one unit of R&D capital stock equals the discounted expected future benefits generated by the knowledge stock. Assume that facing current aggregate input price (at time t), $P_{r,t}$, firm n needs $I_{r,n,t}$ units of aggregate inputs to produce one unit of R&D capital stock. The price for using one unit of R&D capital stock that is i year's old from time t is the unit user cost, denoted by $v_{n,t+i}$. Then we have the following equilibrium condition at time t:

(40) $P_{r,t} I_{r,n,t} = \frac{v_{n,t+1}}{1+r} + \frac{v_{n,t+2}}{(1+r)^2} + \frac{v_{n,t+3}}{(1+r)^3} + \ldots \frac{v_{n,t+L-1}}{(1+r)^{L-1}}$

where r is the constant nominal discount rate and L is the length of R&D capital's useful life. At time t+1, we only have (1-δ_n) units R&D capital stock left because of the impact of depreciation, where δ_n represents the depreciation rate applied for firm n. Consequently firm n only need $I_{r,n,t}(1-\delta_n)$ aggregate inputs to produce (1- δ_n) units of R&D capital stock. Therefore, the equilibrium condition at period t+1 can be written as follows:

(41) $P_{r,t+1} I_{r,n,t}(1-\delta_n) = \frac{v_{n,t+2}}{1+r} + \frac{v_{n,t+3}}{(1+r)^2} + \frac{v_{n,t+4}}{(1+r)^3} + \ldots \frac{v_{n,t+L-1}}{(1+r)^{L-2}}$

The left hand side of equation (41) gives us the cost of producing same amount of R&D capital stock at time t facing price at period t+1. Multiplying both sides of equation (40) by $(1+r)$ and then subtracting equation (41) from it, we have an expression for user cost of firm n's R&D capital stock at time t+1:

(42) $$v_{n,t+1} = \left[(1+r)P_{r,t} - P_{r,t+1}(1-\delta_n)\right]I_{r,n,t}$$

From equation (42), we can see that the user cost of R&D capital actually relies on the technology of creating knowledge asset. If the technology of creating knowledge asset does not change over time, we can assume that one unit of R&D investment can produce one unit of R&D capital stock, thus the expression for user cost can be simplified as follows:

(43) $$v_{n,t+1} = \left[(1+r)P_{r,t} - P_{r,t+1}(1-\delta_n)\right]$$

Similarly, the user cost at time t can be written according to equation (43), that is, we have:

(44) $$v_{n,t} = \left[(1+r)P_{r,t-1} - P_{r,t}(1-\delta_n)\right]$$

From equation (43) and (44), we can see that the user cost of R&D capital also depends on the previous aggregate price of the input factors. If we assume a constant rate of aggregate input price change, ξ, for the industry J, then we can write the previous aggregate input prices, $P_{r,t-m}$, in term of the current price, $P_{r,t}$, that is:

(45) $$P_{r,t-m} = (1+\xi)^{-m} P_{r,t} \qquad \text{for m=1,2,3,…}$$

Applying equation (45) to equation (44), we have:

(46) $$v_{n,t} = P_{r,t}\left[\frac{1+r}{1+\xi} - (1-\delta_n)\right]$$

Applying equation (46), we relate the user cost of R&D capital stock to the observed input price. This is different from the case of physical capital where user cost of the capital service is derived from the prices of capital good of different vintages. In equation (46), we assume constant discount rate, depreciation rate and price change rate. With these assumptions, user cost of R&D capital is actually proportional to the aggregate input prices. Discount rate and price change rate are same for all the firms in an economy, the only element making firm's user cost different from each other is the firm-specific

R&D capital depreciation rate.

Using firm-specific user cost shares as weights to aggregate across R&D capital stocks from different firms effectively takes the heterogeneity of R&D capital into account. User costs shares implicitly reflect the relative marginal productivity of the different R&D capital stock, thus these weights provide a means to effectively incorporate the differences in the productive contribution of heterogeneous R&D capital assets into the overall measure of R&D capital stocks.

