

# Teaching Economics by Teaching 2x2 Game Theory

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## **Abstract**

Although many introductory textbooks now include a few simple games as a minor sub-topic, it is possible to invert convention, introducing economics by starting with a game theoretic approach and restricting oneself to illustrations based on 2x2 games. Based on 8 years of teaching experience, we show that the 2x2 games and Robinson-Goforth topology for the 2x2 games provide an elegantly simple and modern framework for introducing students to social analysis and economics. The paper is likely to appeal most to graduate students whose approach to teaching economics is not yet set.

# 1 Introduction

Economics has made great strides since Samuelson's 1948 classic *Economics: an Introductory Analysis* [31]. Introductory economics has not made similar progress. The standard textbook follows the primary division that Samuelson laid out in his 1947 magnum opus, *Foundations of Economic Analysis* [30], and it applies a slightly watered-down version of the conception of micro-economics that he proposed. Baumol [6], considering how little of twentieth century economics has entered the introductory books, points out

“the startling fact is that there really seem to be only two sets of substantial changes in the microtheory that are found in all standard texts. The first relates to the role of externalities and public goods in the theory of welfare economics. The second is concentrated in what is often a single chapter in current texts: the chapter on oligopoly and monopolistic competition. And, even here, the novelties are not as novel as they may seem.”

Baumol suggests that the reason is “the choice of subject matter by the authors of textbooks (including myself) ... simply follows tradition in the teaching of these subjects and offers material calculated primarily to be attractive to the instructors.”

Avanish Dixit has argued that it is time to update our approach to teaching economics. “I hold a rather radical view: An introduction to game theory should precede, not follow, the introductory economics courses in micro and macro” [10]. We offer a different but equally radical view: It is time to rebuild introductory microeconomics using game theory<sup>1</sup>.

In this paper we argue for an introduction to Micro economics based on game theory and relying heavily on 2x2 games for examples and illustrations. In Appendix 1 we present the the outline of an introductory course that we believe does what a modern introduction to microeconomics should do. The appendix is essentially a proof-by-example that it is possible to construct an introductory course based on game theory and the 2x2 games in particular. Most of the material has been “classroom tested,” but the course described here has not been taught. The appendix is organized as a series of ‘chapters,’ most of which introduce specific core materials beginning with some concepts easily presented in the 2x2 framework, then extend the analysis slightly to provide new concepts and lessons. The consistent use of 2x2 games provides a simple and unified framework. Concepts build logically on what has gone before.

The 2x2 games provide an ideal structure for examining rationality and choice; they are easily understood by students and allow us to isolate specific concepts. they lend them-

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<sup>1</sup>We would also argue, although we think it considerably less contentious, for the introduction of a great deal from behavioural economics.

selves to experimentation and classroom exercises. There is a large behavioural literature, much of which uses variants of 2x2 games. The 2x2 games offer multiple examples of all of the social dilemmas. There is a substantial and accessible literature on many specific games. The periodic table [23] provides a way to organize the games into meaningful groups, escaping the anecdotal approach that texts have followed to this point, allowing relationships among games to be seen easily, and encouraging experimentation with payoff structures, the equivalent of comparative statics in standard theory<sup>2</sup>. The periodic table also offers a very easy way to introduce some useful concepts from topology and group theory, two relatively modern mathematical tools. 2x2 games are easily generalized to 3x3 games or games with more choices and to 2x2x2 games and others with more players.

Games of greater complexity than the 2x2s are seldom needed. (See Table 2.) In the case of the strictly ordinal games for example, the 144 2x2 games appear to embody, in elemental form, virtually all the phenomena observed in games with more players and choices. In exhaustive automated analysis of the 2x3, 2x4 and 3x3 games [13], over 3.6 billion in all, we were unable to find incentive distributions that were not describable as compounds of simpler 2x2 distributions. From the point of view of the proposed course, we were always able to find an appropriate strict ordinal example among the 2x2 games whenever the need arose.

The virtues of the approach are unity, simplicity, rigour and the chance to offer a course conceptually rooted in the late 20th century rather than in the late 19th century. The game theoretic approach has become the way modern economists approach most advanced questions, whether or not they make explicit reference to game theory. Fundamental developments since 1950 such as principle-agent problems, two-stage maximization, incentive compatibility, information economics, mechanism design, Nash equilibrium, coordination problems, expectational models, rational expectations, and even Ricardian equivalence [5], all of which have enhanced our understanding of the properties of optimal allocation mechanisms, are drawn from, based on, or very similar to game theory in approach.

Economists now use game theory to analyze a wide array of economic phenomena, including auctions, bargaining, duopolies, fair division, oligopolies, social network formation, and voting systems, and to model across such broad classifications as mathematical economics, behavioural economics, political economy, and industrial organization.

Introductory texts, however, rely on a collection of highly successful, highly refined and familiar pedagogical tools that have resisted reformulation in more modern and more general terms. The apparatus of supply and demand, clearly a huge accomplishment of economics, and undoubtably a necessary part of an introduction to the discipline, sits like a cathedral across the path of the student. Other topics are attached like sheds around the

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<sup>2</sup>An interactive applet for exploring relationships among the 2x2 games is available at [Dynamic Periodic Table of Strict Ordinal 2x2 Games \(Beta version\)](#)

base of the great edifice.

There are three problems with an introduction to economics focussed on supply and demand. First, it provides an indirect and awkward way to approach the choice-theoretic modelling that is at the heart of modern Microeconomics. The result is that students do not learn to be economists because they are not introduced to an economics in a manner that shows the real structure of how economists think.

Second, a supply and demand based approach obscures the generality and the broad applicability of economic theory. As a consequence, students most often finish their introduction to economics believing that supply and demand is really what economics is.

And third, students are not exposed to many of the easiest and most exciting ideas that economics can offer so they often find economics dry and hard to apply.

Basing the introductory presentation on the 2x2 games circumvents each of these problems and teaching from a single basis like this turns out not to be restrictive. There are far more applications and examples available than can be used in a single course. Teaching in this manner does require that instructors be familiar with game theoretic reasoning and that they have a knack for isolating concepts with simple formal examples.

## 2 Defining economics

Playing with definitions of our field is entertaining and sometimes educational, but not generally very productive. In this case there is real value in reviewing a few familiar definitions: it can help focus on the content and techniques that should be included in a modern introduction. There are two classes of definitions of special interest, one that focusses on the science of resource allocation and one that treats economics as a social science. The first is a mechanic's definitions, which makes economics a branch of management science: "the science that deals with the production, distribution, and consumption of goods and services, or the material welfare of humankind." This is the category that includes the definition attributed to Samuelson, "optimization subject to constraints." A social scientist might prefer something more like "Economics is generally understood to concern behavior that, given the scarcity of means, arises to achieve certain ends" or even Marshall's view that economics is "a review of mankind in ordinary enterprise of everyday life"<sup>3</sup> In this

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<sup>3</sup>The currency of this definition is reflected in the titles of a number of recent books on economics: *Hidden Order: The economics of everyday Life*, David Friedman, Harper Business; *Armchair Economist: Economics & Everyday Life* by Steven E. Landsburg, 1995, and the New York Times columns by David Leonhardt and Catherine Rampell. In a lovely little discussion, Bryan Caplan, [http://econlog.econlib.org/archives/2009/06/in\\_a\\_perfect\\_wo.html#](http://econlog.econlib.org/archives/2009/06/in_a_perfect_wo.html#) argues that "economics is the all-encompassing study of the social world."

view we “examine that portion of individual and social actions that is closely connected with attainment and utilization of materials requisites.”

Paul Samuelson offered a precise synthesis of these two notions when he suggested that “almost all behavior could be understood as maximizing or minimizing subject to constraint.” [30]. What we have in definitions of this sort is an emphasis on using rational agents to model social situations. It is entirely natural to weaken the rationality requirement to, for example, satisficing [34] and when this step is accepted, economics becomes *choice-theoretic* in a more general sense. This in turn opens the way for studies of how people actually choose, the focus of behavioural economics.

Behavioural economics “has rapidly become an important foundation for many areas of economic theory such as bargaining in decentralized markets, contracting and organizational structure, as well as political economy (e.g, candidates choosing platforms and congressional behaviour). At the core of behavioral economics is the conviction that increasing the realism of the psychological underpinnings of economic analysis will improve the field of economics on its own terms.” [8]

One of the most important steps forward in microeconomic modelling was the introduction of game theory with Von Neumann and Morgenstern’s *Theory of Games and Economic Behaviour*[36]. The feature that distinguishes game theory from the classic version of economics is the introduction of strategic play based on awareness on the part of the agent of interdependent payoffs. Intriguingly, the split in the definition of Economics is reproduced in the definitions of game theory. Wikipedia, to mention one of the most influential sources at the moment, however transient it may be, defines game theory as “the mathematical study of optimizing agents”[38] and the Encyclopedia Britannica defines Game Theory as the “branch of mathematics used to analyze competitive situations whose outcomes depend not only on one’s own choice , but also on the choices made by other parties.” Aumann has a much broader vision that “game theory is a sort of umbrella or ‘unified field’ theory for the rational side of social science, where ‘social’ is interpreted broadly, to include human as well as non-human players (computers, animals, plants).” [1].

Whatever definition of economics one prefers, it is clear it will overlap one of the common definition of game theory. The similarity should not be surprising: game theory is essentially a refinement of the methods of the economics that it emerged from, and it inherits the same concerns.

Examining much of the most exciting work in the last 50 years makes it clear that game theory is pervasive in economics. But if this is the case, then an introduction to game theory might serve as an introduction to microeconomics in general, as Dixit suggests, or alternatively, an introduction to microeconomics economics might actually be an introduction to game theory as we suggest here.

### 3 On teaching and the goals of teaching economics

It is likely that anyone who has taught for a few years will have encountered and even tried various approaches to introducing economics. There have been hundreds of introductions written in different styles and from quite different philosophical and pedagogical approaches. The major introductory texts offer a fairly narrow range of choices, based on the model that emerged before 1950 with the then-innovative division of economics into macro and micro and its insight that an enormous range of behaviour can be explored using optimizing models.