Applying equation (38) to aggregate across different types of knowledge capital can also solve the problems caused by similarity of knowledge. It is well known that R&D investment is the major source of the innovation that is essential for a firm's competitive ability. Regardless production innovation or progress innovation, innovation can generate more profits in the future for the firms undertaking R&D investment. All the firms have the incentives to invest in R&D activity. Due to the pressure of competition, some "close" firms may conduct similar research and create very similar technology. If the double research occurs due to the competition, we may face the problem of knowledge similarity. Similarity of the knowledge assets does not have any impact on the R&D capital stock at firm level, but we need to make some adjustments when we aggregate across R&D capital stocks from different firms. Without adjustment, the average technological level of the industry would be overestimate. The aggregate R&D capital constructed based on equation (38) is essentially a weighted average over different firms' R&D capital stocks. This aggregation process can also solve the problem of knowledge similarity. Therefore, we do not need to worry about the impact of similar knowledge on the aggregation of R&D capital stock.

Up to now, we have already examined the aggregation of R&D capital stock without knowledge diffusion. In the next section, we use US manufacturing data to show how our methods different from the traditional method of treating R&D capital.

5. Empirical Results

In this part, we construct measures for knowledge product and knowledge stock based on both conventional approach and the approach we have proposed in the previous section. Because we do not have all the required data for constructing these measures, we construct these measures based on some crucial assumptions and check how sensitive of our measures to the different assumptions. We will then conduct productivity analysis using growth accounting framework based on the constructed measures.

5.1 R&D Stock

Conventionally, R&D stock is constructed based on the perpetual inventory method (PIM) using R&D input data. The essence of the PIM is to form a yearly stock estimate by adding new R&D expenditure in the year to the existing stock and subtracting 'depreciation' or obsolescence of the existing stock. In another word, the series of R&D expenditure can be formed into a stock measure according to the following equation:

(47) $R_t = I_{r,t} + (1-\delta)R_{t-1}$

where R_t is the stock of R&D capital at the end of period t, $I_{r,t}$ is the real R&D investment during period t, and δ is the geometric depreciation rate. This input-based measure of R&D stock is relatively easy to apply. However, because some key assumptions underlying the PIM, such as fixed productivity of R&D activity and constant rate of knowledge decay, do not hold in the real world, the measurement concerns have been raised in both empirical and theoretical work[9].

According to our approach, we need to construct measures of knowledge stock for knowledge production and knowledge stock for non-knowledge production. The construction of these two types of knowledge stock relies on the output of knowledge production—knowledge product, which in turn is affected by the existing knowledge stock served for knowledge production. Knowledge product is measured via equation (22). Because we do not have enough data for obtaining the estimates for the each part of equation (22), we impose the following assumptions to construct the measure of the knowledge product:

(48) $\exp\left\{(1/2)[\frac{\partial \ln g(x_1,t_1)}{\partial t} + \frac{\partial \ln g(x_0,t_0)}{\partial t}](t_1 - t_0)\right\} = 1.005$

(49) $h(R^K_{t-1}) = (R^K_{t-1})^{1/2}$

Equation (48) implicitly assumes that R&D productivity increases at a constant rate. Although this is not necessarily true, we can still examine how R&D productivity affects the creation of knowledge and the construction of R&D stock. As we have discussed in the previous section, the impacts of the existing R&D capital stock to the knowledge creation is subtle, our choice of the h function satisfies important properties of this function, however, we are still open better choice. The stock of R&D capital for knowledge production can be constructed based on equation (27). An important element of determining this measure is the "relevancy index". In order to simplify the problem, we assume that the relevancy index starts from 1 and declines at a constant rate. In

9 Refer to Shanks and Zheng (2006) and Parham (2007) for more detail discussion.

addition, we assume that the average life of general knowledge is 40 years; with the end relevancy index at 0.0001, we can derive the relevancy index as follows:

(50) $\mu = (0.0001)^{1/40} = 0.794$

Based on the above assumptions, we can recursively build the measures of knowledge product and R&D stock for knowledge production by using the following equations:

$$
(51)\quad
\begin{aligned}
K_1 &= Q_1(w_1, x_1, w_0, x_0) \times 1.005 \times \left(\frac{R_0}{R_{-1}^K} \right)^{1/2} \\
R_1^K &= K_1 + 0.794 \times R_0 \\
K_2 &= Q_2(w_2, x_2, w_1, x_1) \times 1.005 \times \left(\frac{R_1^K}{R_0} \right)^{1/2} \times K_1 \\
R_2^K &= K_2 + 0.794 \times R_1^K
\end{aligned}
$$

where R_0 is the initial knowledge stock, which can be constructed using the ordinary procedure as follows:

(52) $R_0 = I_{r,0} / (\delta + \gamma_r)$

where $I_{r,0}$ denotes the constant R&D investment at the first period; δ denotes the depreciation rate of knowledge capital; γ_r denotes the geometric growth rate of R&D investment over the sample period. It can be calculated using the following equation:

(53) $\gamma_r = (I_{r,T} / I_{r,0})^{(1/T)}$

where $I_{r,T}$ is the R&D investment in the last period of the sample period.

The R&D stock for normal production can be constructed by using equation (29) after we have formed the measure of knowledge product. We need to impose assumptions about the frequency of knowledge creation and the utilization path of knowledge because we lack the information to figure out the efficiency index and utilization index. We assume that a new knowledge is created at each period. The technological efficiency of the knowledge asset declines at a constant rate due to the presence of the new knowledge. The initial utilization index is assumed to be 0.8. It increases to 1 at the second period and then starts to decline at a constant rate of 0.15. Thus, knowledge stock for non-knowledge production can be constructed according to the following equations:

$$R_t^{NK} = 0.8 \times 1 \times K_t + 1 \times 0.85 \times K_{t-1} + 0.85 \times 0.85^2 \times K_{t-2} + \ldots$$
$$R_{t-1}^{NK} = 0.8 \times 1 \times K_{t-1} + 1 \times 0.85 \times K_{t-2} + 0.85 \times 0.85^2 \times K_{t-3} + \ldots$$
(54) $\ldots$
$$R_2^{NK} = 0.8 \times K_2 + 1 \times 0.85 \times K_1 + 0.85 \times 0.85^2 0.85 \times R_0$$
$$R_1^{NK} = 0.8 \times K_1 + 1 \times 0.85 \times R_0$$

We use US manufacturing data from 1953 to 2001 to construct R&D stock and conduct productivity analysis. In order to conduct the empirical work outlined above, we need quantity series and price series of input and output for both normal production and R&D investments in US manufacturing. The input and output data other than R&D related data are obtained from the Multifactor Productivity data sets provided by Bureau of Labour Statistics (BLS). R&D related data are derived from the website of National Science Foundation (NSF).