The dominance of this standard model has been challenged in numerous non-standard texts. *Introduction to Modern Economics*[25], by Joan Robinson and John Eatwell is an outstanding example. I have used *A Mathematical Introduction to Economics* by Alasdair Smith [27]<sup>4</sup>, and used materials from Vivian Walsh's choice-theoretic *Introduction to Contemporary Microeconomics* [37]. There are other introductions from the point of view of business; indeed, I have a colleague working on an introduction to economics with an emphasis on sports economics.

While the common practice and these variations on a theme are easy to observe, we have had difficulty deciding what we want to do with beginning students. There are two questions that have to be answered when we think about changing so fundamentally the way we introduce economics: "Can it be done?" and "Should it be done?"

Within the question "Can it be done?" we have to ask what we mean by introducing economics. That leads us to definitions of economics and to an investigation of the practice of introducing economics. Within the question "Should it be done," we find ourselves asking what our goals are: "What do we want to accomplish in introducing economics to students?"

At one level we want simply to provide a valuable first-year experience built on some of the insights that economists value most. This is, perhaps, just teaching students to "think like an economist." It is not entirely clear what "thinking like an economist" means but there are features of the way economists approach problems that are probably reasonable goals for an introductory course. We want students to understand social situations as objects of study. We also want students to understand individual choice as a useful starting point in that study of social situations. We want them to come away with at least some understanding of theory and of modelling and some techniques for understanding certain specific classes of social situations with an emphasis on markets. We want them to begin the development of a habit of deriving prescriptions and predictions from reasonably carefully specified models. Assumptions about the nature of economics, the usual practice,

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<sup>4</sup>This is not the *Mathematical Introduction to Economics* by Griffith V. C. Evans, McGraw-Hill, New York: 1930

and our pedagogical goals in teaching are inevitably folded into any recommendation for changing introductory economics.

### 3.1 The Challenge of Supply and Demand

The most difficult problem in building an introductory course on a game theoretic basis is to make a transition from the more general strategic framework of game theory to the narrower conventional core of introductory Micro, which is clearly supply and demand analysis. The problem is particularly acute if we want to rely on the 2x2 games as the unifying technique because the 2x2 games are at the opposite corner of the domain of economics from market analysis, illustrated in Figure 1. Supply and demand models are of most use in reference to competitive situations characterized by large numbers, price taking (implying an absence of strategic behaviour), and the absence of externalities which are one name for interdependent payoffs, the central feature of game theory. Economists understand competitive markets as a very special case of the general class of economic games.

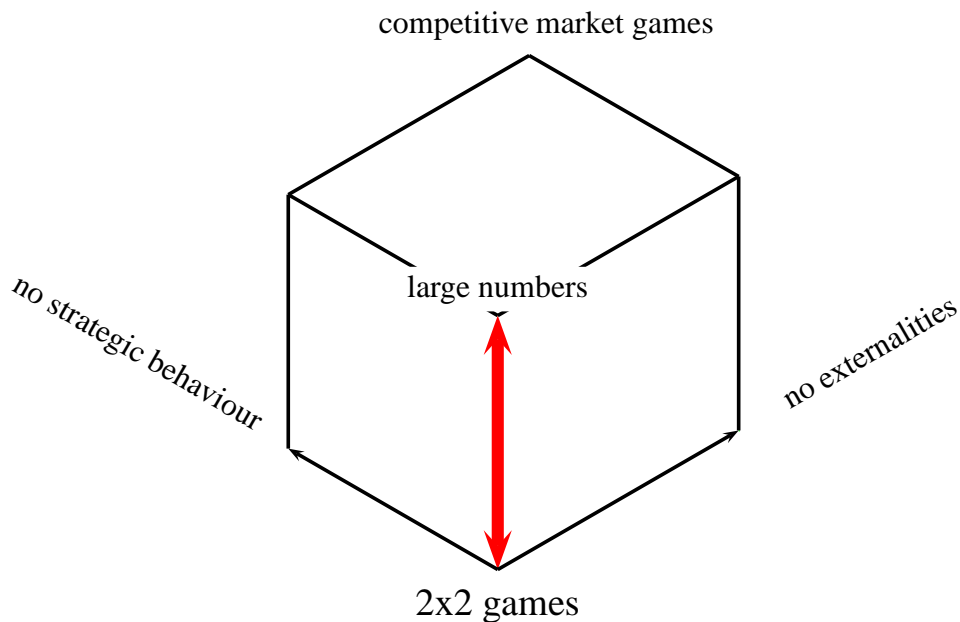


Figure 1: The 2x2 games and competitive markets are at opposite ends of economics

It should be noted that even conventional intermediate courses have logical difficul-

Table 1: Some game-theoretic and economic concepts easily introduced using 2x2 Games

Choice	rationality
rational behaviour	best response analysis
model	theory
prediction	prescription
positive	normative analysis
gamblers	games against nature
opportunity costs	optimality
equilibrium	Nash equilibrium
gains from trade	Pareto dominated outcome
multiple equilibria	symmetry
maximin equilibrium	efficiency
production efficiency	exchange efficiency
expectation	value
externalities	public good
conflict	coalition
collective action	commitment
common interest	distributional conflict
common knowledge	constant sum
cooperation	coordination
credibility	decision node
expected payoff	preferences
fairness	first mover advantage
contingency	Axelrod's tournament[2, ?]
backwards induction(rollback equilibrium)	bargaining set
prisoner's dilemma	Envy-free allocations
repeated games	Characteristic function*
cheap talk	truth-telling
mechanism design	incentive compatibility
credible threats	chicken
decision tree	defect
threat point	distributional conflict
dominance solvable	dominant strategy
Dominant strategy equilibrium	hawk dove game
dominated strategy	efficient set
game tree	evolution of linked populations
Evolutionarily Stable strategy	evolutionary game
focal point	forgiving strategies
incredible threats	independence of irrelevant alternatives
information set	iterated dominance
Kalai-Smorodinski solution	learning and evolution
maximin	mixed strategy
monotonicity	Nash bargaining
Nash equilibrium	Nash solution to a bargaining game
non-transferable utility	norms and cultures
open access resources/cod wars	ordinal values
payoffs	player set
replicator dynamic	strategic form
reputation	retaliatory strategies
risk dominance	salience
screening	second mover advantage
Shapley Value	shrinking pie
signalling	strategy set
strongly dominant strategy	subgame perfect equilibrium
Sucker payoff	Symmetric game
threats and promises	tit for tat
tragedy of the commons	transferable utility
trembling hand equilibrium	unpredictability
Value of a game	variable threat games
zero sum games	weakly dominant strategy



ties with supply and demand. Rigall-I-Torrent [22] describes the contortions involved in teaching supply and demand and market structure following popular texts [19, 20, 35]:

Following the sequence laid down in leading intermediate microeconomics textbooks (Perloff 2007; Pindyck and Rubinfeld 2005; Varian 2006), students first struggle to understand why the demand curve facing an individual competitive firm is given by a horizontal line, whereas the market demand curve is downward sloping. After understanding perfect competition, students are asked to realize that the demand curve facing a monopolist is identical to the market demand curve. Monopolistic competition is then analyzed with the help of short and long run demand curves facing individual firms. Finally, when studying oligopolistic markets reaction curves are used instead of demand curves. Furthermore, the assumptions underlying the perfect competition model (the first model taught to students in typical microeconomics courses) are not in accordance with the real-world experiences of many students.

Rigall-I-Torrent goes on to build an introduction to producer and market theory Microeconomics based on Cournot games. Like us he resolves the problems in the conventional curriculum by resorting to a game-theoretic approach.

The 2x2 games provide a more direct approach yet. There is an easy transition from discrete 2x2 games to 2 person bargaining games. The bargaining games then provide a natural way to treat the division of the surplus from exchange with fixed quantities. It is only necessary to identify a difference in value for a potential seller and a potential buyer. Values are given contingent strategies for each, which is to say an (inverse) supply and an (inverse) demand schedule. Bargaining begins with the surplus for a fixed output ( a *pure exchange* problem) .

The surpluses here, illustrating the gains from trade more generally, are the fundamental drivers of human society. Furthermore, bargaining is philosophically and pedagogically an attractive entry point because it is fundamentally a more basic activity than acting in complete markets. When Smith writes in Chapter 2 of book one of the *Wealth of Nations*

This division of labour, from which so many advantages are derived, is not originally the effect of any human wisdom, which foresees and intends that general opulence to which it gives occasion. It is the necessary, though very slow and gradual consequence of a certain propensity in human nature which has in view no such extensive utility, the propensity to truck, barter, and exchange one thing for another.

Smith clearly places individual trades at the foundation of economic theory, prior even to specialization and markets *per se*.

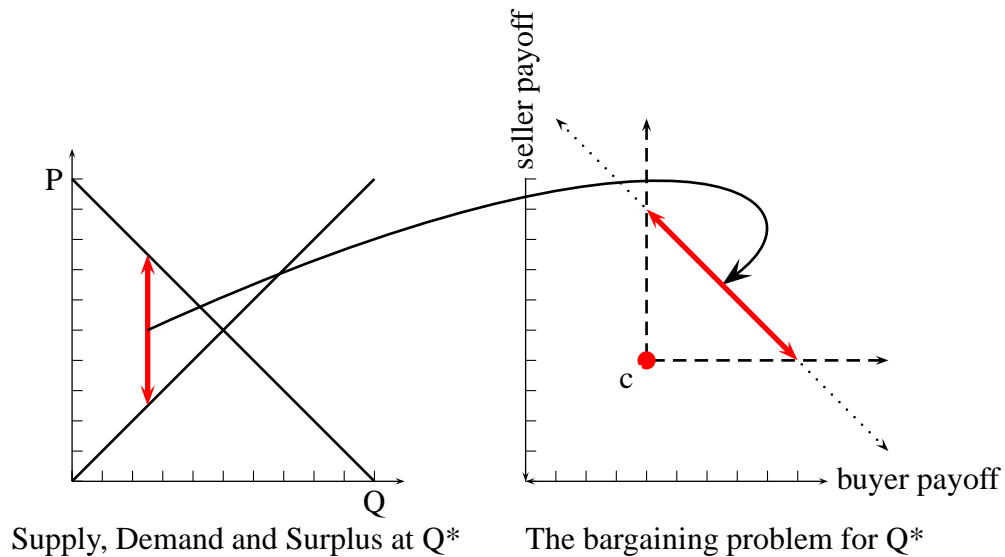


Figure 2: From Demand to Bargains

Similarly, when Ricardo writes in his preface to *On the Principles of Political Economy and Taxation*

The produce of the earth - all that is derived from its surface by the united application of labour, machinery, and capital, is divided among three classes of the community, namely the proprietor of the land, the owner of the stock or capital necessary for its cultivation, and the labourers by whose industry it is cultivated.