Figure 1: Knowledge Product and Knowledge Stock

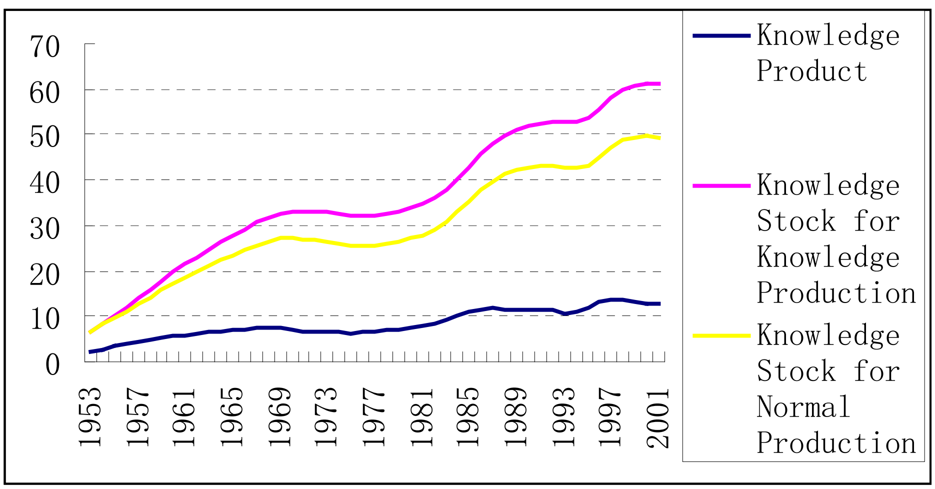


Figure 2: R&D Stock for Normal Production and Conventional R&D Stock

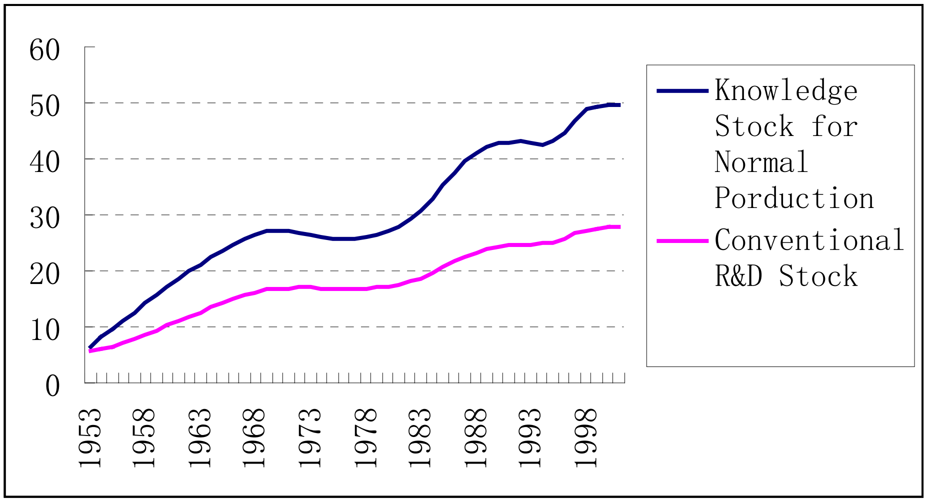


Figure 1 shows the trend of knowledge product and knowledge stock for both knowledge production and normal production. It is obvious that the amount of knowledge stock for knowledge production is greater than that for normal production. Figure 2 shows the difference in R&D capital stock calculated from the traditional approach and our approach. Traditional approach constructs the measure of R&D stock through directly capitalizing R&D expenditure. We propose to incorporate the knowledge production process in the formulation of R&D capital stock. Therefore, our measure of R&D capital stock is built indirectly from R&D expenditures. We want to build the link between R&D inputs and R&D outputs.

Because the measures of knowledge product and knowledge stock are constructed based on the assumptions about important indexes, we also check the sensitivity of these measures to the different assumptions. The following figures (Figure 3-7) show how our measures vary with these assumptions. In general, we can see that the R&D stock for normal production is relatively sensitive to the R&D productivity and the initial knowledge stock, while it is not very sensitive to the service life of the general knowledge capital.

Figure 3: R&D Stock for Normal Production with Different Initial Knowledge Stock

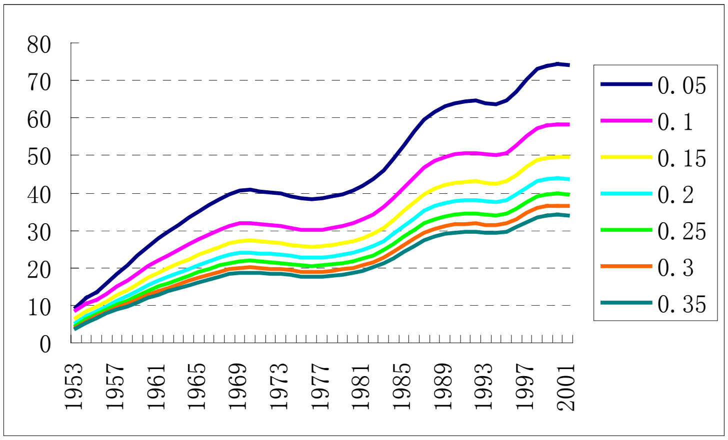


Figure 4: R&D Stock for Normal Production with Different Service Life

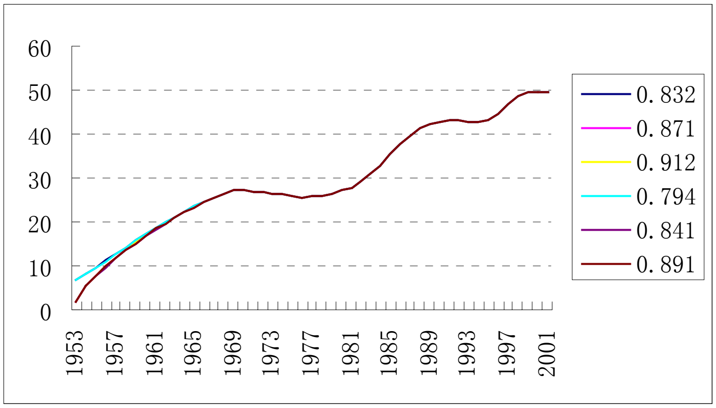


Figure 5: R&D Stock for Normal Production with Different Starting Utilization Level