But in different stages of society, the proportions of the whole produce of the earth which will be allotted to each of these classes, under the names of rent, profit, and wages, will be essentially different depending mainly on the actual fertility of the soil, on the accumulation of capital and population, and on the skill, ingenuity, and instruments employed in agriculture.

To determine the laws which regulate this distribution, is the principal problem in Political Economy.

he is beginning with a surplus to be divided. Modern axiomatic bargaining theory thus provides direct and natural links to the foundations of economics.

With the division of the surplus it is clear that the gains from trade for a producer also provide an incentive for increasing output. The incentive persists until an equilibrium

is achieved. This provides an occasion for discussing adjustment to equilibrium and the notion of *equilibrium* in markets and *allocative efficiency*. The approach has clear roots in the history of economic theory, going back at least to Edgeworth's *Mathematical Psychics* [11].

The discussion can be supported by an exploration of Boehm Bawerk's horse market, described, for example, in Moulin [18].

An advantage of approaching supply and demand through bargaining is that it starts in the heart of economics - the gains from trade. Furthermore, the gap between what a buyer is willing to pay and what a seller will sell for is a measure of the Marshallian surplus at a given quantity. Integrating yields *Consumer plus Producer Surplus*, a quantity that economists recognize and virtually no-one else understands. Surplus is the heart of any efficiency measure, and therefore of the economist's capacity to evaluate markets. In this approach, therefore, supply and demand theory begins with the most fundamental quantity in microeconomic theory.

## 4 Sketch of a course in economics

We return now to "Can it be done?" Can economics be introduced in a satisfactory way using 2x2 games as a foundation of the pedagogical approach? The appendix presents an outline for an introductory course built around 2x2 games. In it we imagine a textbook for a one-term course introducing microeconomics organized as hypothetical chapters. Most of the concepts introduced are, of course, more or less common in existing texts: in fact, the course is designed to introduce most important concepts from a standard course, to introduce them in the context of an integrated and modern view. One feature that distinguishes the approach is that concepts are presented in terms of choices in games in which payoffs are carefully specified. We take this to be basic to microeconomic methodology.

The proposed approach plays heavily on opportunities to get students thinking about examples while gradually building up skill in reading and representing social situations as systems of payoffs. We take the view that student arrive with an individualist bias to analyzing social situations that the game theoretic approach clarifies at the same time as it makes comprehensible the social nature of economic choice. It is the dilemmas of social organization that we need to illustrate and it is the challenge presented by the social dilemmas that explains many social institutions. This is in our view something only economic analysis can do and only a game theoretic approach can do well.

## 5 Conclusion

Introducing microeconomics through the 2x2 games offers unity, simplicity, rigour, direct connections to traditional concerns, and a language that corresponds to the way economics was done in the late 20th century. The change proposed is as significant as the transition from classical to neoclassical economics, and it is a change that has already take place in much graduate training and in the research literature.

A game theoretic approach to microeconomics both emphasizes the use of rationality as a powerful and revealing assumption while making room for alternative choice paradigms and for the lessons we are learning from behavioural and experimental economics. Using the Periodic Table of the 2x2 games provide a systematic and easily learned structure, leaving students with a simple but inclusive mental map of to the space of social situations that interest economists. Equally important for an instructor, perhaps, the 2x2 games support a participatory and experimental approach to economics that students both enjoy and find illuminating.

The approach through the 2x2 games has another advantage: 2x2 games have been used in political science [2, 3, 7, 33, 32], psychology [28, 8], biology [17], physics [?], philosophy [12, 16] and law [15, 4] to elucidate core issues. In every case the analysis is fundamentally economic. The economic approach has proven itself across disciplines. An introduction to economics should, we think, demonstrate the wide applicability and the interdisciplinary use of the economic approach. The 2x2 games open doors our conventional textbooks ignore.

There remains another question that may be more important: “Will it be done?” - will any significant numbers of introductory instructors use an approach like the one we have outlines. teaching of economics? There have been earlier attempts to reform introductory micro, none of which have achieved much. Economists know that a superior technology is not always adopted, as Paul David argued with respect to the QUERTY keyboard [?]. There are many reasons whey another new approach will not be adopted: a very large existing investment in human capital, instructors who have found a local maximum and are too old or too pressed to learn new tricks, a set of customers who are unprepared to see the product repackaged, in part because it would devalue their existing investment, and publishers who have large investments in maintaining market share; standards adapted to the existing approach; network externalities; inertia; and even ignorance of the importance of the developments of the last half-century.

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# Appendix

## Introductory Microeconomics Through 2x2 Games: A Sketch

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### **Abstract**

Although many introductory textbooks now introduce a few simple games as a minor sub-topic, it is possible to invert convention, introducing economics by starting with a game theoretic approach and restricting oneself to illustrations based on 2x2 games. Based on 8 years of teaching experience, we show that the the 2x2 games and Robinson-Goforth topology for the 2x2 games provides an elegantly simple and modern framework for introducing students to social analysis and economics. The paper is likely to appeal most to graduate students whose approach to teaching economics is not yet set.



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## Introduction

This appendix presents the outline of an Introduction to economics course based on game theory and relying heavily on  $2 \times 2$  games for examples and illustrations. The appendix is essentially a proof by example that it is possible to construct an introductory course *based on the  $2 \times 2$  games*. Not attempt has been to provide a complete description: the focus is on presenting the logic of a presentation, with some detail to illustrate the logic. Reasons for beginning with the  $2 \times 2$  games are given in the article.

Most of the material has been “classroom tested,” but the course described here has not been taught. The outline does not provide teaching materials. For each ‘Chapter’ it presents a way to approach specific core materials, usually beginning with some feature of 2x2 game theory and extending the analysis slightly to provide new concepts and lessons. The consistent use of 2x2 games provides a simple unifying framework. Concepts build logically on what has gone before.

The virtues of the approach are unity, simplicity, rigour and modernness. Basing the presentation on the 2x2 games turns out not to be restrictive: there are far more applications and examples available than can be used in a single course. Teaching in this manner does require that instructors be familiar with game theoretic reasoning and that they have a knack for isolating concepts with simple formal examples.

Where familiar topics are introduced without the use of 2x2 games details are left to the reader.

It is important to lay out the goals of a course. This course emphasizes modern approaches, modelling, especially choice-theoretic modelling, and recent behavioural results. Each point is illustrated with a simple example. Pedagogically the course is built around activities and contentious statements. The course should therefore begin with a problem and a model - jumping into an exercise that requires some interpretation. The discussion of rationality in Chapter 1 does that.

# A Introductory Microeconomics Through 2x2 Games

## Chapter 1: Choice and rationality

Start by raising the question of *value*, both as a practical and a philosophical problem. Raise some of the historic issues briefly.

Economics concerns itself with two dimensions: action and values. We want to relate actions to values. The link is *choice*.

**Introduce the notion of choice** by asking student to imagine selecting one of two piles of coins. We start with an image of coins but move quickly to having student choose between numbers on the left or right or numbers that are up or down.

The important features of this exercise are that

Table 0.1: Simple choice

	number of coins
upper box	<b>2</b> coins
lower box	<b>10</b> coins

1. Students recognize that they are facing a problem in which value is well defined. Here is a case where value is not much of a problem. Observe that we are not saying money is the only value, just that we do accept it as a meaningful measure in some cases and that it has the property of being transferrable. Money serves as a store of value.
2. Students recognize that they easily make a *choice* on the basis of perceived *payoffs* or *values*.
3. Introducing a third alternative, students still choose and we have now introduced a more general notion of *optimization*. We have with the most minimal apparatus introduced maximization and optimization. We take time to provide other examples.
4. When asked, students easily recognize that on the basis of the payoffs they are comfortable offering advice to other- for example a blind person - about which to choose. We have introduced the possibility of providing a *prescription* or *policy advice* on the basis of our analysis of the payoffs. This provides the opportunity to refer to the history of economics - the origin of the term, and the common association with *management decisions*. This is the part of economic business programs tend to be interested in (and often critical of economics courses for doing poorly)
5. Students easily recognize that they are comfortable *predicting* the behaviour of others in similar cases. Prediction is what potential makes economics a *Social Science* rather than a form of mathematics or a set of management tools. We point out that students are happy predicting on the *assumption* that others will choose that is best for them - we have introduced the *rationality assumption*<sup>1</sup>.
6. Raising the possibility of choosing among unequal pieces of cake provide the opportunity to discuss that more may not be better, and the possibility that students might consider leaving the larger piece for others. Students easily recognize that it is necessary to understand what agents want to make predictions or prescriptions. We have introduced *preferences* and can introduce the notion of *utility*. We have

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<sup>1</sup>Note that, just as *rationality* is at the core of microeconomics, it is the subject of section 2 of TGEB

protected ourselves from the foolish accusation that the conventional approach to economics is wholly concerned with money or that it assumes that more is always better.

7. Students easily recognize that there is a *class of problems where knowing payoffs is sufficient for both prediction and prescription*, but that the class is limited to cases where we can assume rationality and where we have sufficient knowledge of preferences. We have the central domain of neoclassical economic defined at this point. We might well introduce the view of economics as *optimization subject to constraints*'.

We can extend this example very quickly to make it a social game. Tell students that they can pile the coins, but that another player will then choose the pile she wants and they get what is left. (This is a variant of the familiar cake-cutting game.)

Table 0.2: Giving another person a simple choice

You let her choose from number of coins	or, on the other hand	you let her choose from number of coins
upper box <b>2</b> coins		upper box <b>6</b> coins
lower box <b>10</b> coins		lower box <b>6</b> coins

The results of this exercise are

1. Students recognize that they can determine their own payoff up to one half of the total.
2. Students easily understand *strategic choice* in this situation
3. Students easily make *strategic choice* in this situation
4. Students recognize that the payoffs are predictable even with two players. they can predict how big the piles will be with an even number of coins
5. Students see that they can make predictions in a 2 person game if they specify the choices to be made and the payoffs to be received.
6. Students easily see that there is a kind of indeterminacy with an odd number of coins - it is not possible to predict which pile will be larger but it is possible to predict that the chooser will get one coin extra.

Point out that this is a  $2 \times 2$  *constant sum game* with sequential play. Show it can be represented as a tree.

Emphasize to students that they have learned a lot of economics and are thinking like economists as they analyze these problems.

## Chapter 2: Theory and Model

At this point we can point out that we have begun to build *models* and to use *theories* for predicting and prescribing. It is worthwhile here to discuss, with examples, the meaning of theory and model, as well as specification of a model.

It is very important to make students see that they use models and that the models direct their thinking. The best strategy is to emphasize models they already have - like the rational choices of the previous chapter. It is effective to work through some models that lead to comic results.

For this approach, the theory is one with individuals who choose among packages that have some kind of value to them and whose choices affect other individuals. The models we will illustrate many of the discoveries that interest economists will generally have only two individuals and each individual will have only two choices. Furthermore, values will be expressed as numbers, with big numbers being better and often only four values will be considered - 1, 2, 3, 4.

A very useful example is to discuss ghosts and the movie *Ghostbusters*. Ghosts are *theoretical constructs* that appear in many cultures. They are to some extent immaterial, they inhabit places, usually homes, and they are usually threatening in some way. The *theoretical object* ghost has a relationship to the object building and the object.

Every theory consists of theoretical objects and a set of relations among them. The movie *Ghostbusters* specifies specific forms for the theoretical objects and specific relations among those objects. It presents a specification of the theory - a particular model that represents the theory.

It is very important to make students see that they are creating a model every time they lay out a payoff structure and a set of rules.

Economic theory includes individuals that make choices among actions or collections of other objects. When we say the individuals choose rationally and further specify that to mean choose the largest bundle or the actions with the highest utility we are *specifying a model*. It is important to introduce other specifications of choice, including salience, norm-driven choice, random behaviour, automaticity, and perhaps others. Get students to formulate rules and apply them. Students should have the sense that they can select among models of individual choice. Without this sense they cannot understand model building.

### Chapter 3: Measurement and Value

The notion that individuals choose rationally requires that individuals can rank alternatives. They must have the capacity to *evaluate*. Value has always been a concern of economists, and it is useful to emphasize value as something economics deals with and provides useful insights about. The value question is important and the problem of value should be puffed up in students' minds. One dimension of the most famous diagram in the social sciences, the Supply and Demand figure, is value (represented as price when considering market behaviour).

This chapter focusses attention on the central issue of value and at the same time develops some skill in representing payoff and choices in a manner that makes it easy to introduce the 2x2 games explicitly.

Discuss value in terms of currency. Consider discussing utility as a measure and a *utility function* as an app that calculates a value for every situation. The utility function can be described as a hypothetical device.

Now we begin to complicate the valuation problem. The exercises that follow allow students to explore the concept of value and prepare them to approach many interesting questions. The exercises are time-consuming, but because they raise many important issues they are worth the effort. Notice that the exercises develop students' ability to represent social situations.

Consider a case where students get a combination of coins and checks:

Table 0.3: aggregating payoffs

	you get
upper box	\$2 coins plus a check for \$10
lower box	\$10 coins plus a check for \$2

Vary the problem:

Table 0.4: aggregating payoffs version 2

	contents
upper box	\$2 coins plus \$10 Canadian Tire Money
lower box	\$10 coins plus \$2 Canadian Tire Money

Vary the problem again, this time by introducing time to differentiate the payoffs. Vary the problem again,

This example allows us to introduce *discounting*. The next raises the possibility of *saving*

Table 0.5: time

upper box	<b>\$10</b> today
lower box	<b>\$10</b> tomorrow

Table 0.6: time

upper box	<b>\$5</b> today	<b>\$5</b> tomorrow
lower box	<b>\$10</b> today	<b>\$0</b> tomorrow

The next example allows us to introduce *marginal utility*.

	what you get	
upper box	<b>\$5</b> today	<b>\$5</b> tomorrow
lower box	<b>\$0</b> today	<b>\$10</b> tomorrow

These examples raise the possibility of *intertemporal transfers* and *borrowing*

Table 0.7: a distributed payoff

	for you	for your friend
upper box	<b>\$5</b>	<b>\$5</b>
lower box	<b>\$0</b>	<b>\$10</b>

Table 0.8: a distributed payoff

	for you	for your friend
upper box	<b>\$5</b>	<b>\$5</b>
lower box	<b>\$4</b>	<b>\$7</b>

These raises the issue of *fairness* and the possibility of *altruism*.

	number of coins	
upper box	<b>\$2</b> coins	\$10 Canadian Tire Money
lower box	<b>\$10</b> coins	\$2 Canadian Tire Money

**Opportunity Cost**

Opportunity cost can be introduced here. What is the opportunity cost of choosing the upper box? The lower box? The best alternative is the other box. Op cost is one notion of cost that is more general than money price . Discuss some cases cases.

	number of coins
upper box	<b>2</b> coins
lower box	<b>10</b> coins

**Comparative advantage**

Comparative advantage is actually a cost calculation: even when your absolute prices are higher than a trading partner’s for both goods, one of your relative prices is generally lower. Relative prices determine specialization. The *opportunity cost* of producing the good with the lowest relative cost is lower.

An example: ask students to assume that they and and friend need one red and one blue box for a comping trip, and each has to bring one box. (This can be seen as a coordination problem). Abstract by assuming the value is represented by coins in the boxes as in table 0.9. The pair of students will share the total number of coins in the boxes they choose. The total number of coins is a measure of social welfare produced by their choice.

In the example they can get either 13 or 14 coins. The answer is pretty obvious, but can we use the relative costs to explain the choice? Clearly you give up 2 coins to get 10 by choosing the blue box. your relative price is 2 for 10. Your friend gives up 4 to get 11, which is a higher price.

Table 0.9: A specialization game

	number of coins for Student #1		number of coins for Student #2
red box	<b>2</b> coins	red box	<b>4</b> coins
blue box	<b>10</b> coins	blue box	<b>11</b> coins

Notice that this looks something like a 2x2 game! (The payoffs would be

Table 0.10: Payoffs for the specialization game

	Red	Blue
Red	0, 0	6.5, 6.5
Blue	7, 7	0, 0

Which just happens to be a coordination game.)



In Ricardo's classic example, positive values are bad because they are costs. The price of wine is higher in terms of wool in England than in Portugal, so we would prefer to buy wine in Portugal.

Table 0.11: standard comparative advantage: labour cost of a standard unit

	England		Portugal
wine	<b>4</b> hours	wine	<b>2</b> hours
wool	<b>8</b> hours	wool	<b>6</b> hours

The examples is too familiar to discuss further. The interesting step is the move from expressing cost in terms of input to expressing one outputs cost in terms on another output (op cost).

It might be worth converting the problem form input costs as in Table 0.11 to output possibilities by introducing a resource constraint. Assume each country has 24 hours of labour (the lowest common denominator) to allocate.

Table 0.12: comparative advantage with a resource constraint

	England		Portugal
wine	<b>6</b> barrels	wine	<b>12</b> barrels
wool	<b>3</b> bales	wool	<b>4</b> bales

This exercise reinforces the students understanding that economists work with values and that converting values from one framework to another is useful. It prepares the ground for introducing PPFs.

Show students that by introducing preferences, say such that each country wants one and one half bales of wool and unlimited amounts of wine ( $U = \max(Wine) \text{ s.t. } wool \geq 1.5$ ) we get the same coordination problem as in the first example.

## Chapter 4: Introducing Time

Discounting is an important idea and it is worth working a number of examples. Evaluating a discounted stream involves the same mathematics as summing the series of expenditures in a macro model so it may be worth working a number of examples.

Students in general are not aware of these techniques and find them useful and interesting. The exercises help them focus on the problem of computing utilities. It is also easy to discuss diminishing marginal utility of income as an application of weighted payoffs, especially in the context of insurance problems.

Since human capital is a much more important part of modern economic analysis than it was in the last century, and since students are investing in human capital this offers an opportunity to discuss investment, human capital, and even social capital with highly relevant examples

Table 0.13: time

	number of coins
upper box	<b>\$10</b> today
lower box	<b>\$10</b> tomorrow

In this case students will have views about whether the alternatives are of equal value. Introduce discounting. There is a great deal of empirical research about discounting that may be worth introducing. This question provides an opportunity to conduct some experiments. and to make reference to what the research shows people actually do. It is a good place to introduce the neat results on children who defer gratification and perhaps talk about the evolutionary basis of discounting.

## Chapter 5: Contingent Choice

The notions of contingent choice, plan and strategy are worth spending time on. Ask student to *make a plan*. Tell them their payoff depends on some event ( a vote, the weather). Explain that they can find out if the event has occurred before they decide what to do. The you-cut-I-choose games is a good example.

It usually works to ask for valuations in money terms as in the following example.

Table 0.14: An example of contingent choice

	good weather	violent storms
go to park	<b>\$10</b>	\$0
go shopping	<b>\$5</b>	\$5

This exercise is valuable because it identifies a common choice situation. In this case we could describe this as a game where nature gets to choose the weather (a *game against nature*).