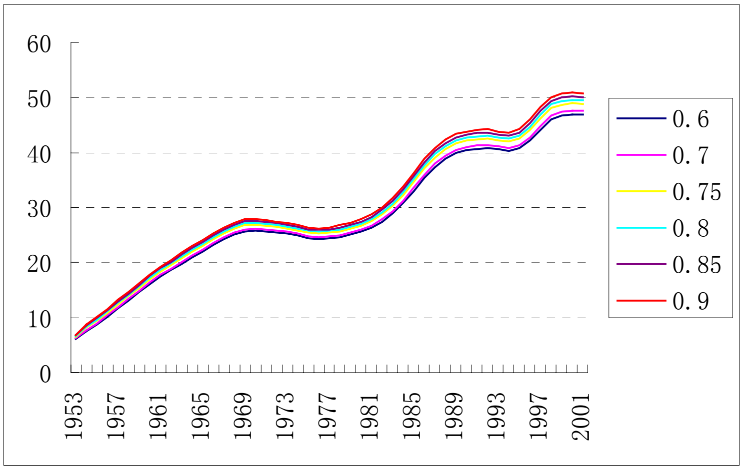
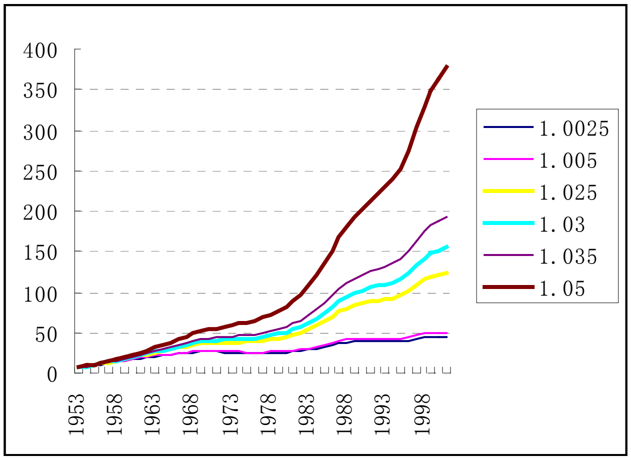


Figure 6: R&D Stock for Normal Production with Different R&D Productivity



5.2 Productivity Analysis

Based on the constructed measures of R&D capital stock, we also conduct simple productivity analysis in the growth accounting framework. We run a simple OLS regression of the following transformed form of equation (3):

$$(55)\quad \frac{Y_t - Y_{t-1}}{Y_{t-1}} = \eta_C \frac{C_t - C_{t-1}}{C_{t-1}} + \eta_L \frac{L_t - L_{t-1}}{L_{t-1}} + \eta_M \frac{M_t - M_{t-1}}{M_{t-1}} + \eta_R \frac{R_{t-1}^{NK} - R_{t-2}^{NK}}{R_{t-2}^{NK}} + tfp$$

where M is the intermediate input. We can use the residual of the above regression as a measure of the productivity growth. Although there are some econometric problems associated with this estimation, such as multi-collinearity problem, we still can investigate the impacts of different treatment of R&D capital on productivity analysis. We do not focus on the accuracy of the estimates; in stead, we are particularly interested in the change pattern of the estimates when we change the assumption.

In order to see how R&D capital affect the productivity analysis, we run preliminary regressions without explicit R&D capital in the equation. In the following table, Column (1), R&D expenditures are not separated from the inputs factors; Column (2) R&D expenditures are separated from the inputs factors but not treated as an explanatory variable in the regression. From Table 1, we can see that capital explains more output growth when R&D capital is not included in the regression.

Table 1: Regressions Without R&D Capital

	(1)	(2)
η_L	0.77339 (0.09751)	0.77959 (0.09928)
η_M	0.29772 (0.1141)	0.25514 (0.1190)

η_C	0.43527 (0.1124)	0.47960 (0.1186)
$\eta_L+\eta_M+\eta_C$	1.50638	1.51433

We then run same regressions by including R&D capital as an explicit explanatory variable. The estimates obtained from the traditional approach are presented in the following table:

Table 2: Regression Using Traditional Approach

	δ=0.05	δ=0.1	δ=0.15	δ=0.2	δ=0.25	δ=0.30	δ=0.35
η_L	0.76139 (0.09214)	0.75805 (0.0937)	0.75726 (0.09405)	0.75688 (0.09426)	0.75678 (0.09452)	0.75700 (0.09486)	0.75755 (0.09524)
η_M	0.28996 (0.1108)	0.29579 (0.1129)	0.30063 (0.1136)	0.30501 (0.1142)	0.30823 (0.1148)	0.30991 (0.1154)	0.31008 (0.1160)
η_C	0.18101 (0.1509)	0.22704 (0.1381)	0.30070 (0.1321)	0.31806 (0.1293)	0.32913 (0.1280)	0.33771 (0.1274)	0.34539 (0.1273)
η_R	0.31764 (0.1101)	0.2206 (0.0868)	0.18933 (0.07448)	0.16823 (0.06716)	0.15317 (0.06243)	0.14098 (0.05913)	0.13042 (0.05666)

The estimates obtained from our proposed approach are shown in Table 3:

Table 3: Regression Using the Proposed Approach

	δ=0.05	δ=0.1	δ=0.15	δ=0.2	δ=0.25	δ=0.30	δ=0.35
η_L	0.76473 (0.09558)	0.75749 (0.09454)	0.76184 (0.0959)	0.76473 (0.09678)	0.76673 (0.09738)	0.76818 (0.0978)	0.76702 (0.09427)
η_M	0.27011 (0.1145)	0. 28626 (0.1135)	0.27299 (0.1148)	0.26535 (0.1157)	0.26061 (0.1164)	0.25747 (0.1169)	0.2595 (0.1168)
η_C	0.36936 (0.1246)	0. 33539 (0.1273)	0.36522 (0.1261)	0.38636 (0.1252)	0.40097 (0.1253)	0.41153 (0.1239)	0.40697 (0.1246)
η_R	0.098287 (0.04525)	0.13394 (0.05480)	0.10188 (0.04792)	0.081536 (0.04292)	0.067901 (0.03917)	0.058247 (0.03626)	0.062281 (0.03812)

Comparing with the above three tables, in general, we find that the size of η_R is relatively smaller in that Table (3), although R&D stock is greater based on our approach. We also find an interesting fact that η_L and η_M are relatively robust comparing with the change of η_C after we incorporate R&D capital in the regression. This fact may imply a close relationship between R&D capital and ordinary physical capital. When we use conventional approach to construct the R&D capital stock and run the regression, we find that η_R decreases with the increase of depreciation rate. While we do not see this monotonic relationship in the regression when we use the measure of R&D capital constructed based on our proposed approach. Investigating the estimates in Table 2, we can find negative relationships between η_R and η_C, and between η_R and η_M. Same relationship between η_R and η_C can be found in Table 3. In addition, we find that η_M increases with η_R and η_L decreases with η_R from the estimates of Table 3.

We show the rate of productivity growth in the following two figures. The lines with different colours in the figures show us the differences in the changes of productivity growth. The pattern of the productivity growth exhibited from both approaches is similar. However, the differences in productivity growth are smaller in the proposed approach. In short, we do not see big difference in the productivity growth rate from these two approaches.

Figure 7: Productivity Growth Using the Proposed Approach

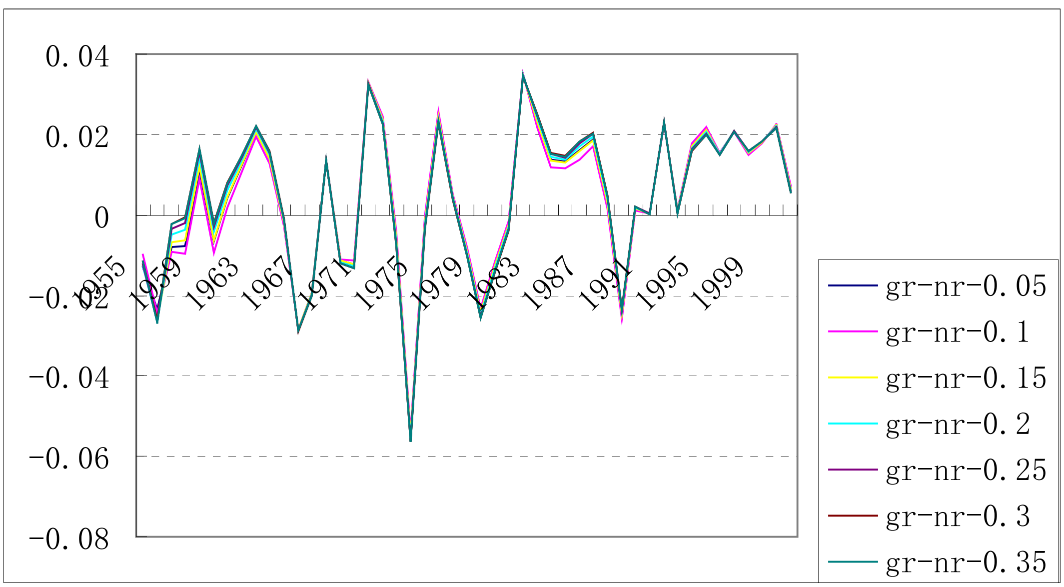
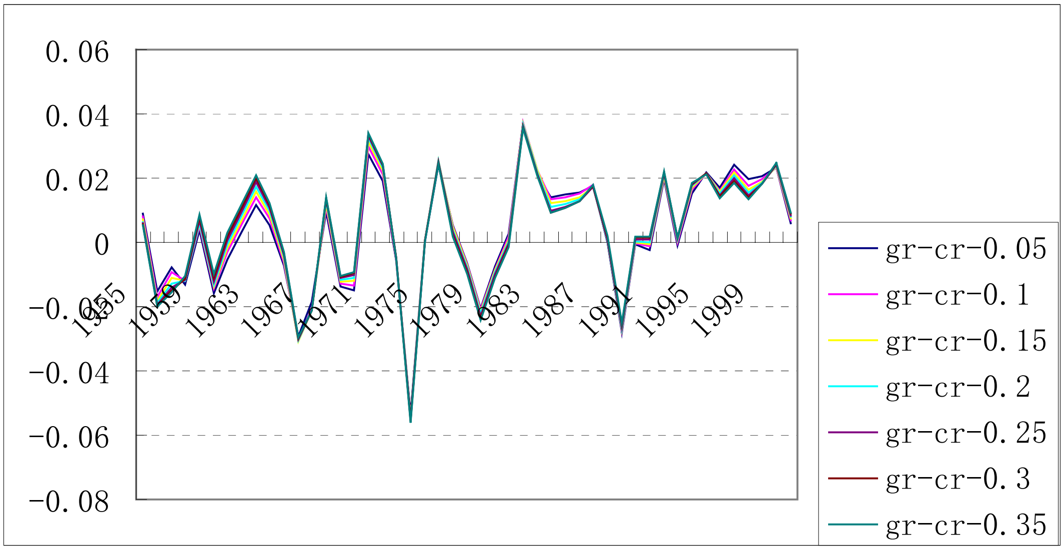


Figure 8: Productivity Growth Using Conventional Approach



6. Conclusion

Implementing the ideal of capitalizing R&D expenditure in the national accounts provides some new changes to the measurement of R&D expenditures and productivity analysis. In this paper, we investigate how to incorporate R&D capital into the growth accounting framework and how to construct an appropriate measure for the stock of R&D capital.

Traditionally, R&D capital is treated as an additional input factor in the production function and the stock of R&D capital is formulated using R&D input data based on the PIM. There is uncertainty about the accuracy with which R&D expenditure represents delivery of R&D outputs. In our paper, we attempt to incorporate the knowledge creation procedure into the construction of the knowledge stock measure. R&D outputs are linked with R&D inputs via a knowledge production process. We assume that each firm conducts two different activities: ordinary product production and knowledge production. Because knowledge capital plays different roles in these two different activities, we propose to distinguish between knowledge stock for knowledge production and knowledge stock for ordinary product production. Our measure of knowledge stock is constructed based on the measure of knowledge product, which is built from R&D expenditures through knowledge production technology. The proposed measures of knowledge product and knowledge stock rely heavily on the quality of the R&D related data. We expect to provide more accurate R&D measures with the development of knowledge market and improvement of data collecting techniques.

The size of R&D stock obtained from our methodology is found to be greater than that conventionally used in the growth accounting framework. In the mean time, the elasticity of R&D capital stock to the aggregate output is, on average, smaller in our estimation. However, we do not find large differences in the rate of productivity growth.

Our current work only deals with the R&D measurement problems in an economy without knowledge diffusion. In the future, we will extend this work to the economy with knowledge spillover, and deal with the measurement issues associated free knowledge. In addition, more work is needed to better understand the knowledge accumulation, and the interaction between R&D investment, knowledge capital and human capital.

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