If you think the world is against you you think that nature will choose the outcome that will give you the lowest payoff. (Von-Neuman and Morgenstern thought this was a reasonable view.) Competitive markets have this property in the sense that players are price-takers. A *demand curve* is a plan to purchase certain amounts at certain prices. A

demand curve is a *contingency plan*. In game-theoretic terms an individual demand function is it is a *strategy*, as is a firm's *supply function*. It is interesting that for a competitive market we can aggregate individual supplies and demand and treat the *aggregates* as two players!

The exercise also allows us to introduce a degenerate version of the *extensive form representation*, the *decision tree*. The decision tree does not include other strategic players. Instead it places probabilities on each outcome.

Introduce some examples of extensive form representations. Do the example above with nature choosing first and with the student choosing first. Be sure to have students work out how to get from a 2x2 matrix to alternative tree-form representations. Emphasize that the order of decisions often matters

### **Demand and supply**

A demand curve can be introduced as an example of contingent choice: "If the price is  $p$  I will buy  $Q(p)$ ". Here the strategy is *continuous* not *discrete*. The buyer has a strategy. It is possible to find out what the buyer's strategy by observations.

Note that the sellers might have a price-contingent strategy too - that would be a *supply curve*.

### **Monopoly**

If you know what the buyer's strategy is, how can you exploit that buyer?: this is the *monopoly problem*. It can be dealt with here. Students enjoy taking the point of view of the monopolist and exploiting knowledge of the other player's strategy. It helps to anchor the notion of strategic choice as well as the importance of information. Ask students why the consumers can't play strategically.

Monopoly is essentially a two-player game. The demand curve represents the known strategy of the representative buyer. The monopolist takes advantage of the information in a demand curve. As a result it is easy to devise a 2x2 example in which the monopolist chooses price and the buyers choose quantity. It is also possible to have students create a revenue table based on a linear demand curve. The students are simply calculating payoffs as in the other problems they are given at this stage. Once they have the table they can easily pick the highest revenue and use the demand curve to recover the price.

Students can also use a demand function and, say a linear *marginal variable cost function* to do profit rather than revenue maximization. The concept of *fixed costs* can be discussed. Fixed costs are ignored as *sunk costs*.

Monopoly provides an ideal introduction to a conventional optimization rule and the chance to introduce a little calculus as an example of how economists use the optimiz-

ing model. Either graphic or mathematical treatment allows an instructor to introduce  $MR=MC$  and then to discuss variants  $MB=MC$ . These in turn can be presented as a general principle and applied to other problems.  $MSB = MSC$  can be played up as the great rule from Microeconomics that everyone should know but only economists do know.

## Chapter 6: Evaluating Gambles

Von Neuman and Morgenstern introduce uncertainty as a measurement problem at the beginning of TGE. Uncertainty is such a fundamental feature of the real world and of modern economic theory that it should be treated as early as possible in even a first course in economics. Uncertainty motivates a great deal of economic behaviour. Insurance expenditures are the largest part of the public budget (health care), for example. Gambling is a major industry. We have evolved with automatic responses to risk. Valuation of uncertainty is vital in an introduction to economic theory. Uncertainty is not simply a sub-area of economic reasoning: it presents a fundamental problem in valuation and a fundamental feature of the world.

Ask students to imagine selecting one of two *gambles*. The gambles are represented in a 2x2 matrix and students choose either the top row or the bottom row. The rows have payoffs of, for example, 1 and 5 and 5 and 1. They will get one of the payoffs in the row they choose. Which one depends on the state of the world, indicated by L or R

Table 0.15: Choosing gambles

	L	R
U	1	5
D	5	1

Student will ask which payoff they get. Reply that you don't know and someone will generally suggest flipping a coin. This is the opportunity to introduce *expected values*, and perhaps to discuss the history of the technique. Applications to *gambling games* and *insurance*, in general can be introduced here and various expected value problems solved using changing *subjective probabilities* and payoffs. The concept of a *fair gamble* should be illustrated. Note that this is part of a general problem of knowing what the payoff is.

Introduce specific probabilities and calculate.

We now have the notion of *probability-weighted payoffs* and time-weighted payoffs as in discounting.

Calculate some problems with both probabilities and discounting - saving and retirement problems with risk of mortality. Illustrate actuarial tables and so on.

There is an opportunity to explain *Decision Theory*, widely used in management and, for example, evaluation of medical procedures. In decision theory the probabilities are attached to each branch of a decision tree, then decisions are made on the basis of expected benefits and costs. This is an interesting and useful model. Exercises are very easy to construct. It is about identifying value.

Introduce the notion of a *game against Nature* and, if of a literary bent, of the general problem of fate, luck, and Chance. If we think of this as a game against Nature, Nature is a strange player who does not care about the outcome for us or for herself. She therefore chooses randomly. At most we can know the probabilities with which she chooses.

These evaluation techniques are useful for students, very easy to teach, they allow us to introduce many current issues, realistic problems, and they are foundational in developing a theoretical understanding<sup>2</sup>. We have by now introduced a number of core concepts and in each case tied the concept to a simple example.

Emphasize that so far we have focussed on individual choice. This is a good place to discuss the Robinson Crusoe models, and even to talk about the novel. We now have individuals that make choices in response to their estimations of value. We can model some choice situations simply by describing payoffs, and with such models we can predict outcomes. This is a remarkable achievement.

## Chapter 7: Other People: Economics as Social Science

In contrast to the previous discussion, many choices involve other people. A great deal of economics deals with social situations. Economists still want to predict and prescribe, and they do not want to abandon the techniques for thinking about individual behaviour.

The simplest genuinely social situation is the 2x2 game. It has the smallest possible number of players and the smallest possible number of choices for each player.

If there are two players and each has two choices, there are 4 possible outcomes (or states of the world). Since each player has to have a payoff in each situation a 2x2 game generally involves keeping track of 8 payoffs. It is convenient to use a matrix with two payoffs in each cell. In all the examples from now on the first payoff will be for the person who selects the row and the second for the person who selects the column.

The Game against Nature had the property that Nature did not care because Nature did not receive a payoff and did not care about the outcome. Explain that in this case the second player cares about the outcome. Her payoffs are listed second in each outcome cell. Explain that this is the Strategic Form, discuss and practice the conventions.

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<sup>2</sup>If there is any doubt that the techniques introduced here are sufficiently theoretical to justify spending a lot of time, review the first chapter of TGEB

<b>1</b>	 Hegemonic Stability 50 55 212	 Samson and Delilah 56 56 213	 Blackmailer 1 49 44 214	 Ideological Hegemony 1 6 18 215	 Blackmailer 2 5 17 216	 Ideological Hegemony 2 35 21 211
<b>6</b>	 Vietnam Bombing 37 50 262	 Cuban Missile Crisis 1 13 41 264	 Blackmailer 1 4 16 265	 Blackmailer 2 2 14 266	 Ideological Hegemony 2 33 19 261	
<b>5</b>	 Cycle 36 49 252	 Pursuit of the Israelites 38 51 253	 Cycle 12 40 254	 Cycle 3 15 255	 Cycle 1 13 256	 Alibi 34 20 251
<b>4</b>	 Cycle 46 70 242	 Cycle 43 74 243	 Pursuit of the Israelites 42 73 244	 Cycle 40 53 245	 Alibi 41 54 246	 Alibi 48 57 241
<b>3</b>	 Cycle 31 78 232	 Cycle 45 76 233	 Total Conflict 44 75 234	 Cycle 23 42 235	 Alibi 24 43 236	 Alibi 27 47 231
<b>2</b>	 Cycle 29 72 222	 Inspector-Evader 47 71 223	 Cuban Missile Crisis 1 30 77 224	 Big Bully 25 45 225	 Hamlet and Claudius 26 46 226	 Alibi 28 48 221

Layer 2  
Layer 1

<b>1</b>	 Called Bluff 22 39 112	 Cuban Missile Crisis 1 20 37 113	 Cuban Missile Crisis 2 21 38 114	 Total Conflict 10 10 115	 Prisoner's Dilemma 11 11 116	 Prisoner's Dilemma 32 12 111
<b>6</b>	 Bully 18 35 162	 Protector 3 14 31 163	 Protector 4 16 33 164	 Anti-Prisoner's Dilemma 8 8 165	 Total Conflict 9 9 166	 Total Conflict 11 11 161
<b>5</b>	 Protector 1 19 36 152	 Protector 2 15 32 153	 Protector 3 17 34 154	 Anti-Chicken 7 7 155	 Protector 4 8 8 156	 Protector 5 10 10 151
<b>4</b>	 Skewed BoS 52 65 142	 BoS 51 64 143	 Hero 55 69 144	 Protector 2 17 34 145	 Protector 4 16 33 146	 Cuban Missile Crisis 2 21 38 141
<b>3</b>	 Asym. BoS 53 67 132	 BoS 54 68 133	 BoS 51 64 134	 Protector 1 15 32 135	 Protector 3 14 31 136	 Protector 5 20 37 131
<b>2</b>	 Chicken 57 66 122	 Asym. BoS 53 67 123	 Skewed BoS 52 65 124	 Protector 4 19 36 125	 Bully 18 35 126	 Called Bluff 22 39 121

**KEY**

	53 67 132	Brams Number Rapoport and Guyer number Layer, Row, Column Index Common name	<b>Solution</b>		Nash
					maxi-min
					both

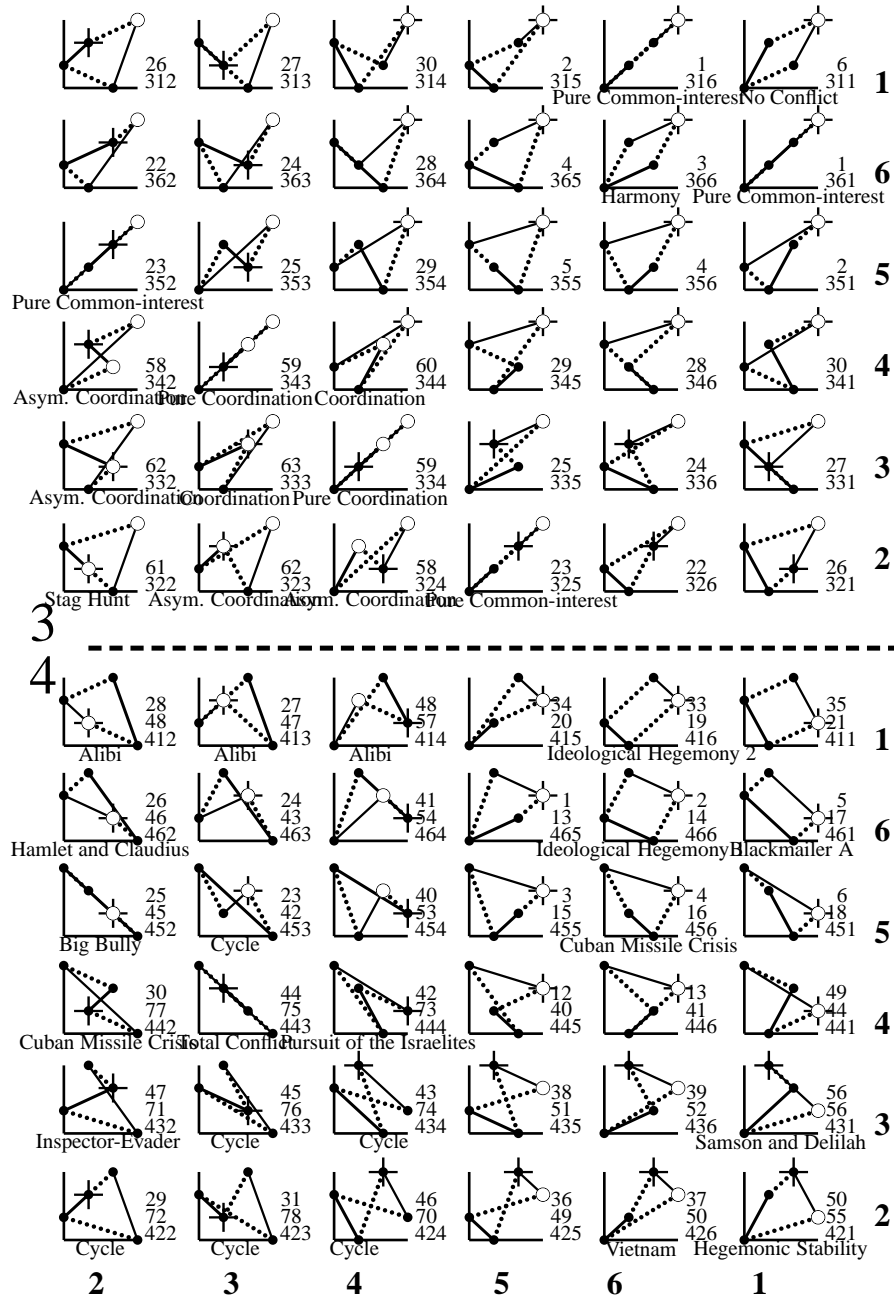


Figure 0.1: The complete Periodic Table of the  $2 \times 2$  games

	your sister	
	picks largest piece	picks smallest piece
cut cake equally	.05, 0.5	0.5, 0.5
cut one piece larger	0.33, 0.66	0.66, 0.33

Table 0.16: the first complete game in matrix form

### Chapter 8: Representation in Payoff Space

It is convenient to examine 2x2 games in payoff space rather than in strategy space<sup>3</sup>

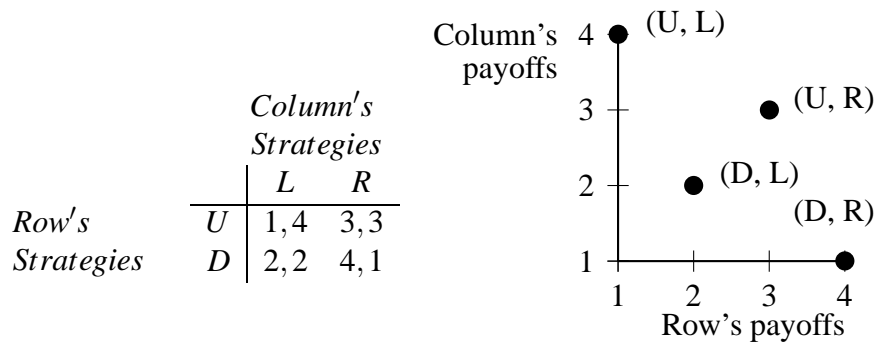


Figure 0.2: Payoff combinations for the Prisoner's Dilemma

We find the payoff representation easier to read and easier to teach from. In addition it relates to representations in utility space and makes it easy to transition to bargaining games and to Production possibility frontiers. Our view is that the payoff space representation is especially convenient for teaching because it employs and reinforces the familiar graphical convention that left and up are 'better.' It also links to the concept of preferences graphically, provides an easy way to see the attraction of an equilibrium, to discuss efficiency and to visualize conflict. an additional pedagogical advantage is that the games presented in payoff space are easily remembered as 'icons.'

The matrix representation is clearly the dominant convention in representing 2x2 games. It is not the most natural for students nor is it the form that presents information most di-

<sup>3</sup>Some instructors find the payoff representation less convenient than numerical versions . Bryan Bruns has created a version of the Periodic table that does not use the order graph representation [?].



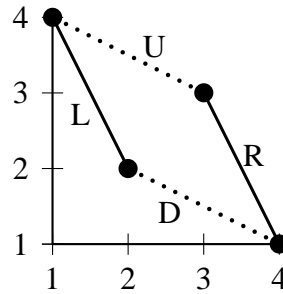


Figure 0.3: Inducement correspondences for the Prisoner's Dilemma

rectly or is most easily remembers. The main argument for using the matrix representation is it is most accessible to economist brought up with that representation.

A major advantage of the payoff space representation is that it is a presentation in vector space and will ultimately make economic analysis in vector spaces easier for students.

Practice translating games in strategic form one representation to another.

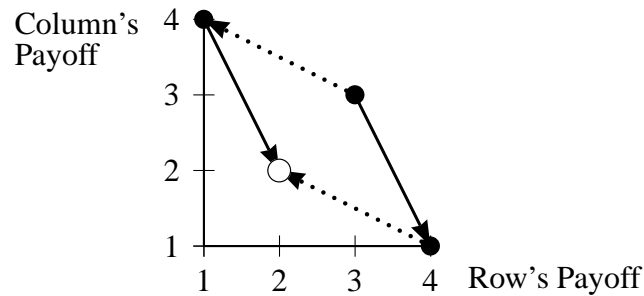


Figure 0.4: Best responses and Nash equilibrium for PD

### The inducement correspondence

The figure partially represents a 2x2 game in *payoff space*. This is the same space as *utility possibility frontier* and *production possibilities frontier*. This step allows both possibility frontiers to be introduced.

Without the labelling the disconnected points in Figure 0.2 do not completely capture the strategic form. The points *UL* and *DL*, for example, are linked in the sense that, by choosing *L*, the column player limits the outcomes available for the row player to *UL* or *DL*. Equivalently, the payoffs pairs associated with *UL* and *DL* appear as a column

of the payoff matrix. The figure now fully represents a 2x2 game in *payoff space* - a representation dual to the *strategy space* representation of the matrix.

### Equilibrium and ‘solving’ games

Equilibrium is a major topic. Elicit or introduce several *solution concepts*. I Introduce the *maximin solution*. Introduce the concept of a *best response* - (a contingent best response). Introduce *Best response payoff* and practice identifying the BRP. Introduce the convention of circling BRPs. Identify a *Nash Equilibrium* as a best response for all players. Nash equilibrium is a solution concept. Solve a lot of examples.

Introduce *Pareto dominance* as a notion of efficiency. Introduce *dominant strategy*.

Examples should include games with no NE and with two NEs and games with Pareto efficient and inefficient outcomes. Introduce and name a number of games and the notion of a *social dilemma*.

Extend the exercise to 3x3 games. The notion of *generalizing a model* should be explained.

The games in the upper right quadrant of the periodic table, for example, are games that offer the possibility of the highest payoff for both players. Rapoport and Guyer called these ‘no-conflict games.’ These are the games in which individual self interested behaviour will usually result in socially attractive solutions. The exceptions in this quadrant are the nine coordination games, for which there are two Nash equilibria. Coordination games present the possibility of an inefficient equilibrium which is stable under the Nash condition that no player gains from a unilateral change of strategy.

## Chapter 9: Classes of social situations

In this Chapter the *periodic table of 2x2 games* is used to talk about the number of possible games, types of games (i.e., types of social situations described by the payoff structures available in the 2x2 games), and some relationships among them. The goal is to have students recognize many familiar situations. A slightly more ambitious goal is to have students learn to construct examples of all the major classes of games. Tell lots of stories.

It is useful to describe the history of game theory as beginning with a focus on games with no Nash Equilibria - the constant sum cyclic games, then moving to recognizing the PD and then on to the coordination games. Discuss gambling games (games of chance) and distinguish economically productive games.

Identify games where we like the results of independent self-interested behaviour ( nice NEs). Identify cases that deserve more thought. Major classes to deal with games with no equilibria, with two equilibria and with inefficient equilibria. Each raises important issues.

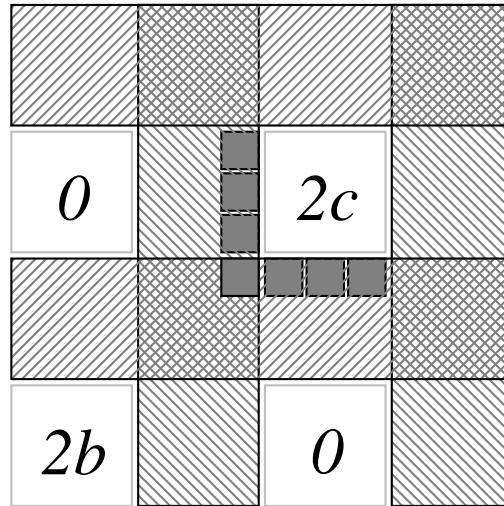


Figure 0.5: The Major Regions: Dominant strategies, number of equilibria, PD family (dark grey)

The games with no equilibria are of interest because they correspond to gambling and theft and were the first really studied. They demonstrate a problems with the Nash equilibrium concept for some analysts. Alternatively, and more interestingly, they represent real situations that may not have stable points. In fact in games of chance it may be important that they do not. The number of pizza parlours may not be stable, in the real world, for example.

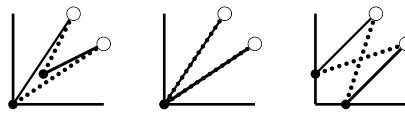


Figure 0.6: Variants of the symmetric and quasi-symmetric Battles of the Sexes

Among the games with two equilibria there are two families: Coordination games and Battle of the Sexes games. Each represents a represent a major class of social problems.

Battles of the Sexes games, as in Figure 0.6, offer alternative equilibria which are equally efficient but distributionally distinct. This raises interesting questions. Gender questions can be raised.

One possibility is that players may make side payments (share) and another is that they may employ correlated strategies. In both cases we are moving to cooperative solutions - which may require enforcibility. The effect of an agreement is to make available points on a line between the ‘horns’ of theBoS. This opens the way to a discussion of bargaining,

The coordination games are another class of real social situations represented by simple payoff structures.

There are also pure common interest games in which every move that is good for the chooser is good for the other player. Even in this set there are interesting games. The coordination and Battle of the Sexes games famous examples with good stories to tell and they present social dilemmas

Note that there are regions with similar games. There are, for example, two regions with no NE, and two regions with two equilibria. There are regions with inefficient equilibria and regions with dominant strategy equilibria.

### Chapter 10: Nasty equilibria: The Prisoner’s Dilemma

This is the most famous game. Elster called it the fundamental problem in political science and the e-coli of social sciences. The PD is not simply curiosity or an entertaining story: it is the classic demonstration that rational, self-interested, individual choice can lead to nasty outcomes. As such it is the fundamental motivation for mechanism design and for many social institutions.

Define the PD as a game with a nasty (Pareto inefficient) dominant strategy equilibrium. Have students look for other games with the same property. There are 6 dominance-

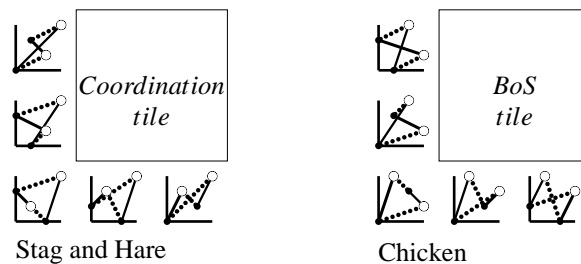


Figure 0.7: Chicken and asymmetric BoS; Stag and Hare and asymmetric Coordination games

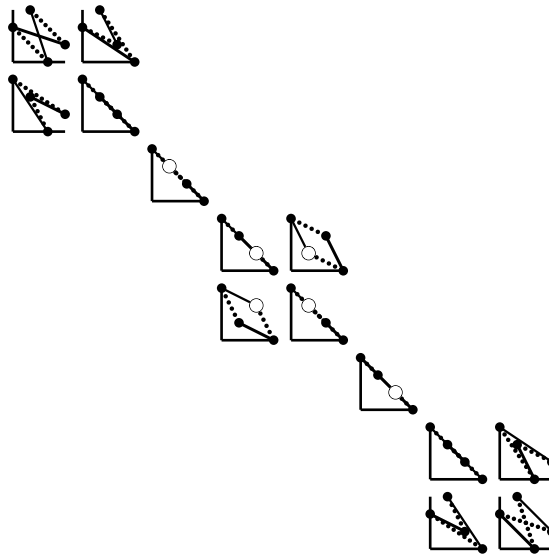


Figure 0.8: The pure conflict games

solvable games with nasty equilibria. We (Robinson and Goforth) call them the alibi games and they make us a set we call the prisoner's Dilemma family.

Tell the history of the game. Emphasize the challenge to first theorem of welfare economics. Tell the story of the game. The best version has the prosecutor designing and incentive structure - teach students to construct PD with various conditions of the payoff values.

Show how the game involves an *externality*. Discuss pollution problems and the tragedy of the commons. The entire class of tragedy of the commons games can be treated as 2-person games by introducing a *representative agent*. Cod wars and *free-rider problems* should be discussed as very important applications. They raise the the important issue of *social capital, trust* and *morality* in economic efficiency, and provide a rationale for *regulation* in some cases.

Public goods can be introduced as a social dilemma with, in some views, the structure of a *multi-person PD*. A solution that will emerge form a class is the possibility of *taxation*, which allows for a lively discussion of social rights and responsibilities.

*Generalize* the PD by introducing a third player (2 2x2 matrices with three payoff in each cell and the third player chooses which game to play) and by introducing a third choice for each player. This demonstrates that the model can be generalized.

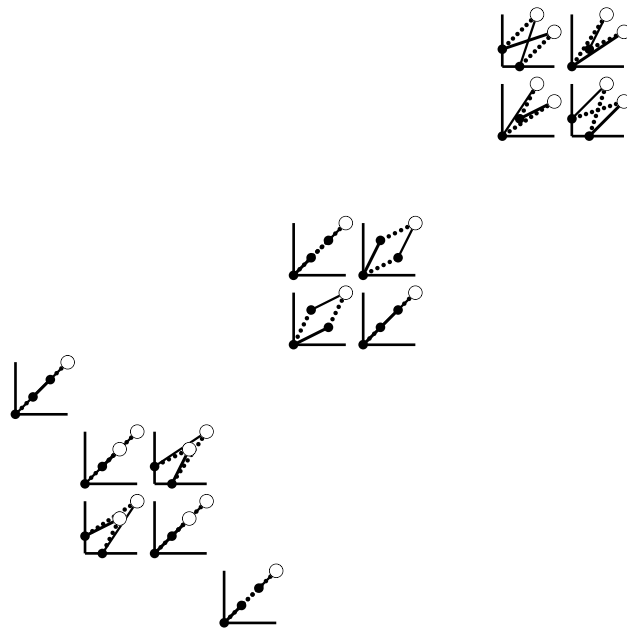


Figure 0.9: The pure common-interest games

Generalize by introducing a third player.  
 Discuss some behavioural results of interest.

### Evolutionary games

Axelrod's tournament provides an excellent way to introduce simulation games and to discuss many social issues. It can easily be simulated on a small scale in class. It opens the door to evolutionary games, which in turn provide a wide range of economic lessons. It also provides an opportunity to demonstrate simulation methods.

This is potentially a large topic and provides some very valuable insights into social organization.

## Chapter 11: Two person cooperative games: Bargaining

Bargaining is non-market economic behaviour that is usefully described using the *Nash bargaining model*. It is of great interest to students but is rarely considered as an elementary topic. It is also a very convenient transition to cooperative games, understood as games with *enforceable agreements*.

Bargaining is easily introduced using the *order graph* representation of a 2x2 game. We begin by selecting, say the Prisoner's Dilemma or the *Battle of the Sexes*, then showing any *convex combination* of points is accessible using *correlated strategies*. Talk about real world correlated strategies.

It is also possible to introduce the notion of a *side-payment* here as a way of getting to a point on the convex hull of a payoff set. It is easy to introduce the notion of *transactions costs* and to discuss the efficiency implications if it is necessary to hire a lawyer to achieve an agreement.

We then identify the *feasible set*, and the *efficient set*.

What happens if there is no cooperation? Players probably end up at the NE. Define this as the *Disagreement point* or *Conflict payoffs*. Identify the *rational set*. (Payoffs at least as high as the disagreement payoff for all players).

Identify the intersection of the feasible, efficient and rational sets as the *bargaining set*. Ask for solutions in this set. This is a good place to run an experiment with pairs of students.

There are three good candidates that appeal to students: the *Nash Bargaining Solution*, the *Kalai-Smorodinski Bargaining Solution* and the *egalitarian solution*.

The K-S solution is easy to show. It is particularly easy to explain using a Battle of the Sexes Game, since in those game players might reasonably use their highest payoff as a reference point. The Nash solution is useful because it provides a reason to develop a *rectangular hyperbola* to find the solution graphically. A rectangular hyperbola is useful later in sketching *average fixed costs* and hence average cost curves, and it can be used as a *unit-elastic demand curve*.

There is lots of research to cite and describe about what happens in laboratory games.

Extract some practical lessons about bargaining.

## Chapter 12: Auctions and Matching Games

Auctions can be treated as 2-person games by introducing a *representative agent*. The key problem is the division of a surplus arising from trade, as in the bargaining game and the monopoly game. The presence of many other bidders changes the game and constrains the outcome. This is an argument that can be applied in dealing with supply and demand later or saved until after supply and demand have been discussed.

### Matching Games

Matching games provide another non-market approach to economic allocation that is instructive and entertaining. Students encounter matching games in real life and we have

useful tools to impart. Three classes of matching games can be explored in class. Individuals with different preferences are matched with otherwise identical goods. A useful source is *Cooperative Microeconomics* by Henri Moulin [4]. Bohem Bawerk's horse market, described, in Moulin page 49 [4] is easily simulated in a class.

In the second type, individuals with different preferences are matched with goods that are not identical. Moulin, page 104 [4] describes a market for houses. Using a mixed box of donuts in class lots of fun in a class or 12-24. The Top-cycle algorithm is easy to teach and a good example of algorithmic solutions.

Matching players with preferences with other player with preferences is another matching game students encounter. The marriage game Moulin, page 111 [4] is an excellent class activity. Student love to set up a high-school class of, say, 10 males and ten females and to give each a character and a set of preferences. If one does this exercise it provides an opportunity to discuss the Gale-Shapley theorem [4]. The theorem is a remarkable result from economics that should be known by all Sociology and Women's Studies students and isn't.

The core can be described as the case where any person can go out in public and will not meet a person they would prefer to be with who would also prefer to be with him (or her). There are no blocking coalitions.

### Chapter 13: Supply and Demand

The 2x2 games provide an easy transition to 2 person Nash bargaining games. Nash bargaining games provide a natural way to treat the division of the surplus from exchange with fixed quantities. The surplus here, and the gains from trade more generally, are the fundamental drivers of human society.

Contingency plans, or strategies, have been introduced earlier. It is easy to get students to describe a contingency plan for a buyer when the contingency is price - i.e., to say what they think would happen at different prices. This introduces *continuous strategy sets* and the *inverse demand function*,  $P^d = f^{-1}(Q)$ . The Cournot model also assumes that consumers are described by an inverse demand function [?] [4].

The gains from trade for a producer determined in the bargaining process also provide an incentive for increasing output. The process can be repeated until an equilibrium is achieved. The approach uses the so-called *short-side rule*, which is really a property of voluntary trades.

This stage of the game is a Battle of the Sexes (See Figure 0.6) if all the surplus goes to one player or the other. If they can cooperate, there are intermediate solutions.

The presence realized surplus for the seller provides an occasion for discussing adjustment to equilibrium and the notion of equilibrium in markets. The approach has clear



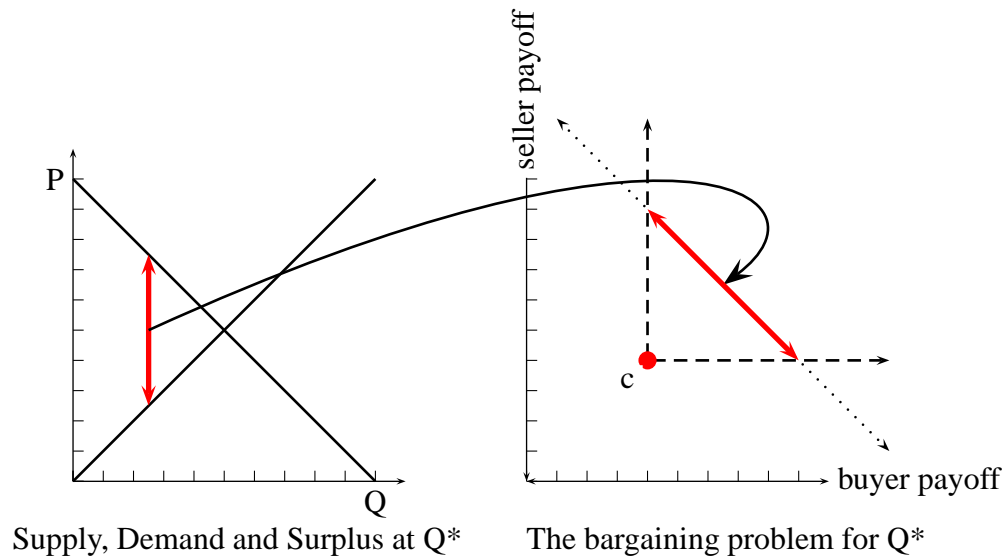


Figure 0.10: From Demand to Bargains

roots in the history of economic theory - going back at least to Edgeworth's *Mathematical Psychics* [1].

With the division of the surplus it is clear that the gains from trade for a producer also provide an incentive for increasing output. The incentive persists until an equilibrium is achieved. This provides an occasion for discussing adjustment to equilibrium and the notion of *equilibrium* in markets and *allocative efficiency*. The approach has clear roots in the history of economic theory, going back at least to Edgeworth's *Mathematical Psychics* [1].

The bargaining game results in *exchange efficiency*. The adjustment to equilibrium tends to economy wide *allocative efficiency*. The two-stage treatment makes it easy to make these distinctions.

The discussion can be supported by an exploration of Bohem Bawerk's horse market, described, for example, in Moulin

An advantage of approaching supply and demand through bargaining is that it starts in the heart of economics - the gains from trade. Furthermore, the gap between what a buyer is willing to pay and what a seller will sell for is a measure of the Marshallian surplus at a given quantity. Integrating yields *Consumer plus Producer Surplus*, a quantity that economists recognize and virtually no-one else understands. Students enjoy talking about

invisible (Harberger [2]) triangles that only economics students can see. Even business students find interest in the fact that there is a value measure that the accountants always ignore. Consumer surplus provides an opportunity to enthuse about the way theory makes visible what would otherwise not be seen. A brave instructor might sing the Beatles' song "The Fool on the Hill," in which the fool "sees the sun going down, but the eyes in his head see the world turning 'round," as an example of theory guiding perception.

The notion of an invisible surplus is the heart of any efficiency measure, and therefore of the economist's capacity to evaluate markets. In this approach, therefore, supply and demand theory begins with the most fundamental quantity in microeconomic theory.

### **Taxation and Transaction Costs**

Consider an accountant who steals 20% of the revenue of a firm. This third party intervention causes the producer to stop producing at a quantity that is too low. Show the *deadweight loss*. *Transaction costs* can be introduced here.

Repeat the exercise with a tax. Again a third party has been introduced. The resulting DWL is called *excess burden*.

These are important results that are only available if we can see "invisible triangles."

### **Consumer surplus and Externalities**

Note that the inverse demand curve  $P^d = f^{-1}(Q)$  measures value - taking us back to the discussion in earlier chapters about measuring value. Emphasize that the demand curve respects the preferences of buyer, given their income .

Now ask what happens if there are other effects of the transaction that do not fall on buyer or seller. They will not appear in the price system. These externalities can be illustrated using any game, like the PD, in which the actions on one player affect the payoffs of another.

It is natural to "correct" the demand curve if there is a consumption externality. It is easy to do the comparative statics. Students will be able to supply many examples.

### **Public Goods and Vertical Summation**

Public goods can be approached in the same way as externalities - by discussing whether an individual's demand is a measure of social value generated.

### **Monopoly and Surplus**

The monopoly problem can be re-examined and the solution evaluated using the concept of consumer surplus. The exercise focusses student attention on the possibility of *market*

*failure* and corrective public policies. The monopoly outcome should be described as a game with an inferior outcome.

### Overview of demand

An advantage of approaching supply and demand through bargaining is that it starts in the heart of economics - the gains from trade. Furthermore, the gap between what a buyer is willing to pay and what a seller will sell for at a given quantity is a measure of the Marshallian surplus. Integrating yields Consumer plus Producer Surplus, a quantity that economists recognize and virtually no-one else understands. Introducing surplus focuses on the capacity of economic analysis to make invisible quantities visible. Surplus is the heart of any efficiency measure, and therefore of the economist's capacity to evaluate markets. In this approach, therefore, supply and demand theory begins with the most fundamental quantity in microeconomic theory. Surplus is what makes society desirable.

## Chapter 14: Mechanism Design and Constitutional Games

Mechanism design is a modern topic of overwhelming mathematical difficulty for first-year students. The 2007 Nobel Memorial Prize in Economic Sciences was awarded to Leonid Hurwicz, Eric Maskin, and Roger Myerson "for having laid the foundations of mechanism design theory".

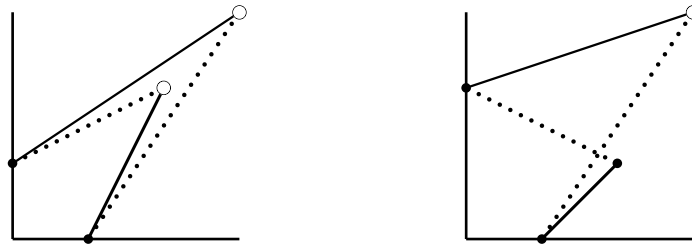


Figure 0.11: Redesigning a Coordination Game

The concept is simple, however. A game "designer" chooses the game structure with in order to influence the outcome of the game that others will play. Students find this an interesting and entirely natural topic to study. And no wonder, when all the talk about the virtues of capitalism or competitive markets are discussions of mechanism design and

when the notion of selecting a constitution or a set of legal rules is fundamentally about mechanism design.

For instructors so inclined, this topic provides an opportunity to introduce issues of the ideal social organization and, for example, Rawl's *A Theory of Justice*[5]. The focus in the literature is usually on motivating agents to disclose their private information as in public goods games. Such games are of course social dilemmas that can be modelled as 2x2 games. Mechanism design can then be illustrated as modifying the payoff structure to produce a game with a satisfactory outcome.

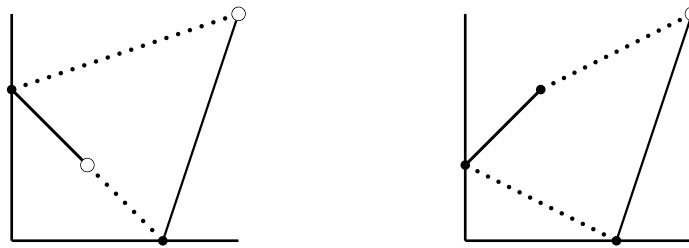


Figure 0.12: Engineering the Stag Hunt

A very simple approach is to begin with a social dilemma and swap the low payoffs for one or both players. These minimal changes transform a game into an adjacent game. This also keeps the move on a layer in the periodic table.

Consider for example the Coordination game (344 in our notation). Changing the 2 for 3 for the column player yields game 345, which has a single equilibrium which is Pareto efficient<sup>4</sup>. The resulting game is asymmetric and dominance solvable. The possibility of an inefficient coordination equilibrium has been eliminated.

Similarly, swapping only the lowest payoffs for Row converts the Stag Hunt, a relative of the coordination games, into a game with a single, efficient solution.

The Prisoner's Dilemma can be converted into the anti-dilemma (166). There are also examples of problem games that are not easy to fix by swaps to adjacent games. Of the eighteen cyclic games, six can be turned into games in which both players get maximal payoffs with a single change, 10 can be changed to yield a best outcome for one player and a second best for the other. Only one requires more than one swap to make it dominance solvable and yield a Pareto efficient solution, don't have as many good neighbours - e.g., 242, 232, 222 are adjacent to asymmetric PDs via C12 swaps.

<sup>4</sup>In Figure 0.11 the leftmost point, with a value of 2 for Column, moves up to three and the lower equilibrium moves down to give 2 to the column layer.



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